PLACE-VALUE FLUENCY

Examples of Child-Centered Practice and Assessment

presentation at the
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FACT

Math may be eternal…

but the standards,
They are a'changin'

FACT

Creating new standards
is not a magic wand.

It does not transform
who children are
or what they are capable of.

You have to teach the children you have,
not the children you wish you had.

FACT

Math is not something we do to children, it is something children do.

FACT

A good learning task
is a good diagnostic task.

A good diagnostic task
is a good learning task.

FACT

Math flows from the inside out,
not from the outside in.

"You can't make
a flower grow
by pulling on it."

old Japanese proverb

Where do today's early childhood math standards come from?

- NAEP (National Assessment of Education Progress)
- TIMSS (Third International Math and Science Study)
- NCTM Standards & focus points

(National Council of Teachers of Mathematics)

- NRC studies (National Research Council)
  - *From Neurons to Neighborhoods* (2001)
  - *Adding It Up* (2001)
  - *Mathematics Learning in Early Childhood* (2009)
- Common Core State Standards

(National Governors Association, 2010)

Keeping Our Eye on the Prize

On the one hand, we need to provide:

1. More math made available in early childhood classrooms.
2. Better math made available in early childhood classrooms.
3. Better scaffolding of all the mathematical moments that occur in early childhood classrooms.

On the other hand, we want to make sure we don't:

1. Violate proven principles of appropriate and effective early childhood practice.
2. Ignore children's innate mathematical intelligence and curiosity.
3. Teach *arithmetic* (i.e., something you are shown and remember) instead of *mathematics* (something you figure out and understand).

Points to take into account as you adjust to the new standards

Point #1:

They are DEVELOPMENTAL.

Point #2:

They are CONCEPTUAL, targeting understanding rather than memorization.

Point #3:

They are AMBITIOUS,
asking more of the children
(and of us)
in the early childhood years
than the previous standards.

Point #4:

They WORK.
Look at what other countries
are accomplishing
using our supposed standards!

Point #5:

Traditional teaching methods
will not get the job done.
Remember:

Arithmeticians plod…
mathematicians soar.

I have come to believe the following:

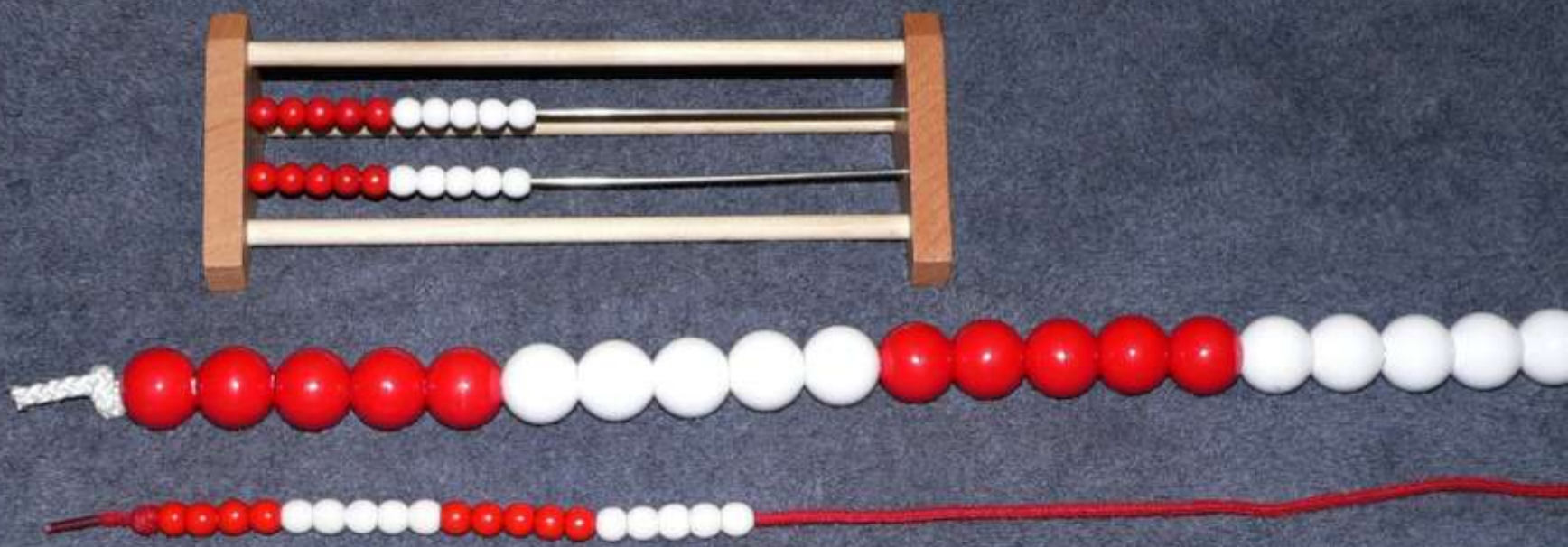
In order to teach less, we need to "seed" the environment with more math-rich stimuli, so we are fostering more child-initiated mathematical events. In other words:

Use the ENVIRONMENT more effectively as your co-teacher.

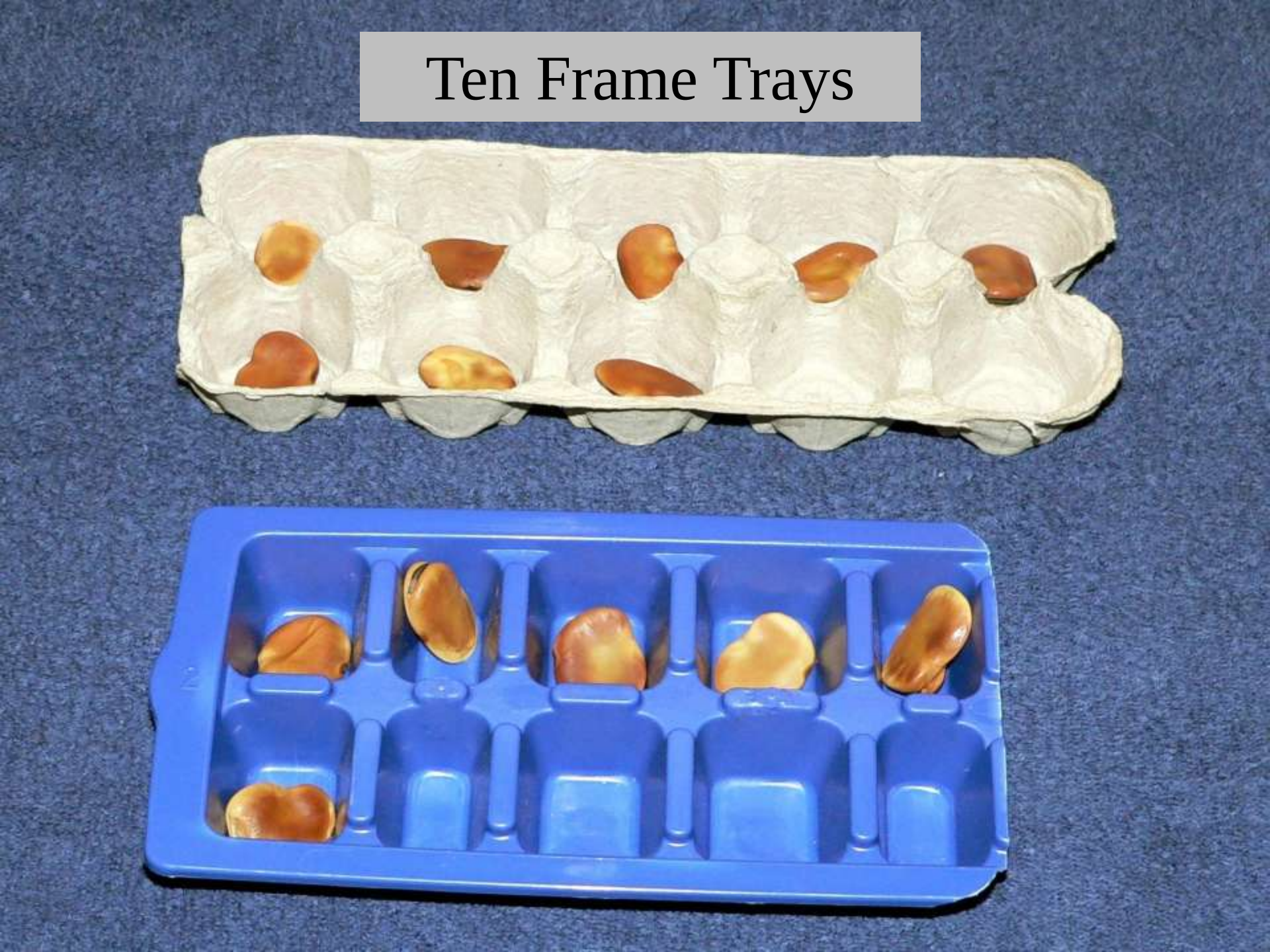
TEACH SMARTER, NOT HARDER.

Beginnings of
Place-Value Awareness

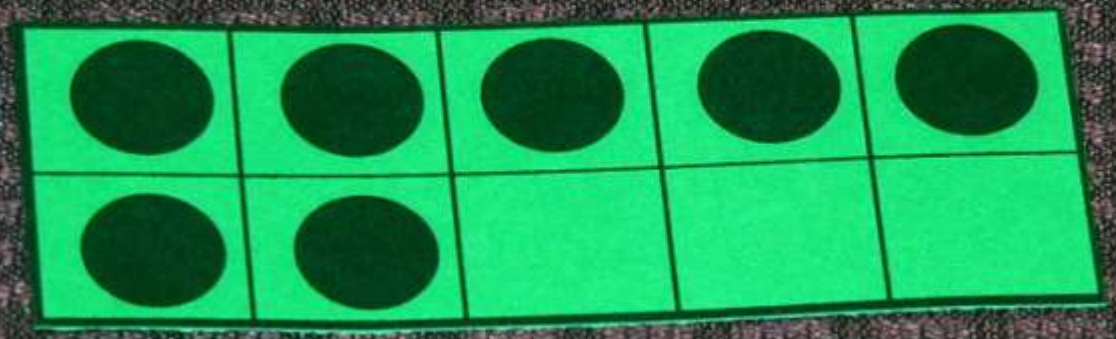
Conceptually Useful Early Counting Models:
Tens and Fives as Anchor Points

