Exploring AP Calculus With Colorful Calculator Investigations

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Explore Limits, Derivatives, and Integration through hands-on activities that involve color-enhanced graphing calculators and data collection devices.

Introduction to Limits

What is a limit? _____

<u>Formal Definition</u>: If $f(x)$ becomes arbitrarily close to a unique number L as x approaches c from either side, then the **limit** of $f(x)$ as x approaches c is L. This is written as $\lim\limits_{x \to c} f(x) = L$.

Our goals today are to: 1) Understand the limit concept.
2) Use the definition of a limit to estimate limits.
3) Determine whether limits of functions exist.

I. Estimate a Limit Numerically

1. Estimate $\lim\limits_{x \to 3}(2x - 4)$ by creating a table by using the table feature of your graphing calculator.

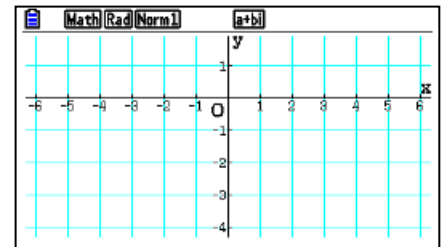(Since $x \to 3$, that is x is <u>approaching</u> a value of 3, use values that greater and less than 3)
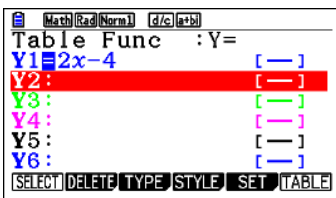
x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$							

From the table, it appears that the closer x gets to 3, the closer $f(x) =$ _____.

Look at the graph of $f(x) = 2x - 4$ to verify the information from the chart.

$$\lim\limits_{x \to 3}(2x - 4) = \underline{\qquad}$$

For all **polynomials**, such as this linear example, limits can be found through **direct substitution** because polynomial functions have unrestricted domains.
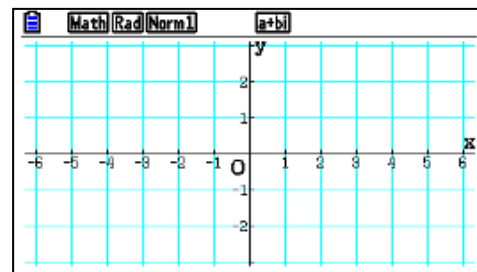
2. Find the limit of $\lim\limits_{x \to 0} \dfrac{x}{\sqrt{x+1} - 1}$ by using your graphing calculator to complete the table listed below.
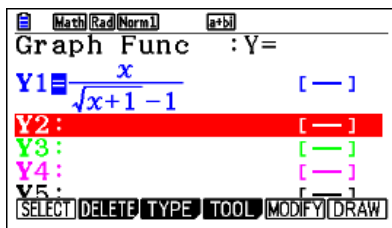
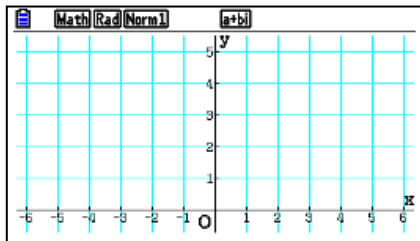| x	-.01	-.001	-.0001	0	.0001	.001	.01
$f(x)$							

Although $f(0)$ is undefined, as the values approach 0 from the left and right, $f(x)$ approaches _____. Graph and use the trace

feature to look at $f(0)$. $\lim\limits_{x \to 0} \dfrac{x}{\sqrt{x+1} - 1} = \underline{\qquad}$

II. Limits that fail to exist.

3. <u>Unbounded Behavior</u>: Investigate $f(x) = \dfrac{1}{x^2}$ graphically. For what value is $f(x)$ undefined? $x =$ _____

This tells you where the limit **_MIGHT_** fail to exist.
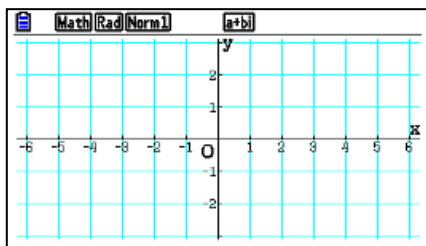Investigate the limit using a table as x values approach this value.

