

Presentation 117. Kamii. "Measuring Length and Time in the Common Core, Grades K-1."  
(A presentation at the NCTM Annual Conference, Denver, April 18, 2013)

**I. What CCSS says about the measurement of length** in first grade

**"1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.**

**2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps."**

(CCSS, 2010, p. 16 about Grade 1)

The first of the above two standards concerns *transitivity*, and the second is about *unit iteration*. Piaget published his research about both in 1948 in *The Child's Conception of Geometry*. Both are part of logico-mathematical knowledge, which each child constructs (creates) from within and is not teachable. My reasons for saying this are explained below.

The three kinds of knowledge Piaget distinguished according to their ultimate sources

Physical knowledge: Knowledge about objects such as the fact that marbles roll, and counters stay put. The fact that glasses are likely to break when they are dropped on the floor is also an example of physical knowledge. The ultimate source of physical knowledge is objects.

Social-conventional knowledge: Knowledge made by conventions that people make over time, such as languages (e.g., "uno-dos-tres" and "one-two-three"), holidays like Christmas, and the fact that the right hand is used to shake hands. The ultimate source of social-conventional knowledge is conventions.

Logico-mathematical knowledge: Mental relationships that each individual constructs from within such as the relationships of "different," "similar," and "two." If I showed you a red marble and a blue one, we could think about them as being "different," "similar," or "two." These relationships do not exist in the external world. *Only when we think about them as being "different" do they become different for us at that moment. Likewise, only when we think about the marbles as "two" do they become two for us at that moment.* The ultimate source of logico-mathematical knowledge is in each individual's head. Each individual must make (construct) his/her own mental relationships from within. This is the hardest kind of knowledge for empiricists to understand, and the authors of CCSS are totally unaware of the nature of logico-mathematical knowledge. **Transitivity** and **unit iteration** are part of logico-mathematical knowledge.

Transitivity: The ability to **deduce by thinking** that if  $A=B$  (empirically) and  $B=C$  (empirically),  $A$  must be equal to  $C$  (a deduction).

A \_\_\_\_\_  
B \_\_\_\_\_

B \_\_\_\_\_  
C \_\_\_\_\_

The ability to make this deduction is constructed by each child from within and appears among 75% of children by grade 2 (Kamii & Clark, 1997). The authors of CCSS think that transitivity can be taught. However, logico-mathematical relationships cannot be taught from the outside.

Unit iteration: The ability to use **one object** and iterate it. The authors of CCSS incorrectly believe that unit iteration means the ability to use **many short objects** and align them end to end.

The ability to iterate a unit out of transitive reasoning appears in grade 4 (Kamii & Clark, 1997). Only small percentages of first graders can make the logico-mathematical relationships of transitivity (29%) and unit iteration (10%). CCSS forces the premature teaching of both in first grade.

[A videotape of a second graders will be shown to illustrate these points.]

## II. What CCSS says about the measurement of time in first grade

**“Tell and write time in hours and half-hours using analog and digital clocks.”**

(CCSS, 2010, p. 16 about Grade 1)

Knowledge of analog and digital clocks is **social-conventional knowledge**. The fact that there are 60 minutes in an hour, and 24 hours in a day is social-conventional knowledge. The fact that, on an analog clock, “0 to 1” means “5 minutes” is also an example of social-conventional knowledge.

To make sense of clocks, children first need to have **transitive reasoning** and **unit iteration** (logico-mathematical knowledge). Transitivity is constructed by 75% of children between 2<sup>nd</sup> and 4<sup>th</sup> grade, and unit iteration is constructed by 70% by 6<sup>th</sup> grade (Long & Kamii, 2001).

The measurement of time involves an additional factor: Conservation of speed. Before 6<sup>th</sup> grade, children think that the instrument used to measure time changes its speed when their action changes. For example, children were asked to transfer marbles one by one from one bowl to another “as quickly as possible” and then “as slowly as possible.” They thought the water (or sand) used to measure time moved at different speeds, depending on whether they performed the action quickly or slowly.

[A videotape of a fourth grader will be shown to illustrate these points.]

In conclusion, CCSS requires first graders to “tell and write time in hours and half-hours” without any connection with the logico-mathematical knowledge of transitivity and unit iteration.

### References

Kamii, C., & Clark, F. B. (1997). Measurement of length: The need for a better approach to teaching, *School Science and Mathematics*, 97(3), 116-121, which can be downloaded from my website (Constancekamii.org).

Long, K., & Kamii, C. (2001). The measurement of time: Children's construction of transitivity, unit iteration, and conservation of speed. *School Science and Mathematics*, 101(3), 125-132, which can be downloaded from my website (Constancekamii.org).

Piaget, J. (1960). *The child's conception of geometry*. New York: Basic Books. (First published in Paris by Presses Universitaires de France in 1948)