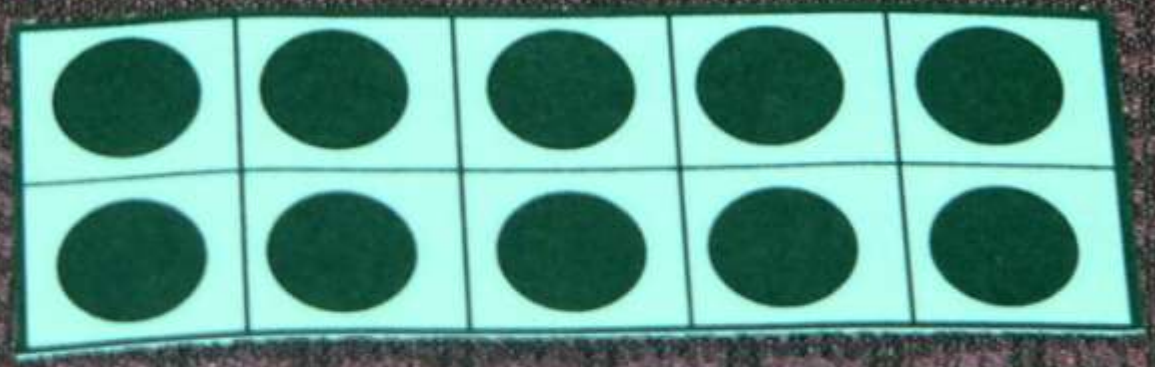


Ten Frame Trays



Ten Frame Cards

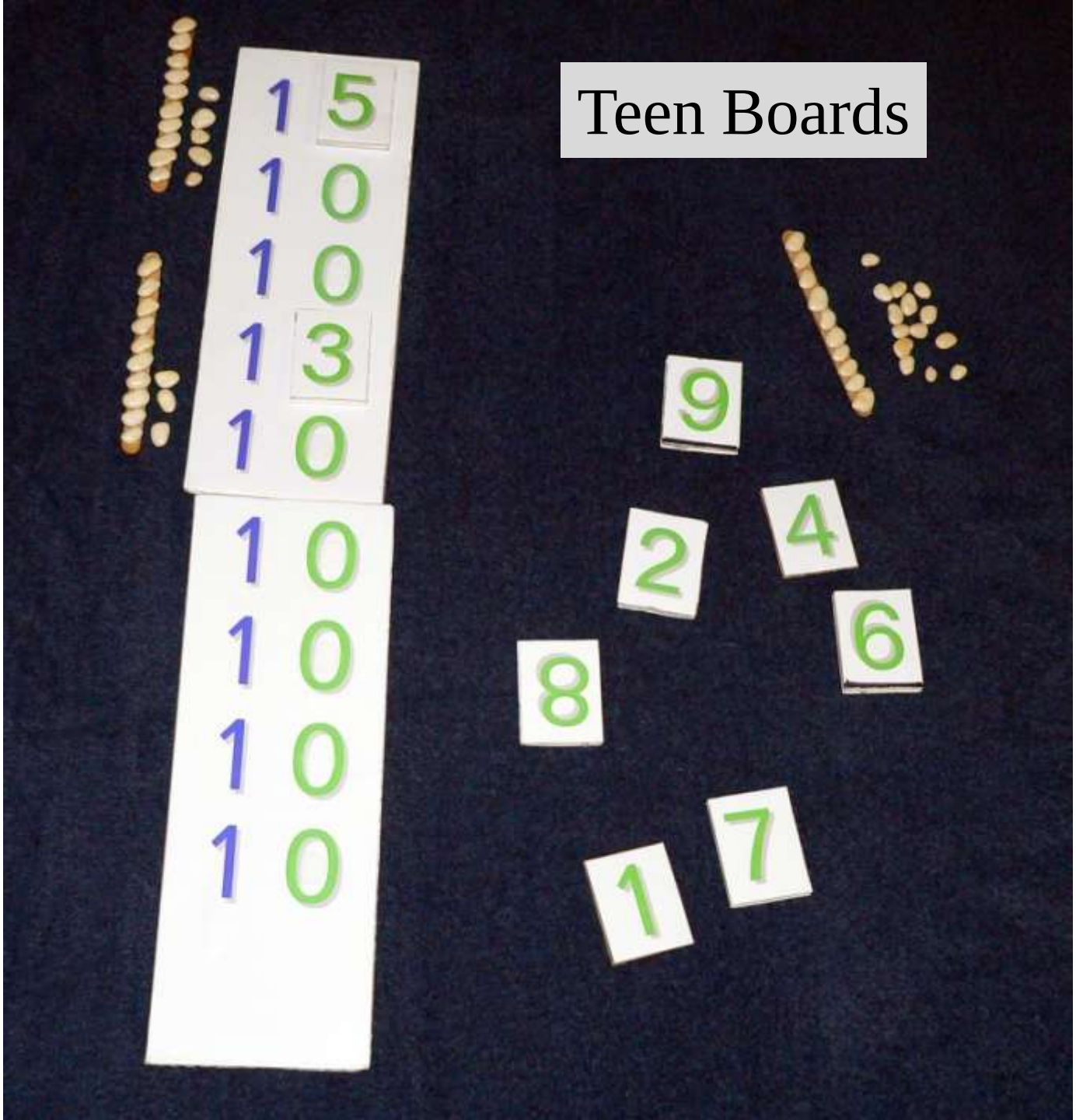




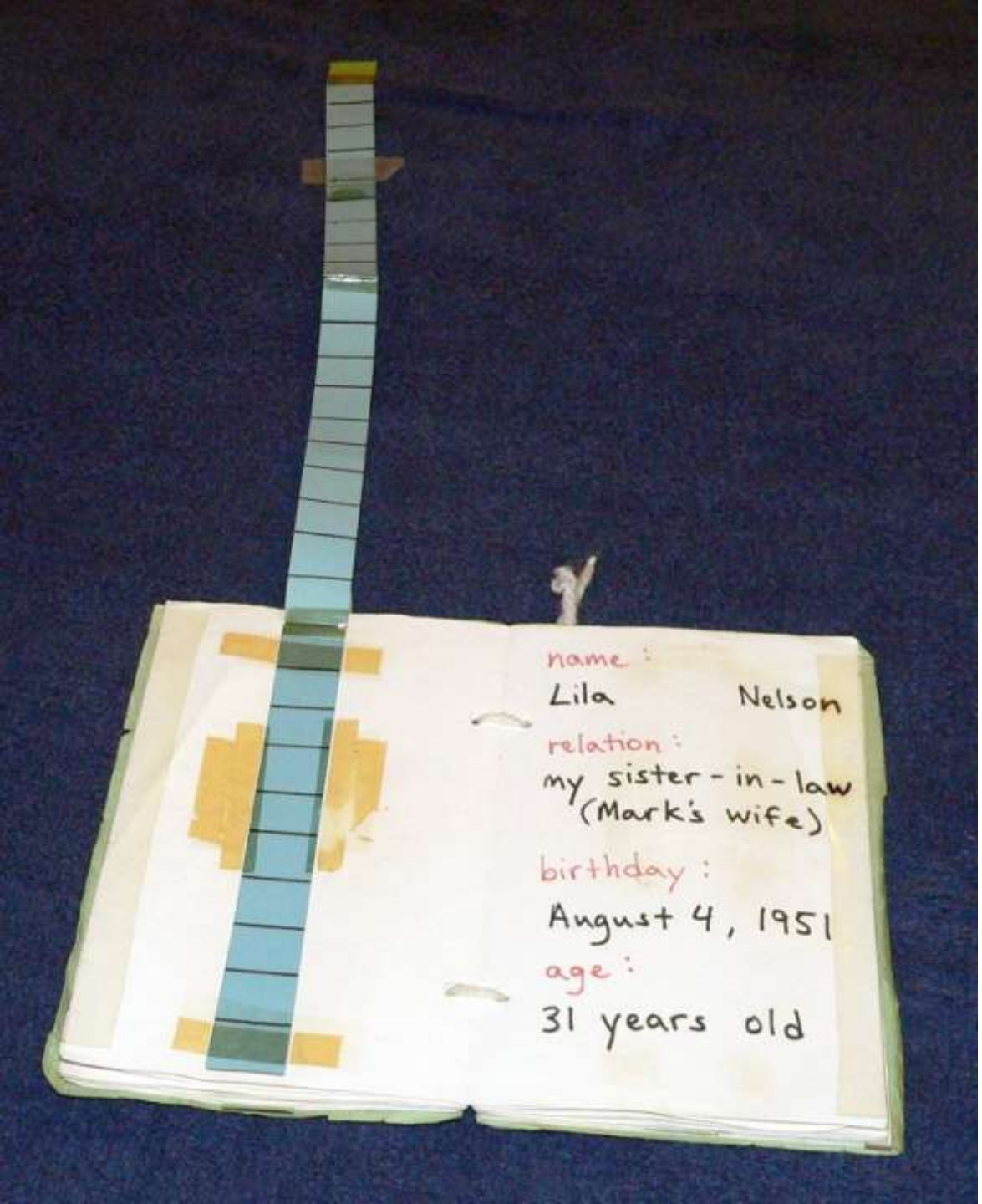
1-10 Finger Cards

eight

ten


Teen Boards
15
10
10
13
10
10
10
10
10
10
9
4
2
6
8
7
1

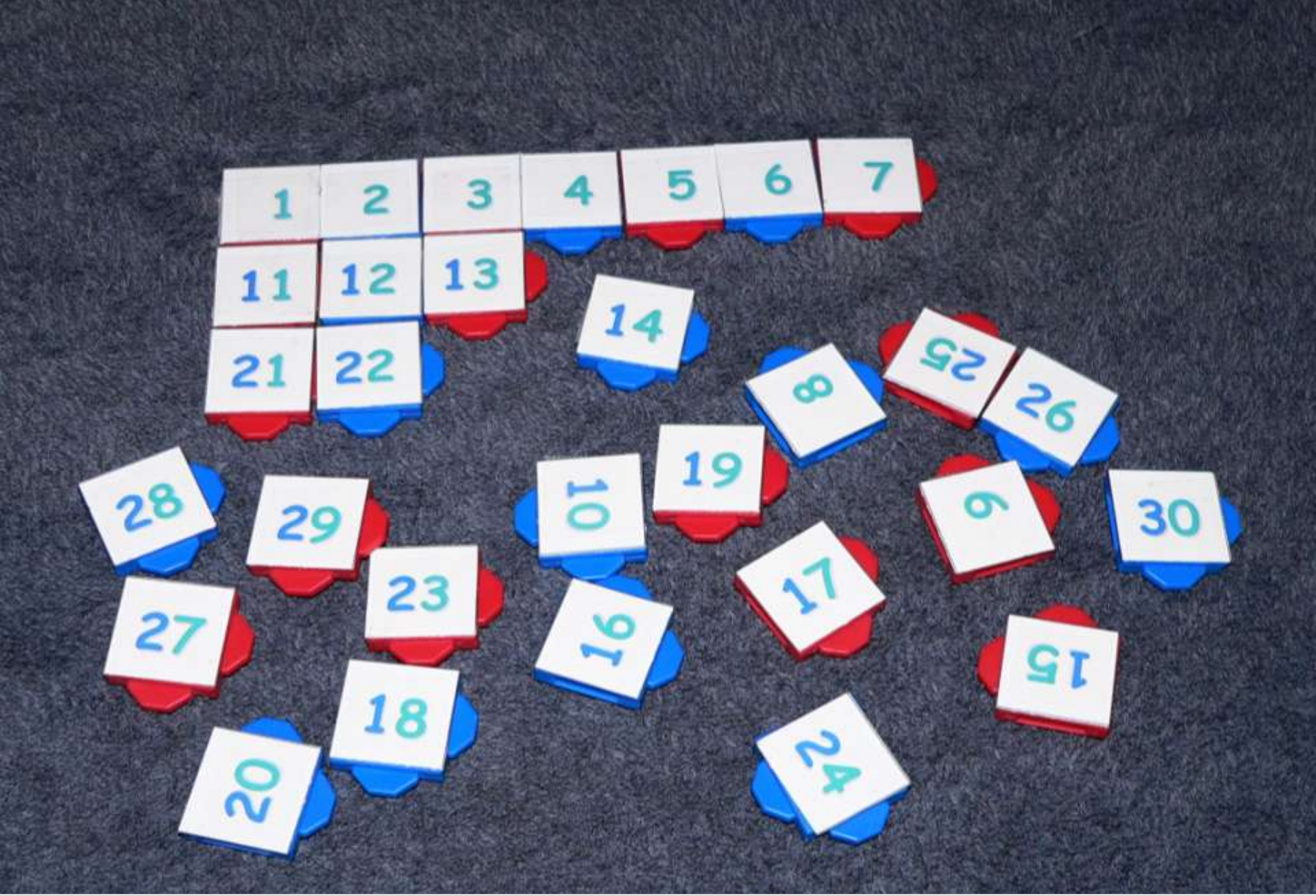
(materials for “How Old We Are” pages)