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$							

From the graph and the table, what does the graph appear to be approaching? _____

Recall our formal definition: If f(x) becomes arbitrarily close to a unique _____ L as x approaches c from either side, then the **limit** of f(x) as x approaches c is L. This is written as $\lim\limits_{x \to c} f(x) = L$

What does this imply about $\lim\limits_{x \to 0} \dfrac{1}{x^2}$? _____Why? _____

4. <u>Left and Right Hand Behavior</u>: Investigate the graph of $f(x) = \dfrac{|x|}{x}$. For what value is $f(x)$ undefined? $x =$ _____

This tells you where the limit **_MIGHT_** fail to exist.
Investigate the limit using a table as x values approach this value.

x	-2	-1	-0.5	0	0.5	1	2
$f(x)$							

What does the graph appear to be <u>approaching from the left</u>? $x \to 0^- =$ _____.

What does the graph appear to be <u>approaching from the right</u>? $x \to 0^+ =$ _____.

Recall our formal definition AGAIN!! If f(x) becomes arbitrarily close to a _____ number L as x approaches c from either side, then the **limit** of f(x) as x approaches c is L. This is written as $\lim\limits_{x \to c} f(x) = L$
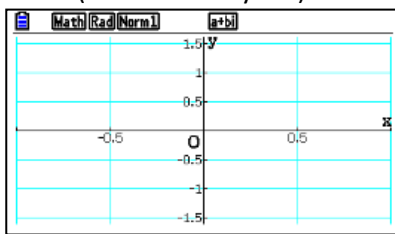
What does this imply about $\lim\limits_{x \to 0} \dfrac{|x|}{x}$? _____Why? _____

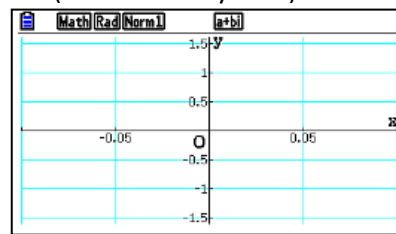5. <u>Oscillating Behavior</u>: Investigate the graph of $f(x) = \sin\dfrac{1}{x}$.

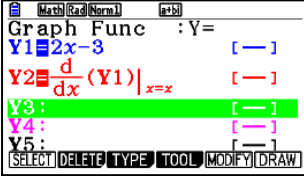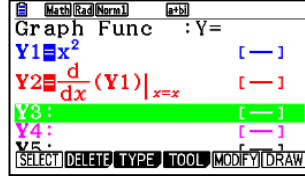| Adjust to a Trig View Window	Adjust view window between -1 & 1 (scale x-axis by 0.5)	Adjust view window between -0.1 & 0.1 (scale x-axis by 0.05)
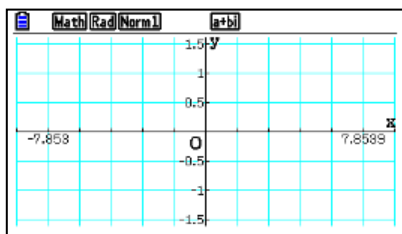
ONE LAST TIME…recall the formal definition: If f(x) becomes arbitrarily close to a unique number L as x approaches c from _____, then the **limit** of f(x) as x approaches c is L. This is written as $\lim\limits_{x \to c} f(x) = L$
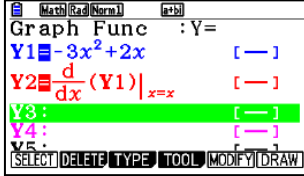
What does this imply about $\lim\limits_{x \to 0} \sin\dfrac{1}{x}$? _____Why? _____
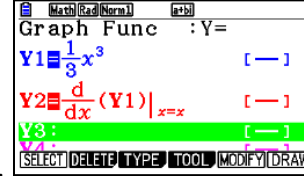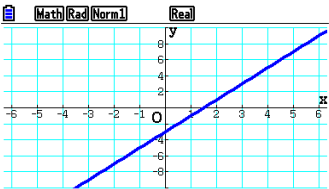
Discovering Derivative Rules

The derivative of a function represents the behavior of the slope of the function at each point along its domain. A) Use a graphing calculator to graph a function and its derivative. Write what you equation you _think_ would produce the graph of the derivative and then check by graphing it as your Y3 graph. B) Repeat process to predict the equation of the second derivative.

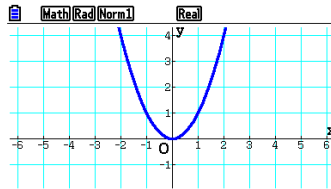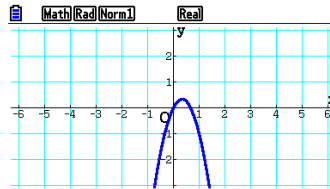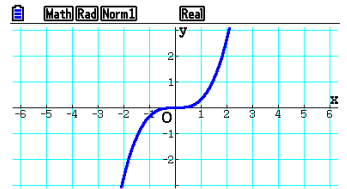**1.**
Graph Func :Y=
Y1=2x−3   [—]
Y2=$\frac{d}{dx}$(Y1)$|_{x=x}$ [—]
Y3:   [—]
Y4:   [—]
Y5:
SELECT DELETE TYPE TOOL MODIFY DRAW