Assembled
"How Old We Are"
page

Hundreds Board

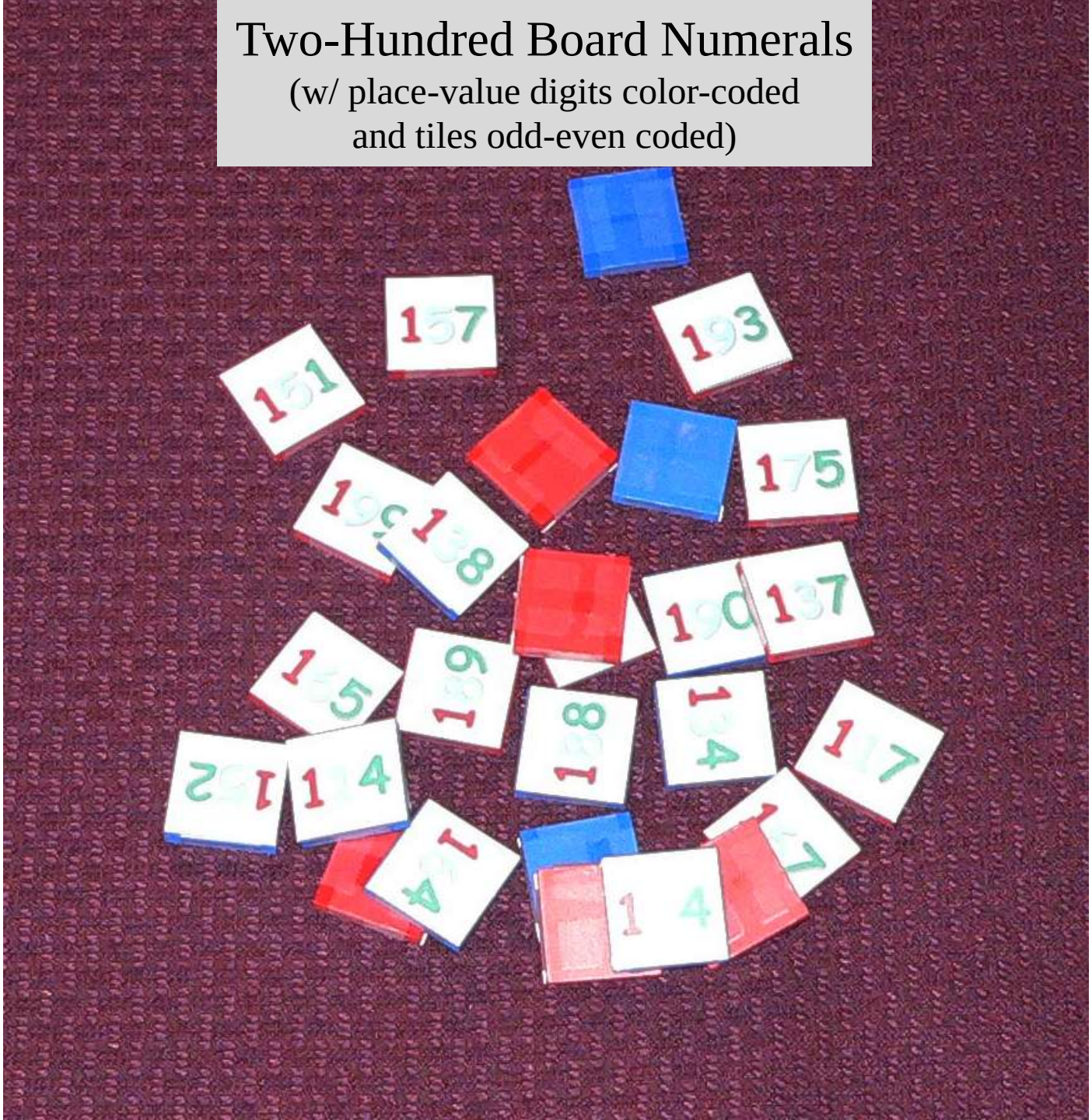
(w/ place-value digits color-coded
And odd-even coded tiles)

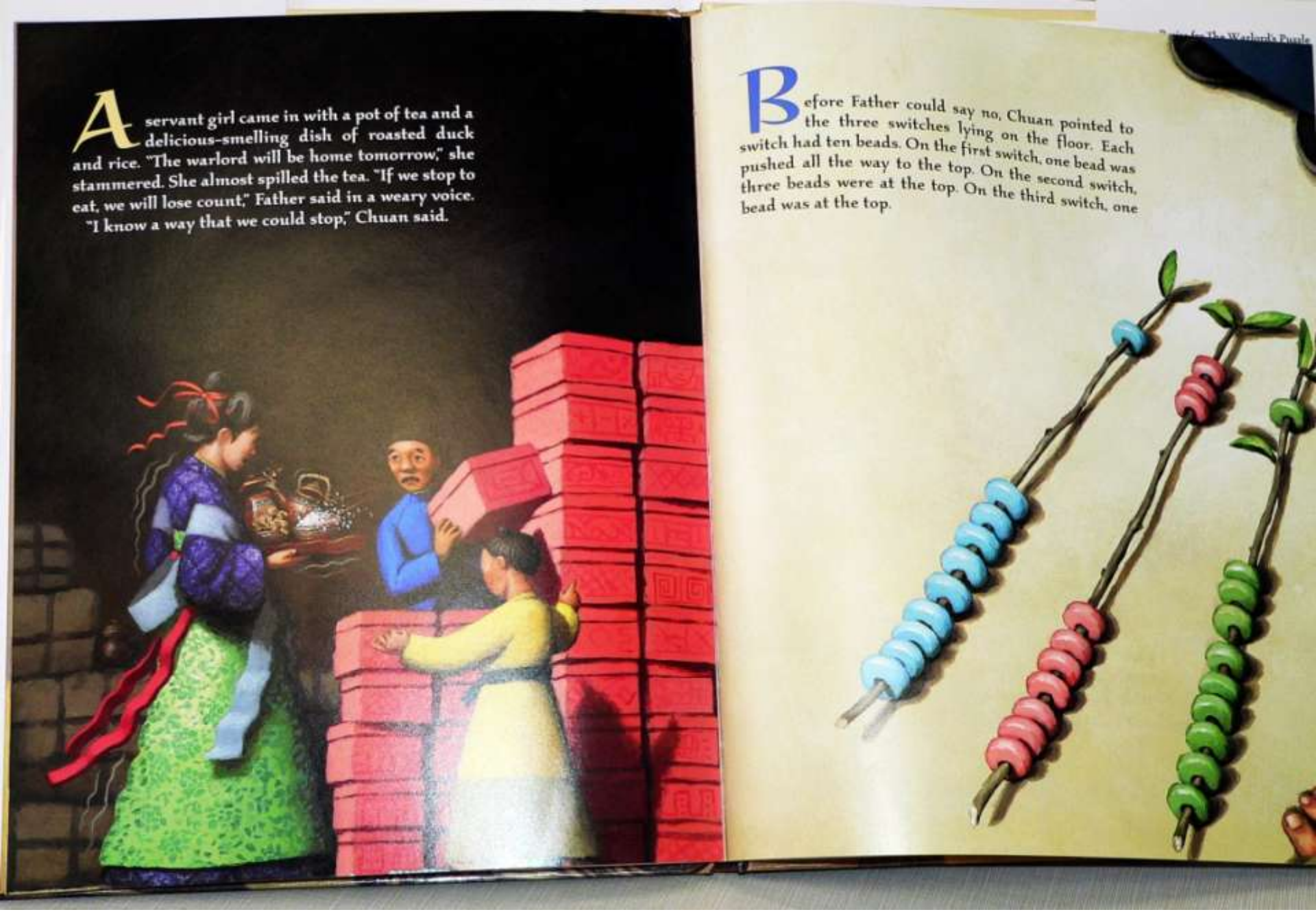


Puzzle-Tiles (1-25)

Two-Hundred Board Numerals

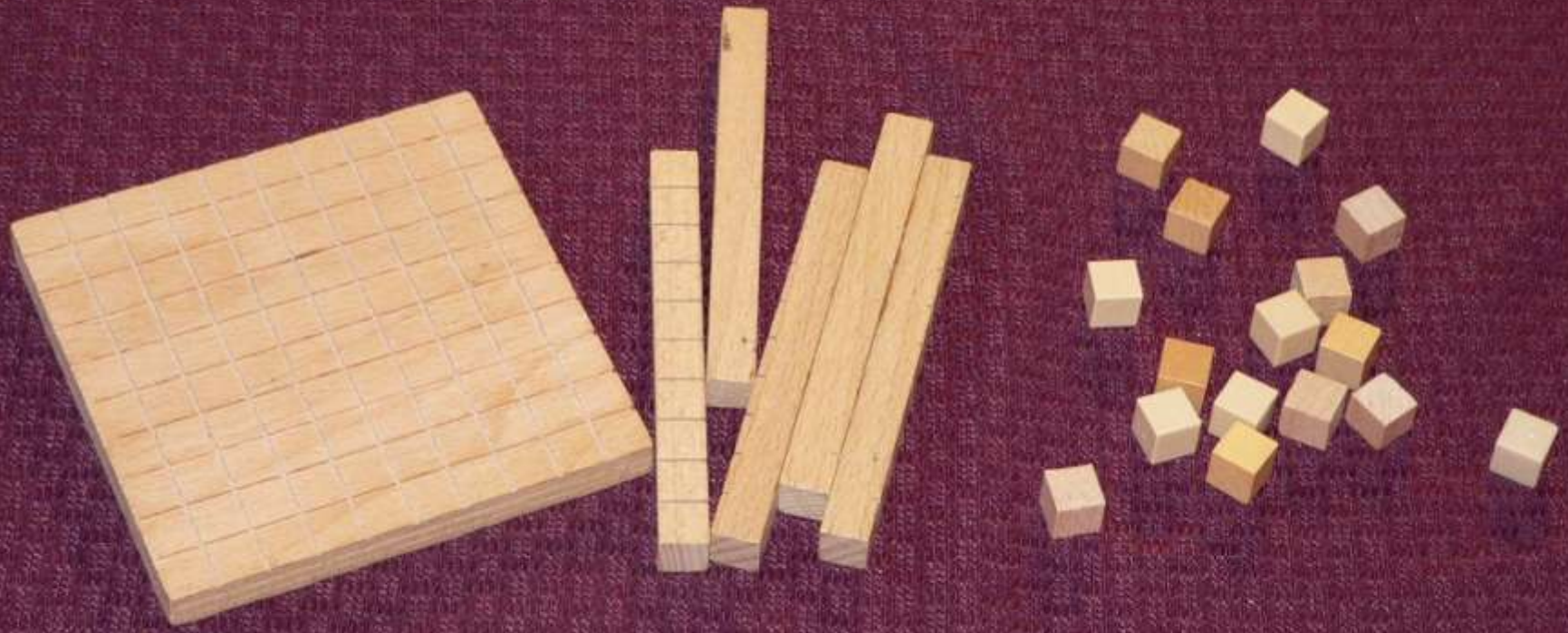
(w/ place-value digits color-coded
and tiles odd-even coded)





The Warlord's Beads (Virginia Pilegard, 2001).