2.
Graph Func :Y=
Y1=x²   [—]
Y2=$\frac{d}{dx}$(Y1)$|_{x=x}$ [—]
Y3:   [—]
Y4:   [—]
Y5:
SELECT DELETE TYPE TOOL MODIFY DRAW

3.
Graph Func :Y=
Y1=−3x²+2x   [—]
Y2=$\frac{d}{dx}$(Y1)$|_{x=x}$ [—]
Y3:   [—]
Y4:   [—]
Y5:
SELECT DELETE TYPE TOOL MODIFY DRAW

4.
Graph Func :Y=
Y1=$\frac{1}{3}x^3$ [—]
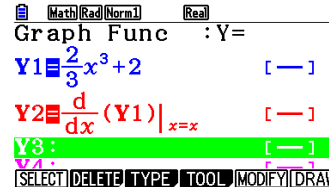Y2=$\frac{d}{dx}$(Y1)$|_{x=x}$ [—]
Y3:   [—]
SELECT DELETE TYPE TOOL MODIFY DRAW

$y' = $ _____
$y'' = $ _____

$y' = $ _____
$y'' = $ _____

$y' = $ _____
$y'' = $ _____

$y' = $ _____
$y'' = $ _____

4.
Graph Func   :Y=
Y1=$\frac{2}{3}x^3$+2   [—]
Y2=$\frac{d}{dx}$(Y1)$|_{x=x}$ [—]
Y3:   [—]
Y4:
SELECT DELETE TYPE TOOL MODIFY DRAW

5.
Graph Func :Y=
Y1=sin x   [—]
Y2=$\frac{d}{dx}$(Y1)$|_{x=x}$ [—]
Y3:   [—]
Y4:   [—]
Y5:
SELECT DELETE TYPE TOOL MODIFY DRAW

6.
Graph Func :Y=
Y1=cos x   [—]
Y2=$\frac{d}{dx}$(Y1)$|_{x=x}$ [—]
Y3:   [—]
Y4:   [—]
Y5:
SELECT DELETE TYPE TOOL MODIFY DRAW

7.
Graph Func :Y=
Y1=2sin x   [—]
Y2=$\frac{d}{dx}$(Y1)$|_{x=x}$ [—]
Y3:   [—]
Y4:   [—]
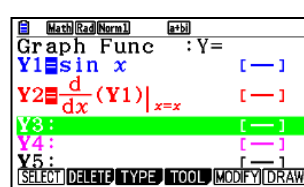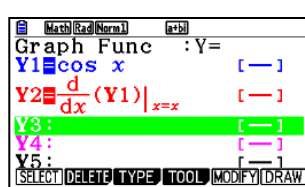SELECT DELETE TYPE TOOL MODIFY DRAW

$y' = $ _____
$y'' = $ _____

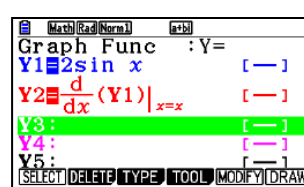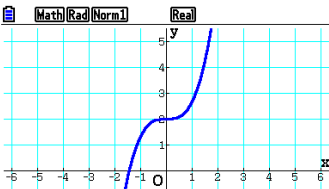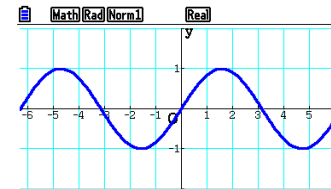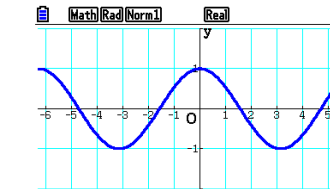$y' = $ _____
$y'' = $ _____

$y' = $ _____
$y'' = $ _____

$y' = $ _____
$y'' = $ _____

8.
Graph Func :Y=
Y1=cos (2x)   [—]
Y2=$\frac{d}{dx}$(Y1)$|_{x=x}$ [—]
Y3:   [—]
Y4:   [—]
Y5:
SELECT DELETE TYPE TOOL MODIFY DRAW

9.
Graph Func :Y=
Y1=tan x   [—]
Y2=$\frac{d}{dx}$(Y1)$|_{x=x}$ [—]
Y3:   [—]
Y4:   [—]
Y5:
SELECT DELETE TYPE TOOL MODIFY DRAW

10.
Graph Func :Y=
Y1=e^x   [—]
Y2=$\frac{d}{dx}$(Y1)$|_{x=x}$ [—]
Y3:   [—]
Y4:   [—]
Y5:
SELECT DELETE TYPE TOOL MODIFY DRAW

11.
Graph Func :Y=
Y1=|x| [—]
Y2=$\frac{d}{dx}$(Y1)$|_{x=x}$ [—]
Y3:   [—]
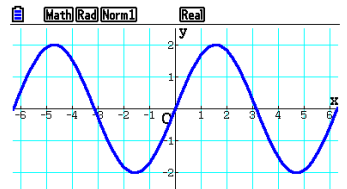Y4:   [—]
Y5:
SELECT DELETE TYPE TOOL MODIFY DRAW
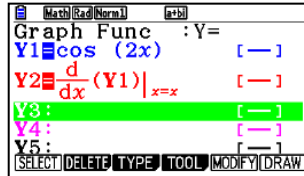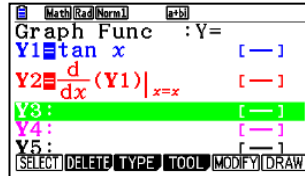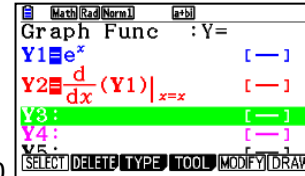
$y' = $ _____
$y'' = $ _____
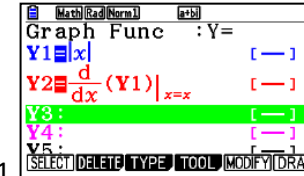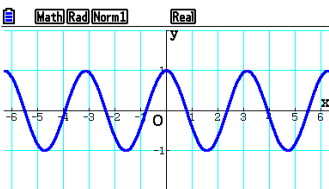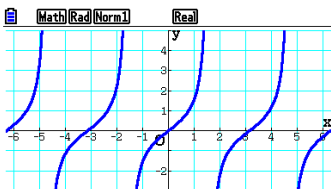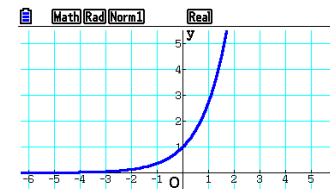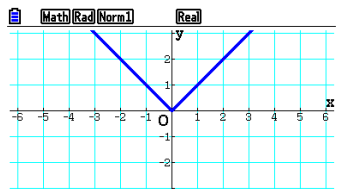
$y' = $ _____
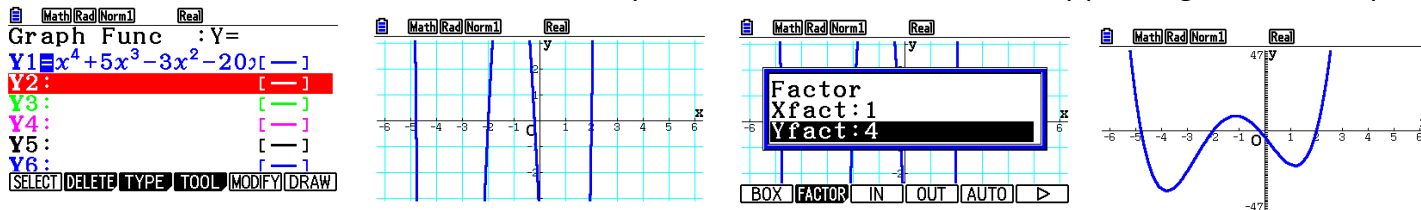$y'' = $ _____

$y' = $ _____
$y'' = $ _____

$y' = $ _____
$y'' = $ _____

Comparing characteristics of f, f', and f"

From the Main Menu, enter the Graph Mode and enter the function $f(x) = x^4 + 5x^3 - 3x^2 - 20x - 4$.

Under the initial view window, the 4 roots are visible on the x-axis but the y-axis needs to be scaled in order to "see" the extrema.

Set the scale factor for the x-axis to 1 and the y-axis to 4. Zoom out two times by pressing the minus key twice.