Base-Ten Blocks

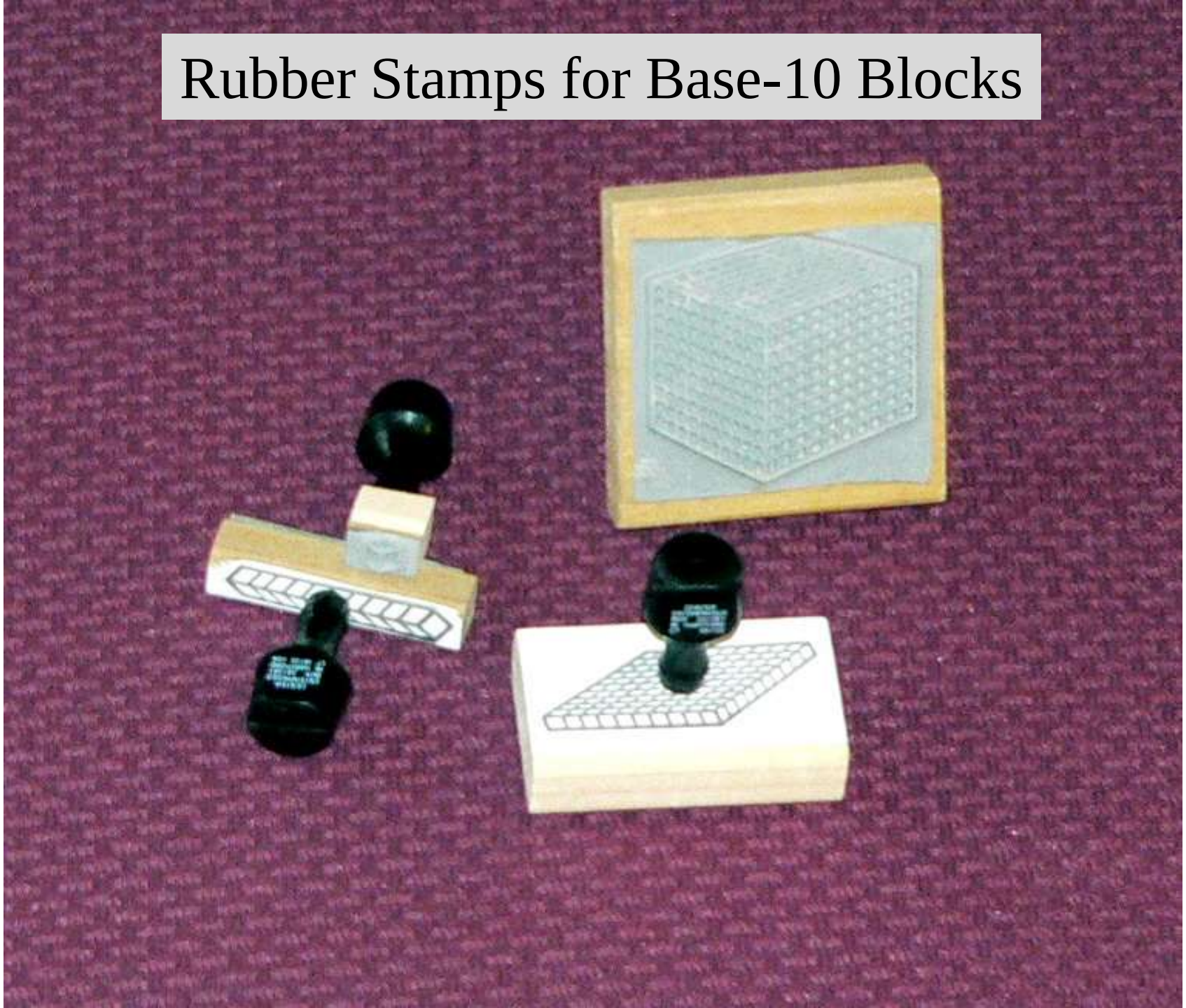


hundreds

tens

units

Rubber Stamps for Base-10 Blocks

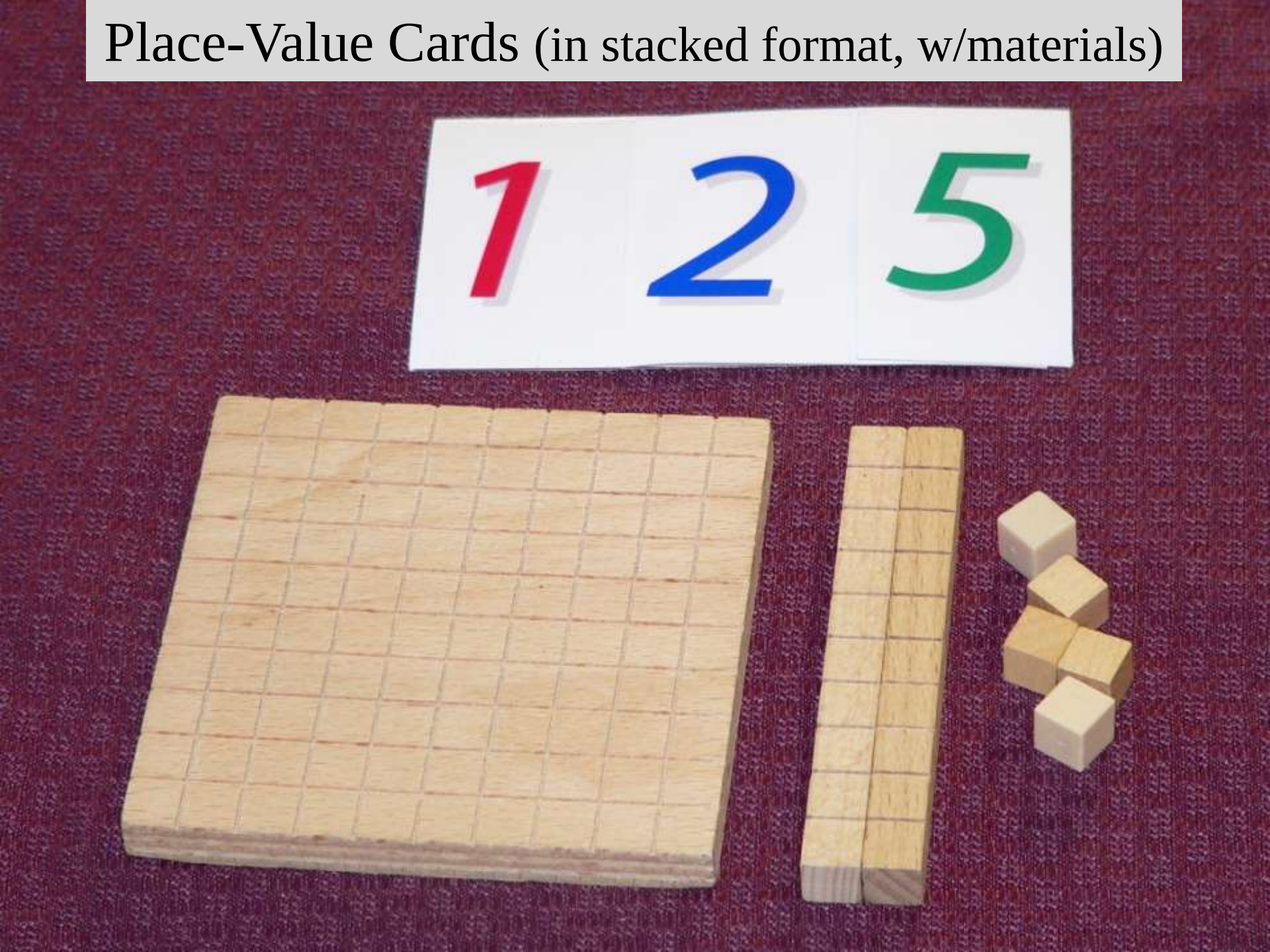


Place-Value Cards
(note color-coding of place values)

Place-Value Cards
on binder ring
(0-9, 10-90, 100-900, and 1000-9000)

Place-Value Cards
(in expanded notation layout)

Place-Value Cards (in stacked format, w/materials)



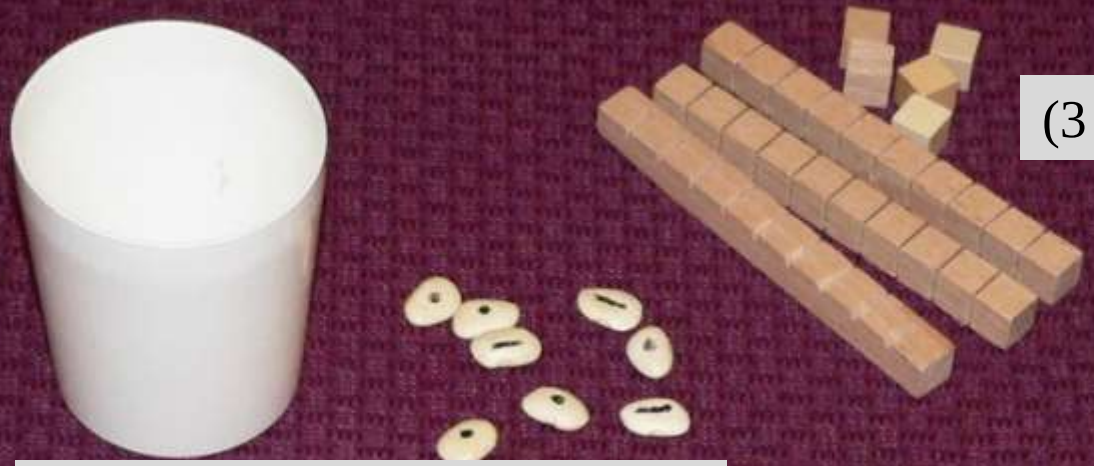
Place-Value Response Wheels

Deepening Understanding of the models

Exchange Game Board (w/ place-value materials)

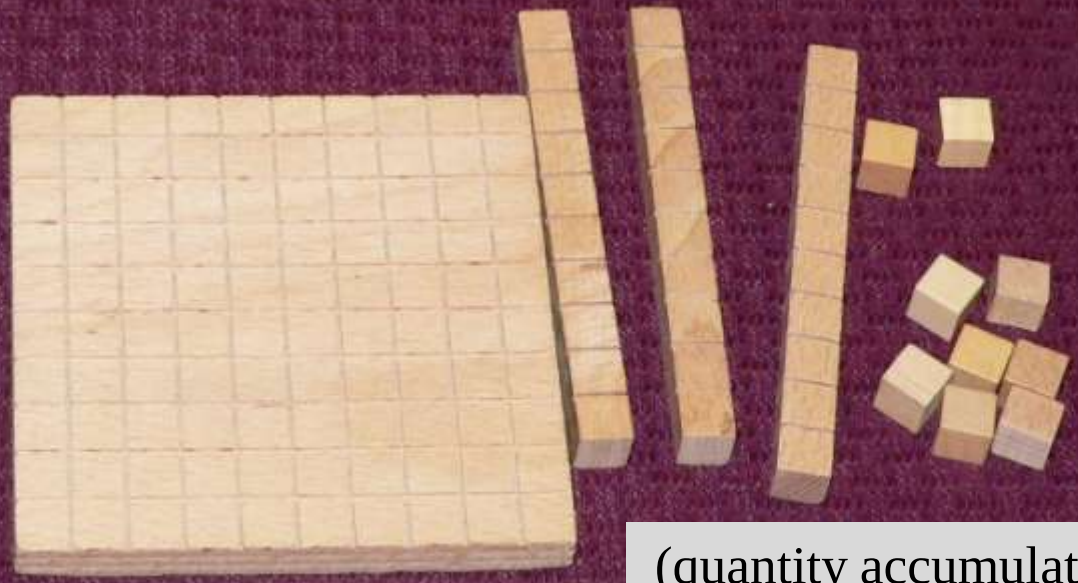
(land on *get* __, *give* __, or *exchange*)