1. Graph the function and the first derivative. Sketch vertical lines by using the sketch feature at the points where the first derivative is zero.
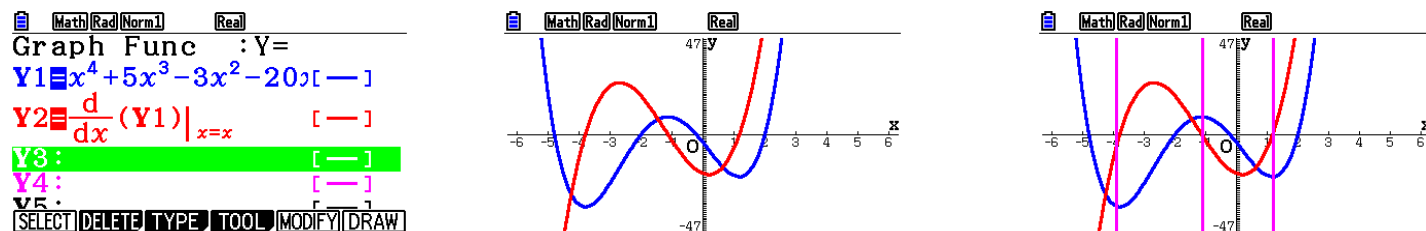
What do you notice? _____

2. Now, return to the graph entry screen and graph the function and the second derivative only. Sketch vertical lines where the second derivative is zero.
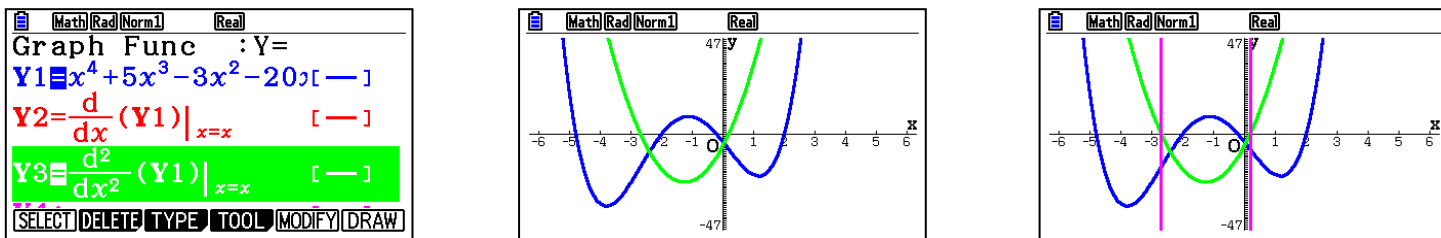
What relationship exist between the two graphs? _____

3. Lastly, graph the first derivative and the second derivative. Sketch vertical lines where the second derivative has a value of zero.

What do you notice?

_____
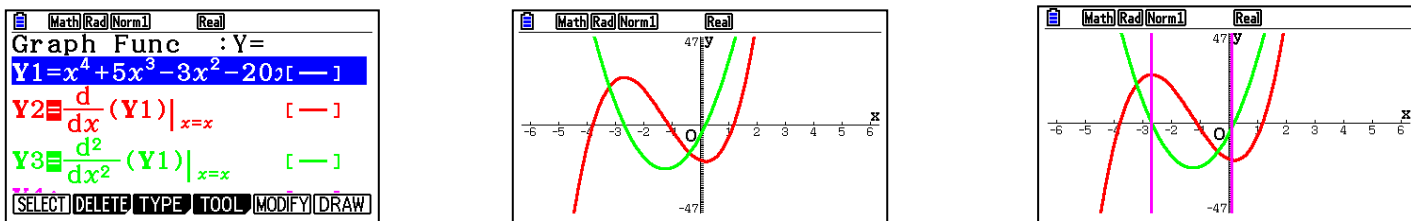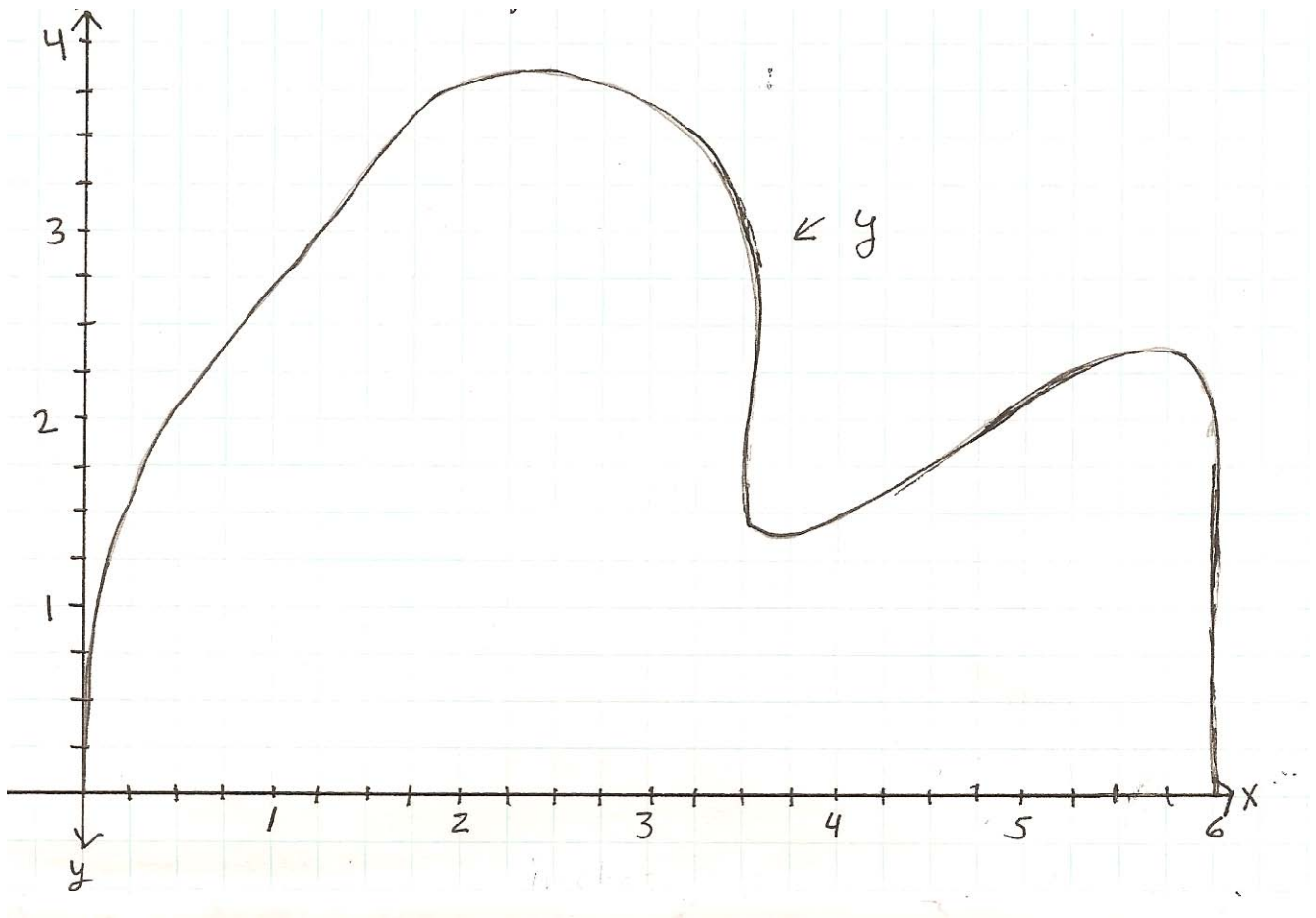
2. Approximate area by using 6 sub-intervals (at 1 inch marks) by using the indicated Riemann Sums.

Group 1: Left-Hand   Group 2: Right-Hand   Group 3: Midpoint   Group 4: Trapezoids

3. Find the quartic equation that best fits the mushroom by using the regression feature of your calculator. Use the points that are labeled on the graph. Round the coefficients to the thousandths place.

Y1=_____

4. Enter the equation under the graph mode and find the area by integrating between the roots.

Area = $\int_{0}^{6} Y1 dx =$_____

5. Find the volume formed if the shape is revolved about the x-axis using the same 6 slices as in step 2 with the height of the graph at each midpoint is used as the radius length of each cylinder.

Volume of a cylinder: V=_____

6. Using the quartic equation from step 3, find the volume of this solid if revolved about the x-axis.

Volume of a solid revolved about the x-axis: V= $\pi \int_{0}^{6} (Y1)^2 \, dx$ = _____

7. Using the equation from step 3, find the volume formed by cross-sections perpendicular to the x-axis that are:

A) Squares   B) Equilateral Triangles   C) Rectangles with a height of 3

Using the Casio Picture Converter, any picture can by imported and then many explorations can be done