Race to 1000 Game



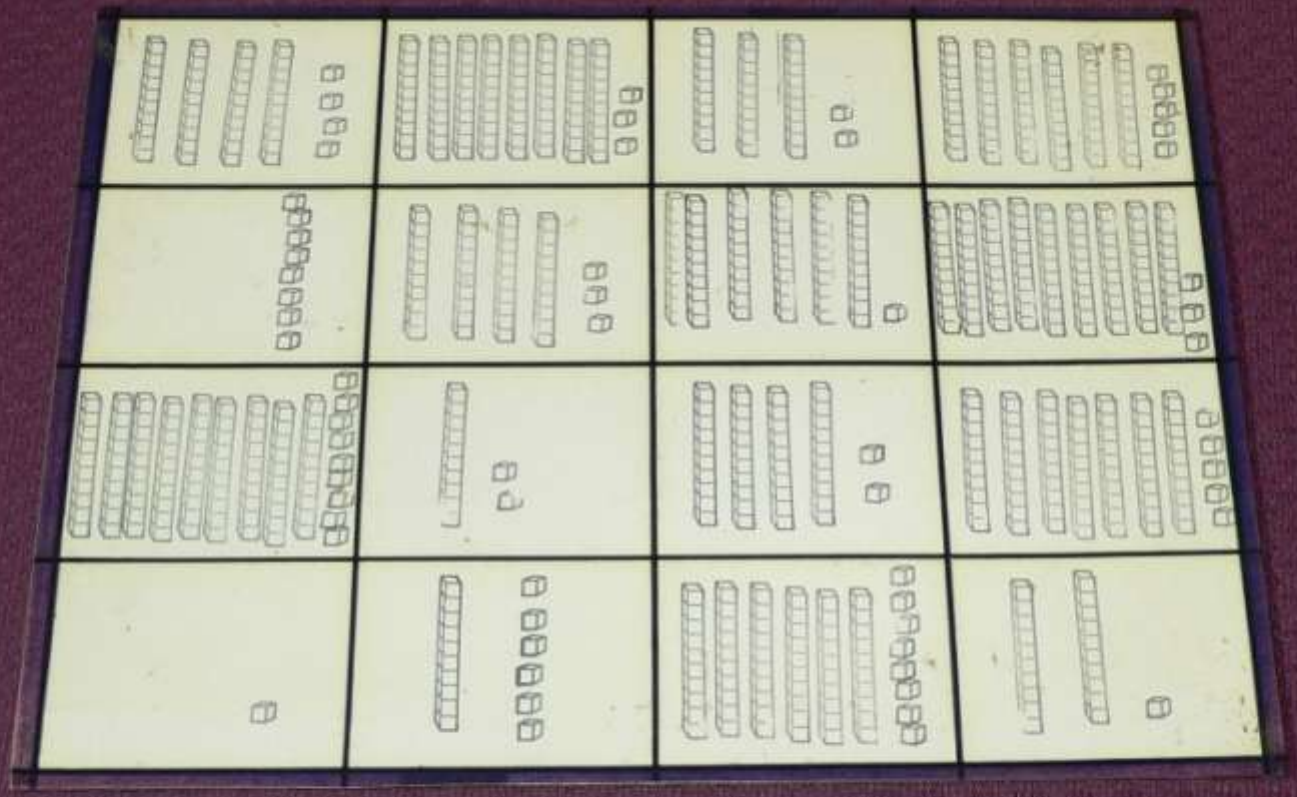
(3 lines, 5 dots = 35)

Shaker beans (dot on one side, line on the other)



(quantity accumulated so far)

Place-Value Bingo



50

8

57

43

32

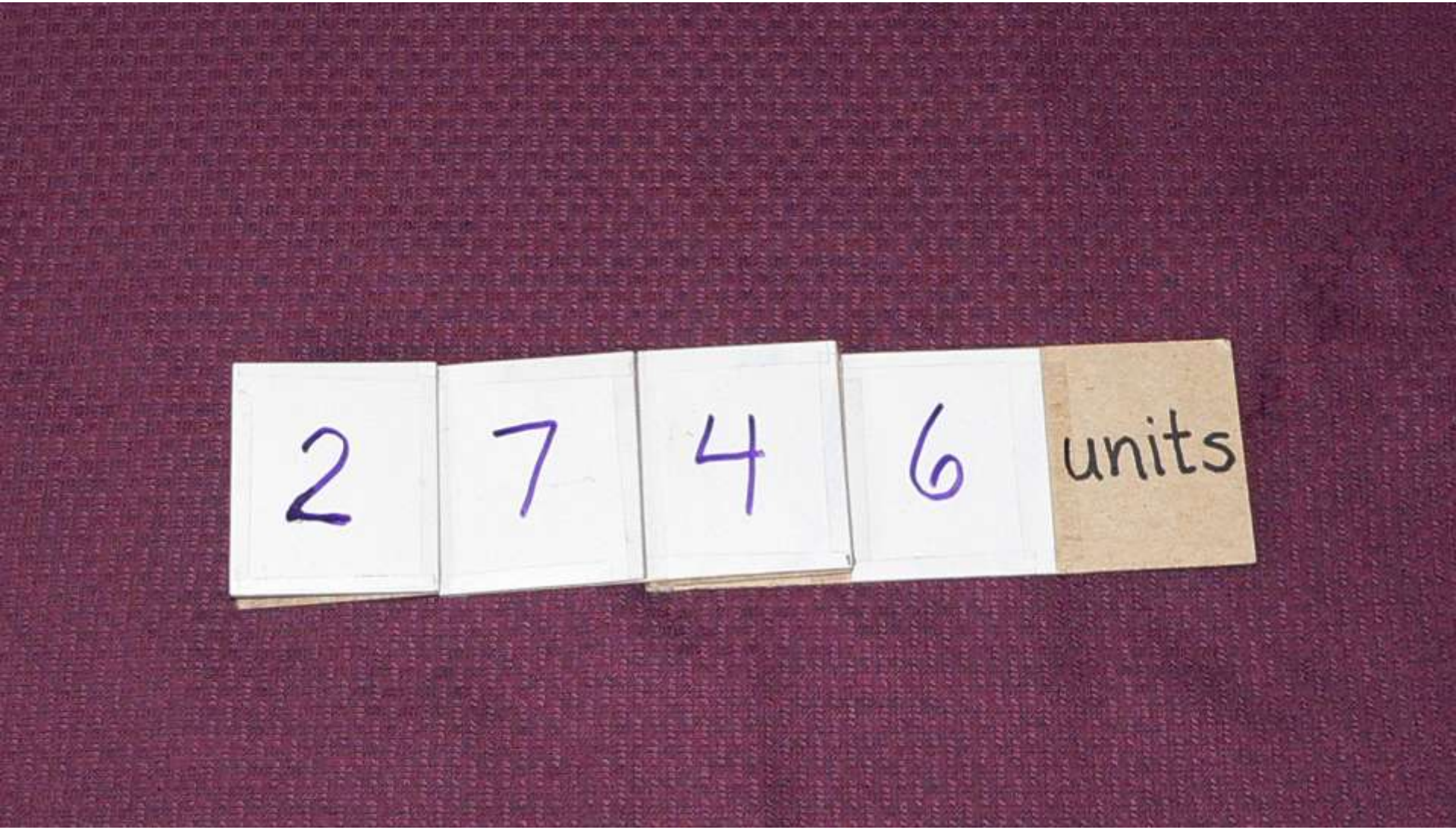

Make-Your-Own
Place-Value Materials


Place Value
Accumulator Tray
(quantity to be
estimated shown)

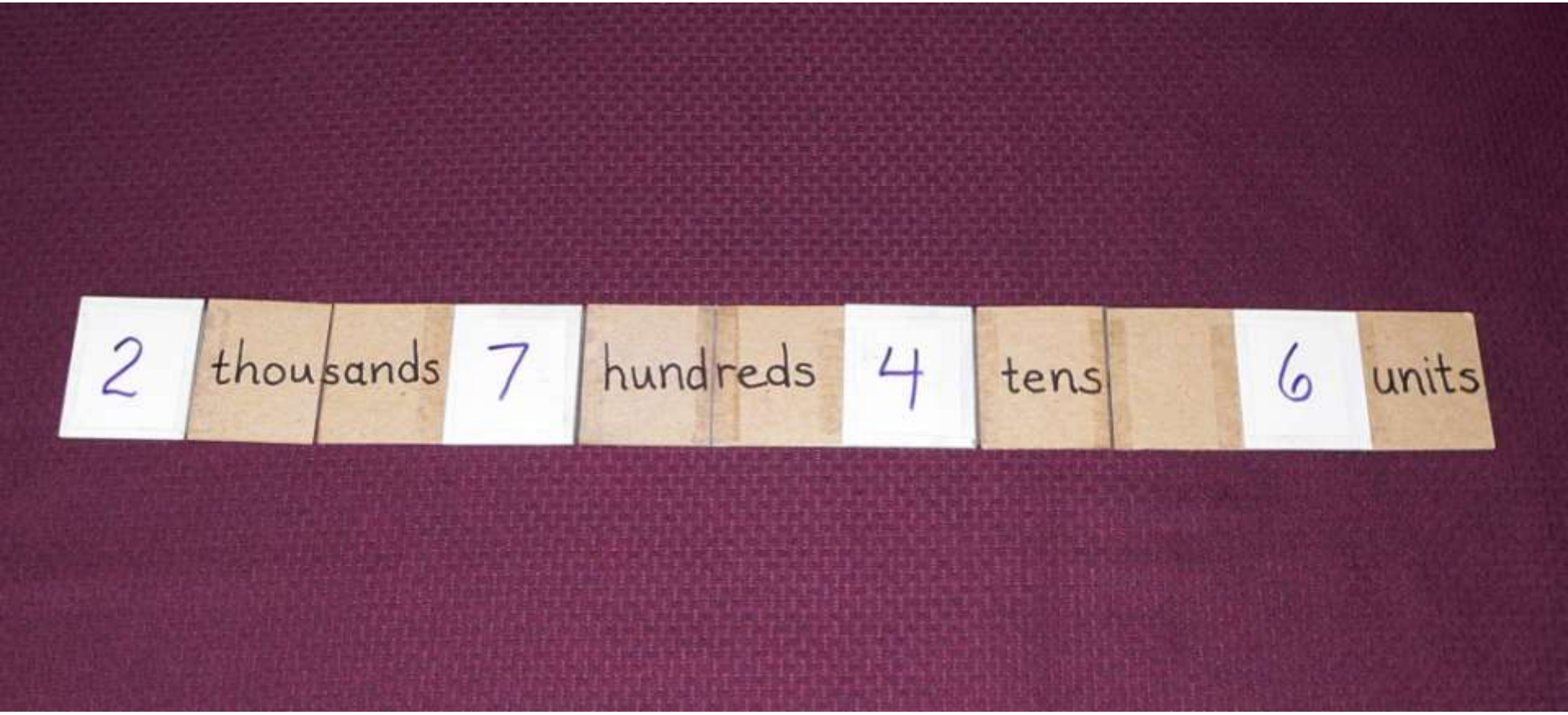
Place Value Accumulator Tray

(quantity shaken into
place-value arrangement)

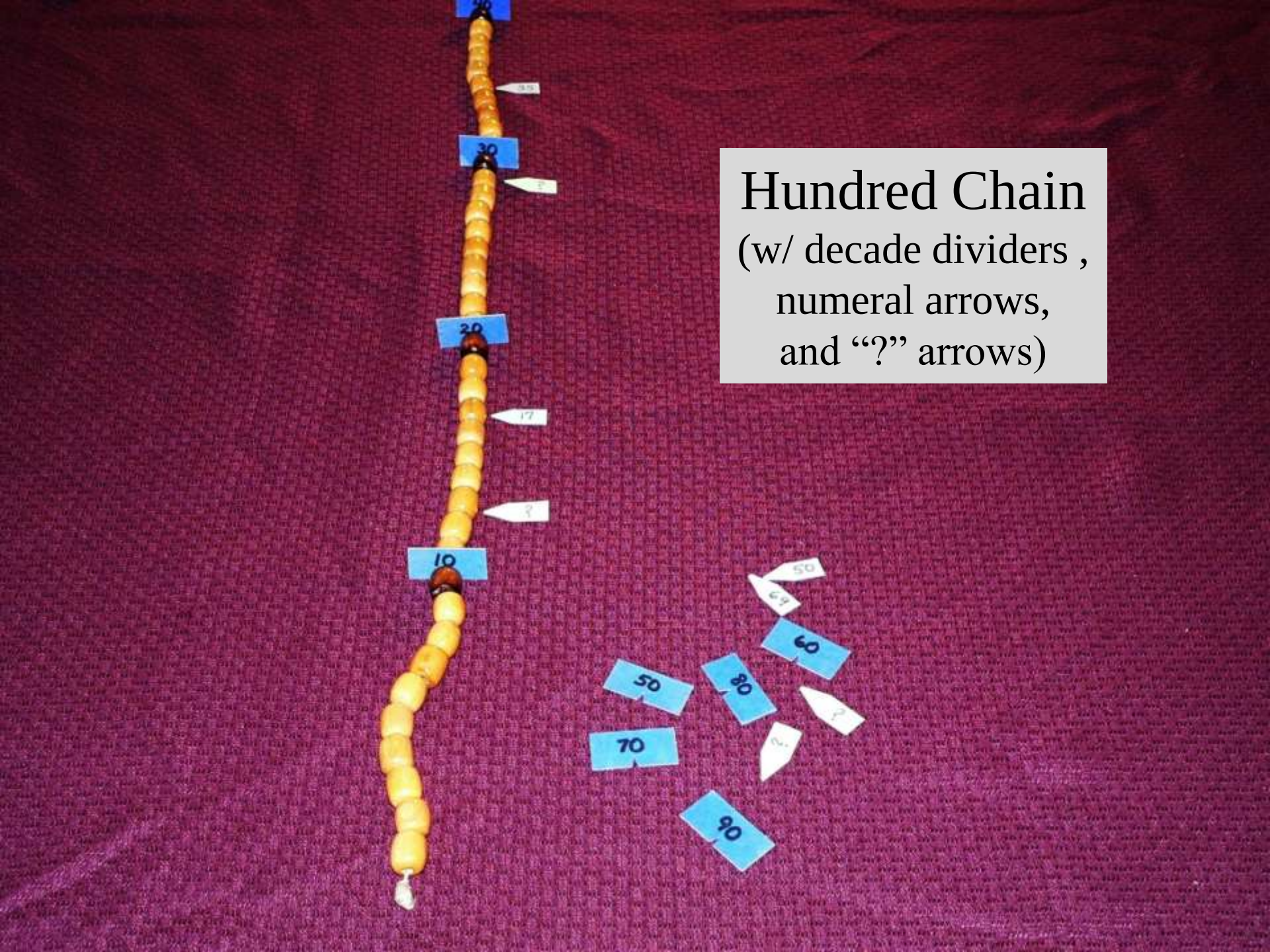
Expanded Notation Strip (folded)



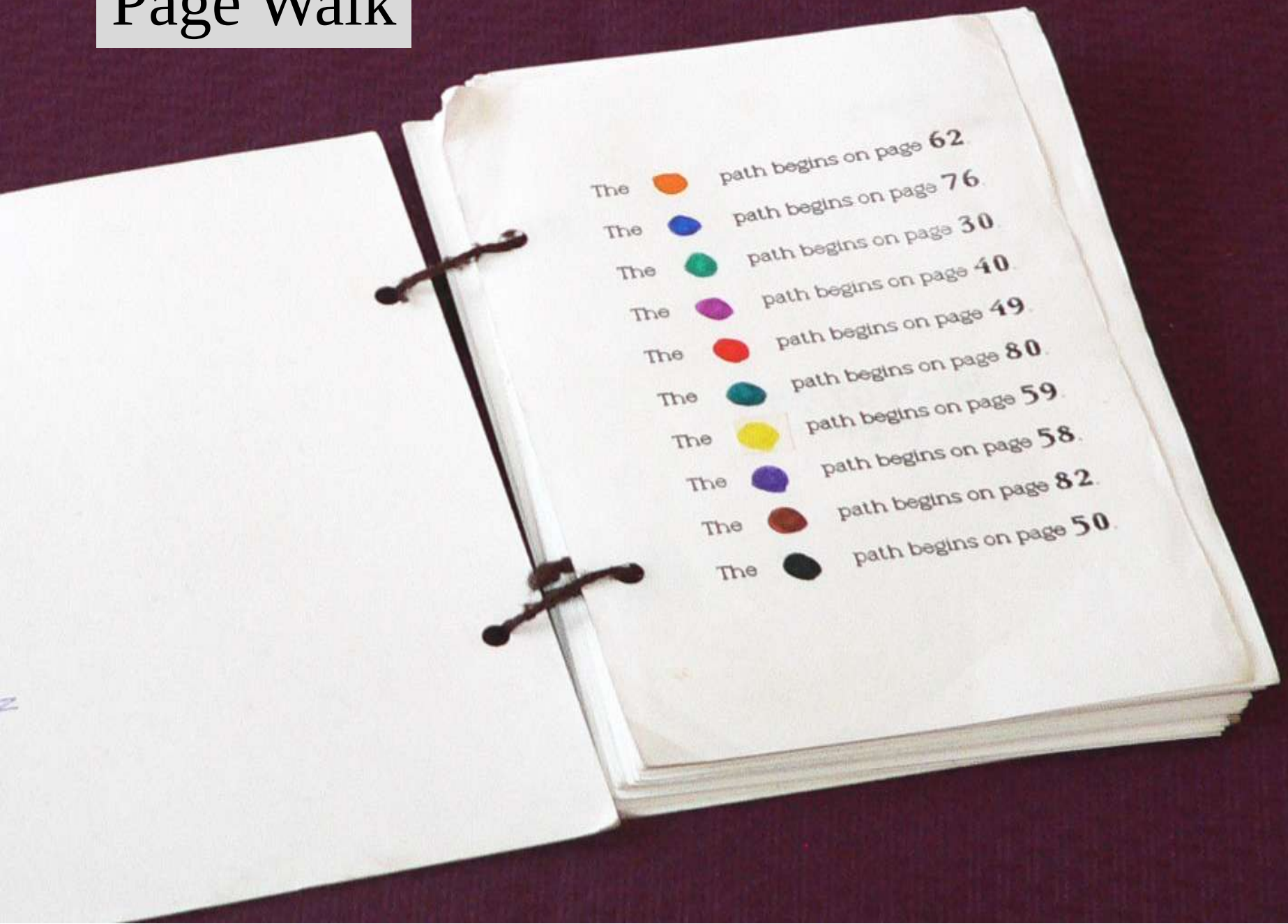
Expanded Notation Strip (unfolded)



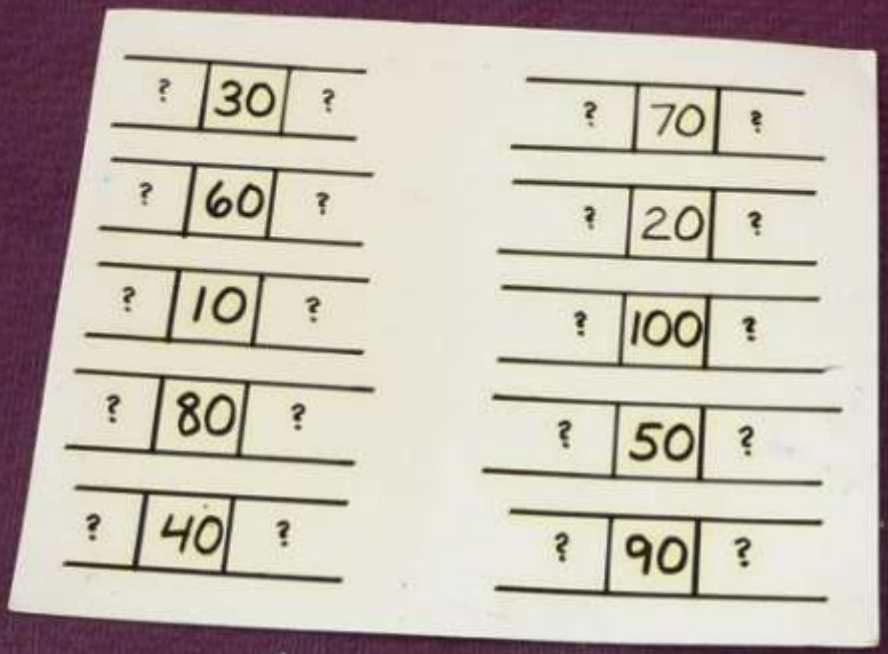
Do Your Children
Really Understand
Place Value?

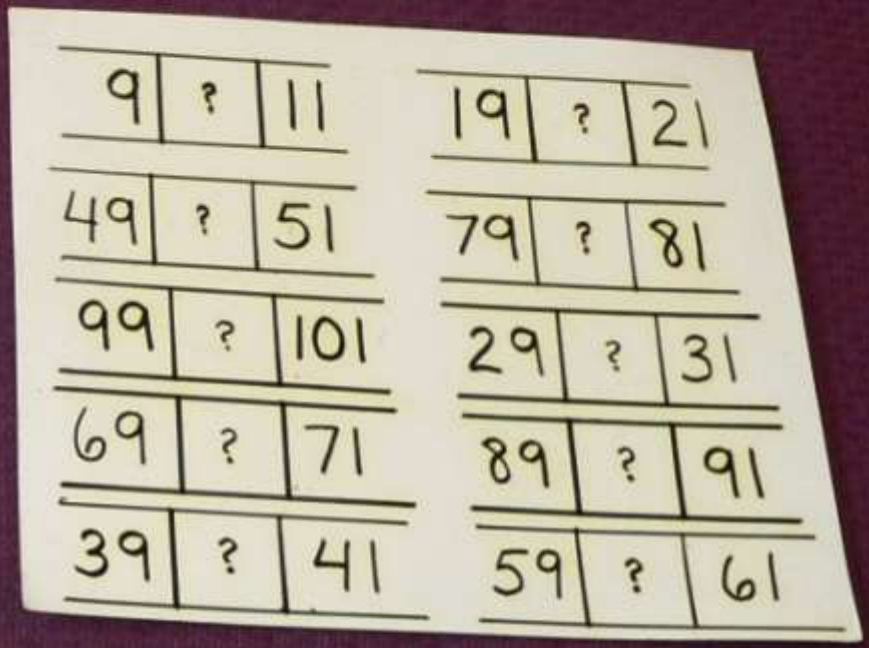

Hundred Chain
(w/ decade dividers ,
numeral arrows,
and "?" arrows)