FILE | Plot | List | DefG | MODIFY | ▷

LOWER=0.0528   UPPER=12.247
∫dx=51.757843   =51.757843
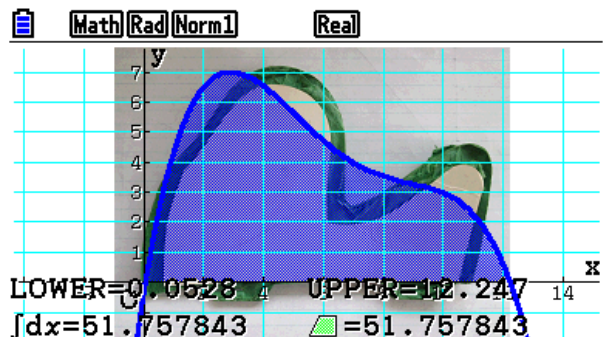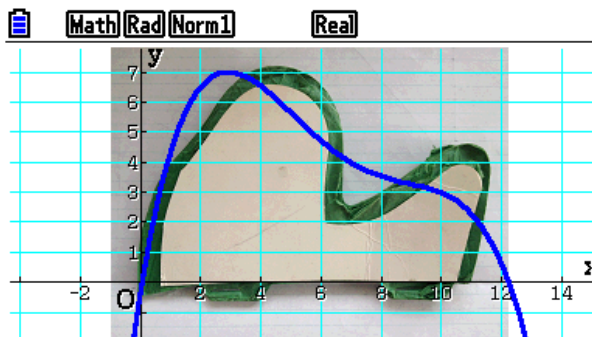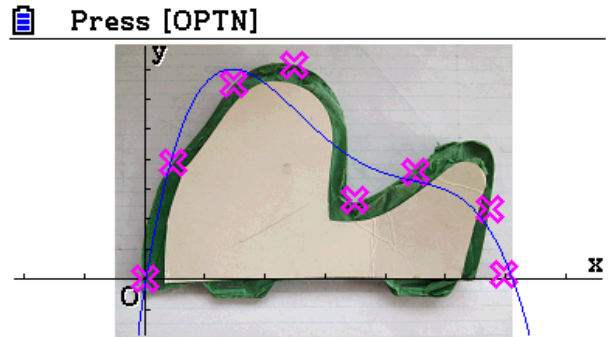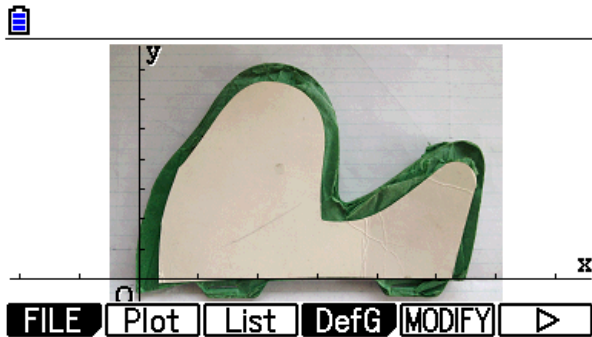
Ready….Set….Go!!!!

Everyone has experienced speed and velocity in their everyday life, but not everyone has seen graphs depicting the characteristics of motion with respect to time! Let's do a little experimenting with a motion detector in order to develop a deeper understanding of how these graphs relate to derivative graphs.

1. Begin by setting up the calculator, motion detector, and data collection device to collect <u>distance</u> (8 feet) data <u>over</u> an interval of <u>time</u> (5 seconds). The graph produced will be a model relating to motion and you are to analyze what movements cause the graph to increase, decrease, or remain constant.

```
        MAIN MENU
   5   6   7   8
 Graph Dyna Graph Table Recursion
   9   aX²+bX+c A   B   C
ConicGraphs Equation Program Financial
   D   E   F   G
 E-Con2   Link   Memory   System
```

```
EA-200 Controller

F1:Setup EA-200
F2:Setup Memory
F3:Program Converter
F4:Start Sampling
F5:Graph Analysis
SET MEMORY PROGRAM START GRAPH HELP
```

```
EA-200 Controller

F1:Setup EA-200
F2:Setup Memory
F3:Program Converter
F4:Start Sampling
F5:Graph Analysis
WIZARD ADVANCE
```

```
======== EA-200 ========

    Select Sensor

   CASIO  :[F1]
   VERNIER:[F2]
```

```
======== EA-200 ========
Vernier Sensors
 Gas Pressure      ↑
 Heart Rate
 MagneticField
 Microphone
 Motion            ↓
```

```
======== EA-200 ========
Motion
 Meters
 Feet
```

```
======== EA-200 ========
   Input Total
Sampling Interval

  1sec - 30days
      :5|
```

```
======== EA-200 ========
   Select Unit
     1:sec
     2:min
     3:hour
     4:day
```

```
======== EA-200 ========
 SONIC:Motion
 total:5sec

    OK  :[F1]
  Cancel:[F6]
```

```
===== SETUP WIZARD =====
    Complete!

1:Start Setup
2:Save Setup-MEM
3:Convert Program
```

```
======== EA-200 ========
 *IS THE SENSOR CONNECTED?
 *CONNECT LINK-CABLE FIRMLY?
 *IS SAMPLING DONE?

   Press:[EXE]
```

```
======== EA-200 ========

 Setting EA-200...

   Cancel:[AC]
```

2. Determine "how" you must move to create graphs that are similar to the ones listed through experimenting with the motion detector. Write your explanation below each graph. Label each axis with respect to what it is measuring.