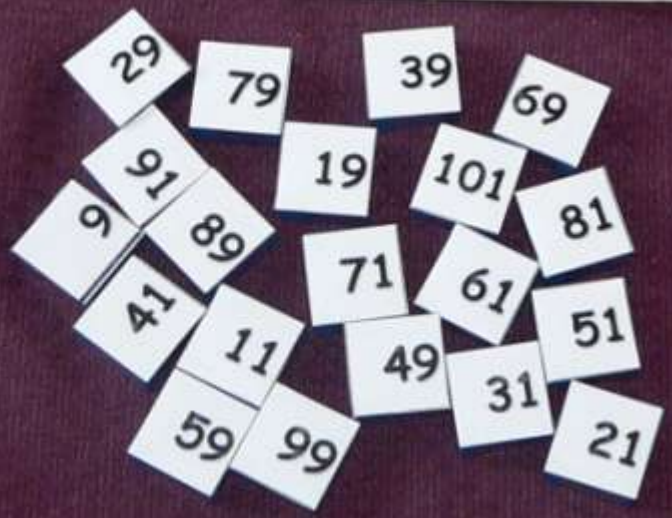
Page Walk

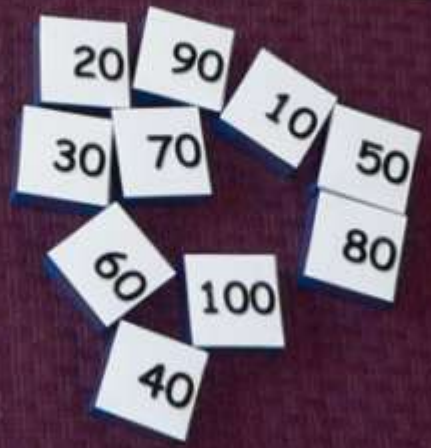

The path begins on page 62.
The path begins on page 76.
The path begins on page 30.
The path begins on page 40.
The path begins on page 49.
The path begins on page 80.
The path begins on page 59.
The path begins on page 58.
The path begins on page 82.
The path begins on page 50.

Decade Transition Boards

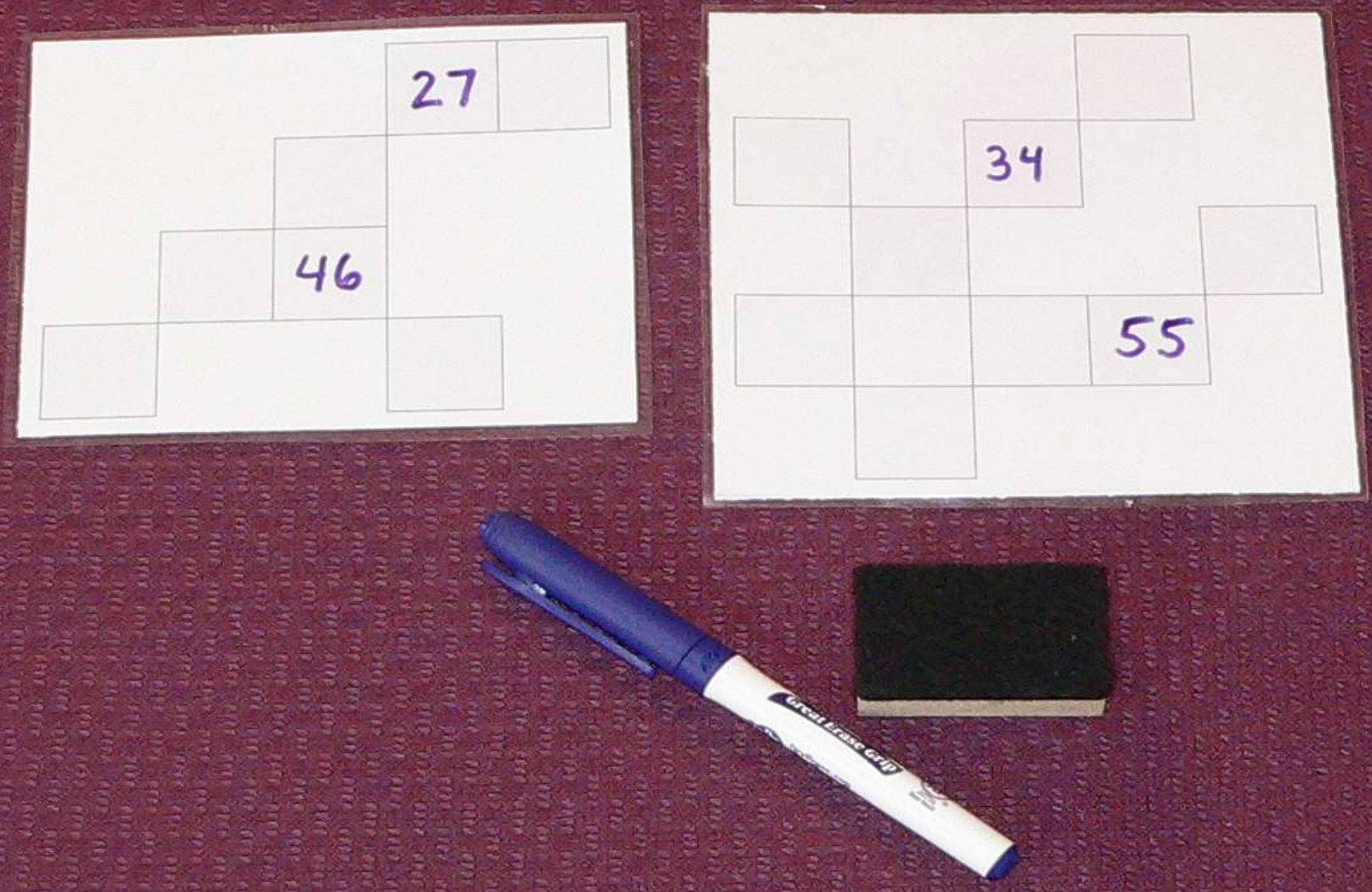








Magic Windows



Encourage Use of

Base-Ten Strategies

In Mentally Computing Sums

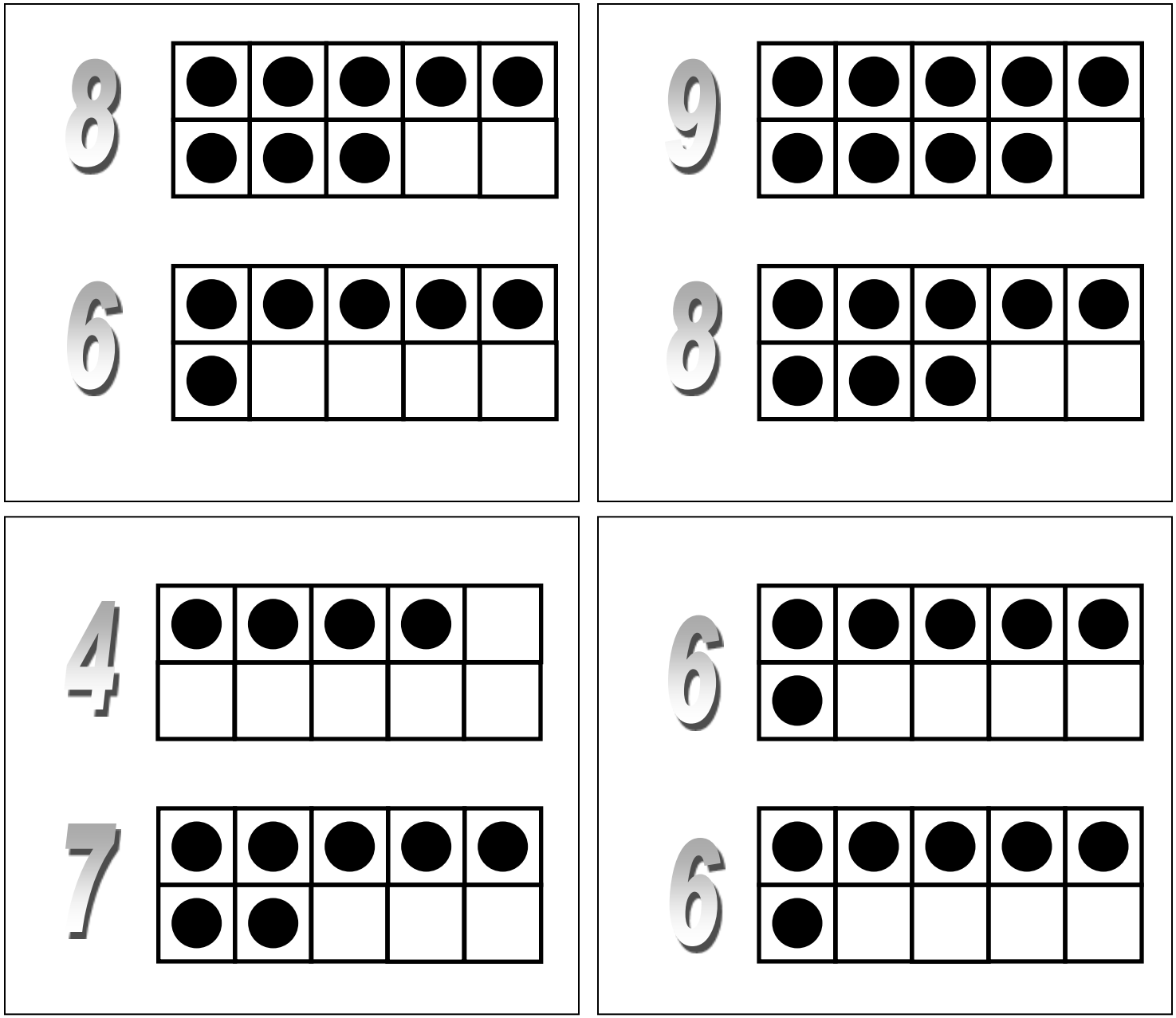

Converting Sums
To Teen Numbers on
Cuisenaire Hundreds Rack


Ten-Frame
Addition
(two addends)