a)

b)

c)

d)
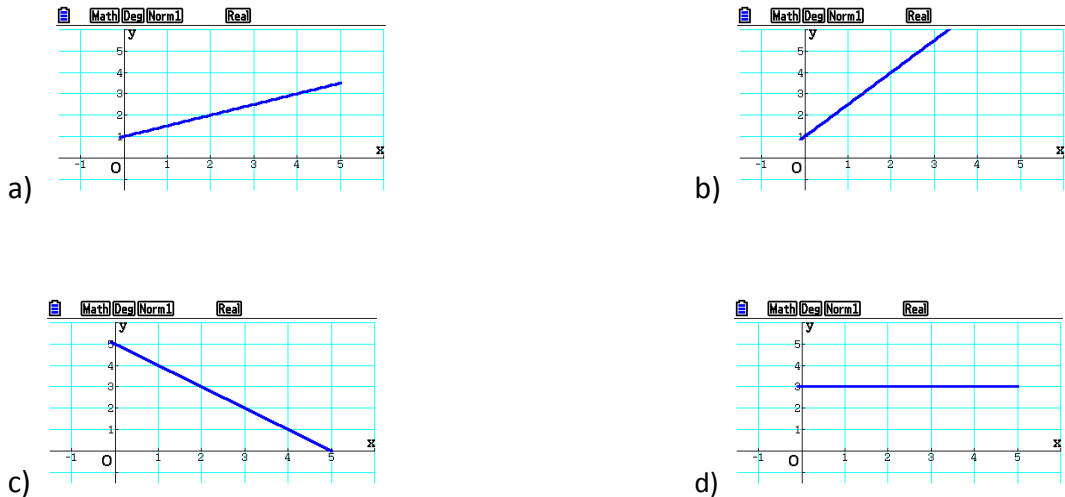
What can be said about the velocity required to produce linear graphs? _____

What caused the slope of the graphs to be positive, negative, and zero? _____

Using the Calculation feature, create a linear equation that models a "good" section of the graph and explore the graph of its derivative. What does this imply? _____

3. The following graphs are piecewise functions. Try to create these models with the motion detector and then describe the movement that produces these graphs.
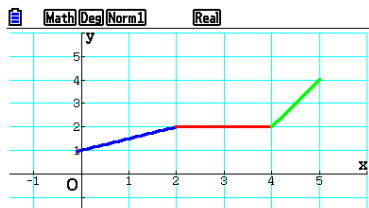
a)
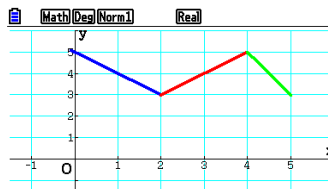
b)

What is happening at points on these two graphs where the graphs have "corners"? _____

Velocity is considered positive when the distance between the motion detector is increasing, negative when the distance is decreasing, and zero when the remaining still or changing direction.

On graph 3a, how close where you to the motion detector at t=0 second? _____ And at t=2 seconds? _____

On graph 3b, what point represents when you were closest to the motion detector? _____ What time did that occur? _____
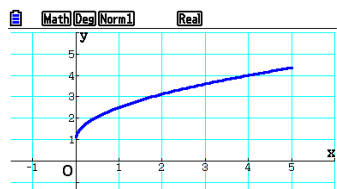
On graph 3b, at what time interval where you farthest from the motion detector? _____ How far? _____

These graphs are also linear, so the velocity had to be virtually _____ to create these graphs. It is hard to create these graphs because…..

4. The following graphs are **_not_** linear models. Describe how to adjust your movement to create these graphs.
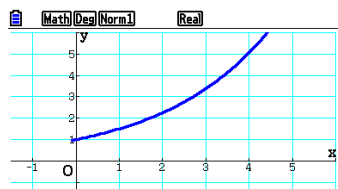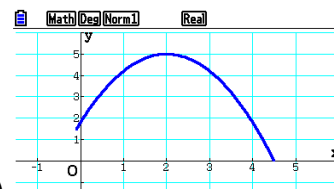

a)

b)

c)

d)

When the rate of your velocity is NOT constant graphs are not _____! When the change in velocity is increasing or decreasing, your graph will appear either concave up or concave down. _____ is the measurement of the rate of change of velocity with respect to the change in time.

What seems to cause the graph to be concave upward? _____

What seems to cause the graph to be concave downward? _____

While in the graph mode of the EA200, notice what you can do with sections of the graphs where the data is representative of the desired motion.

(Data can be collected using any Verneir Probe and then analyzed. Using the temperature probes, Newton's Law of Cooling can be explored and verified.)