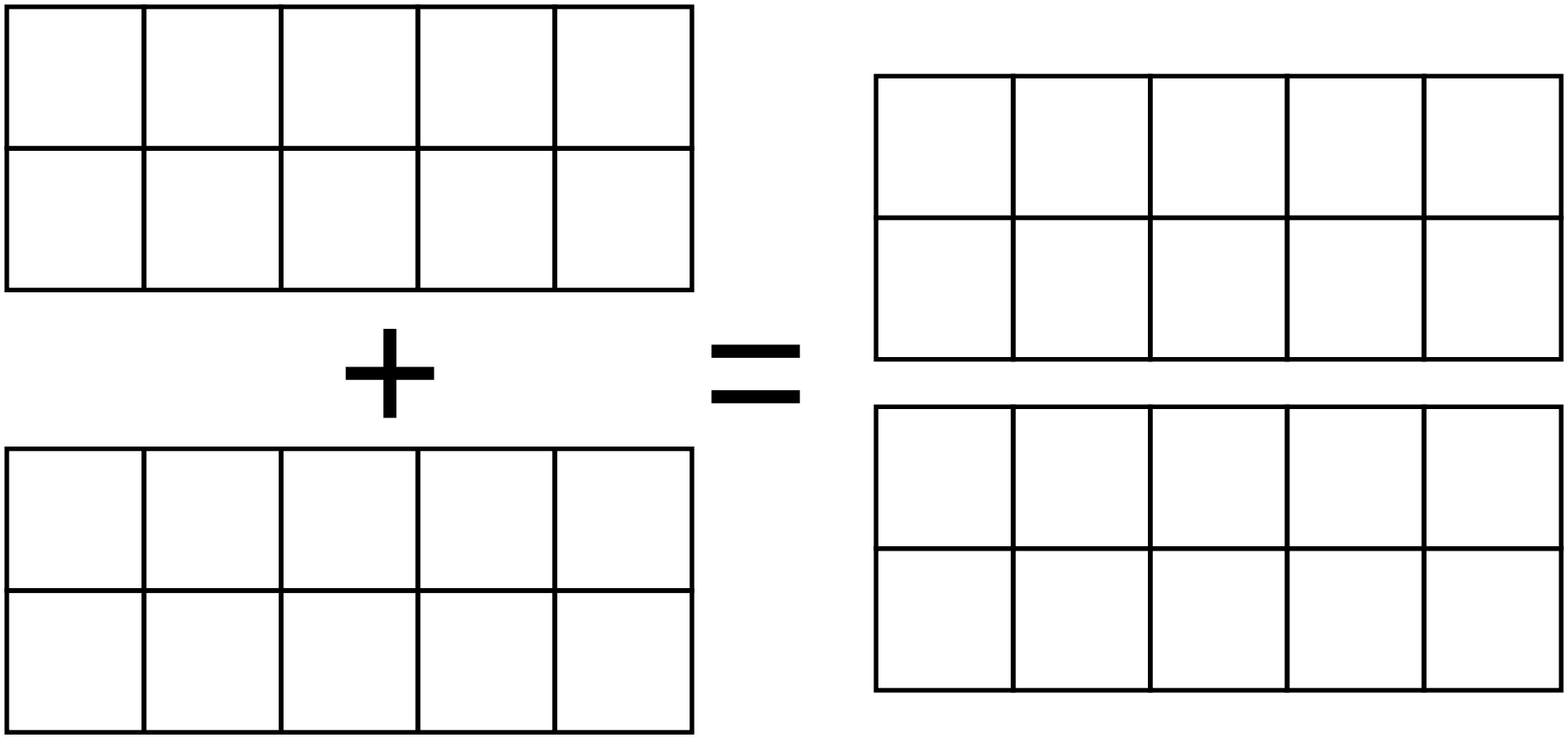
Ten-Frame Addition

(sum)

Ten-Frame Addition Flash Cards (sums on back, in numeral and ten-frame notation)

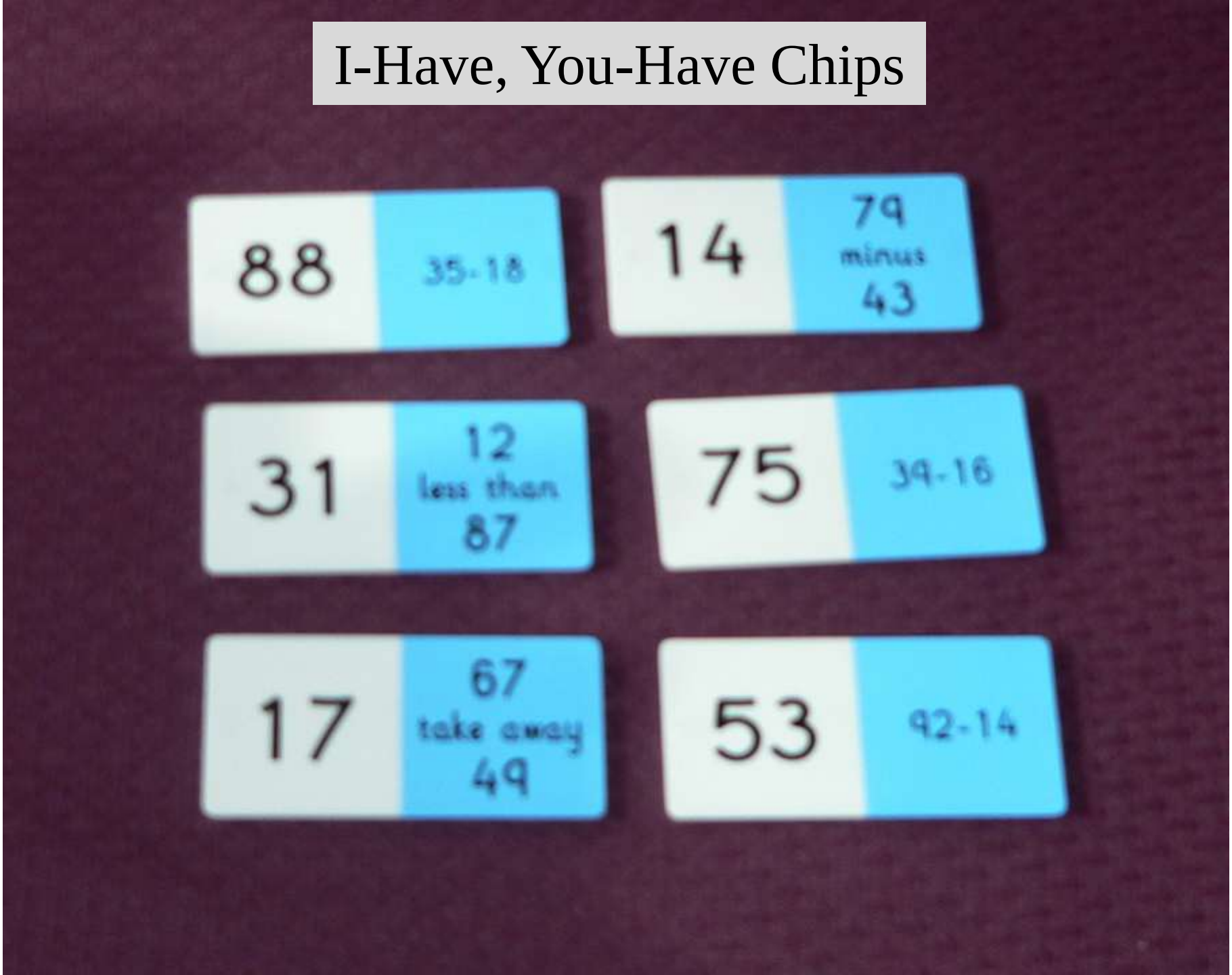


Ten-Frame Addition Worksheet



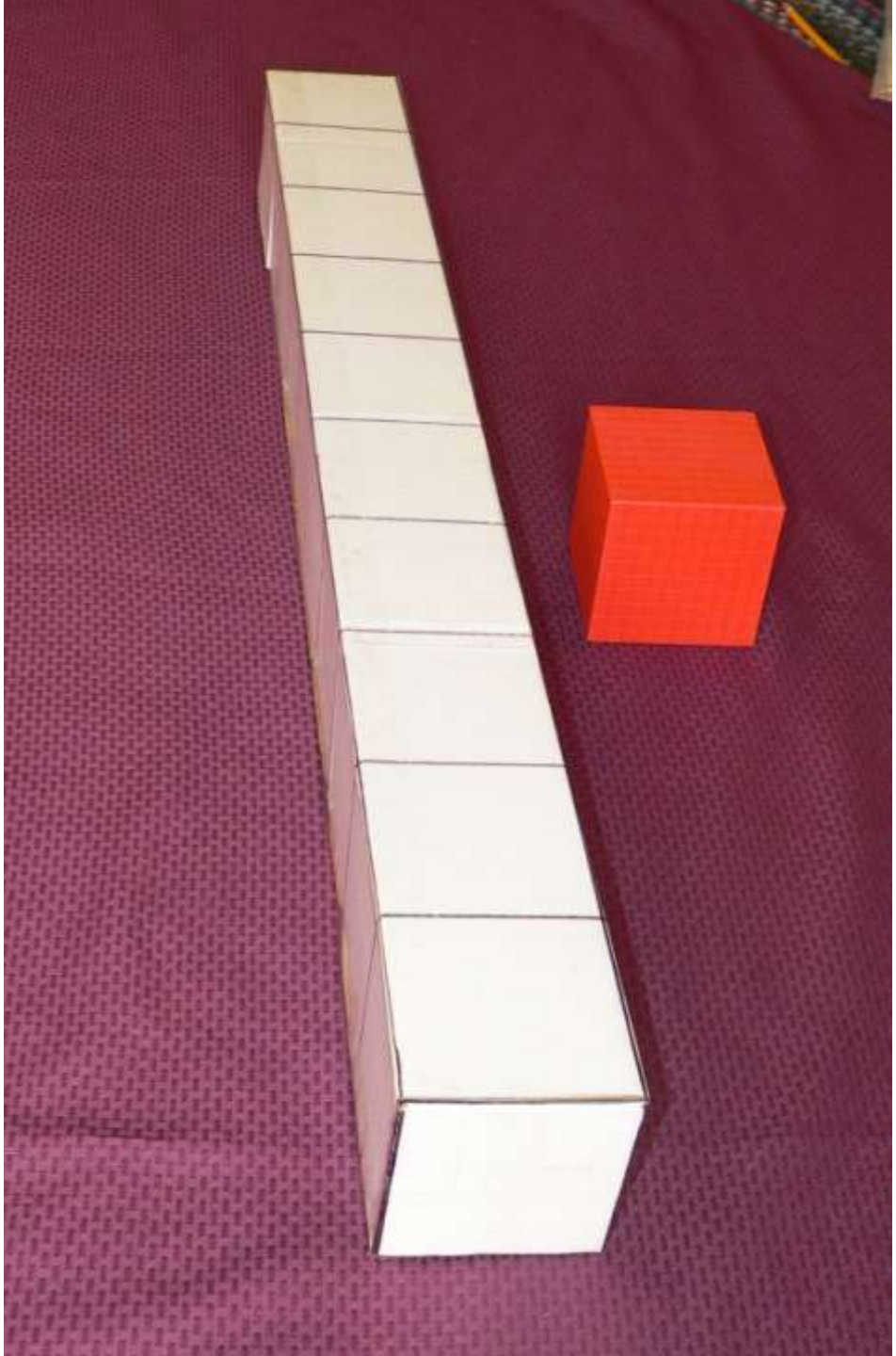
___ + ___ = ___

I-Have, You-Have Chips

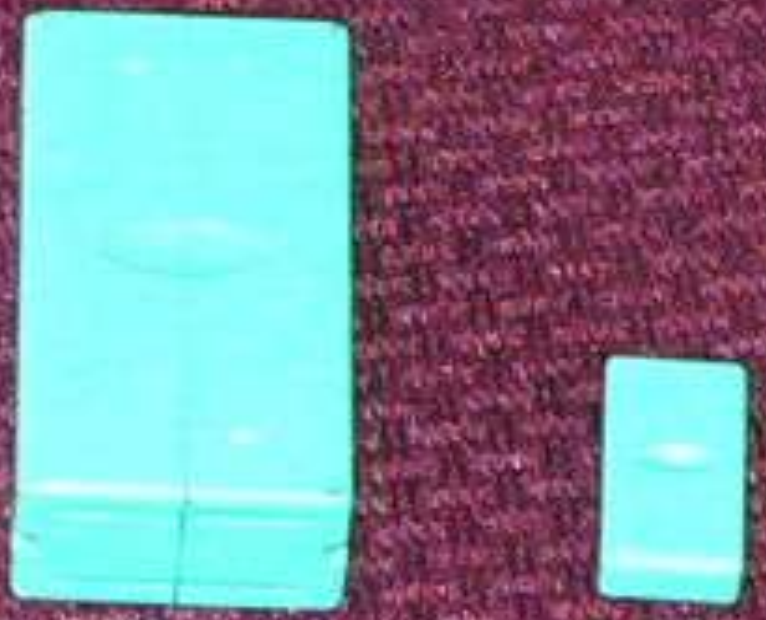


Other Useful Base-Ten Models

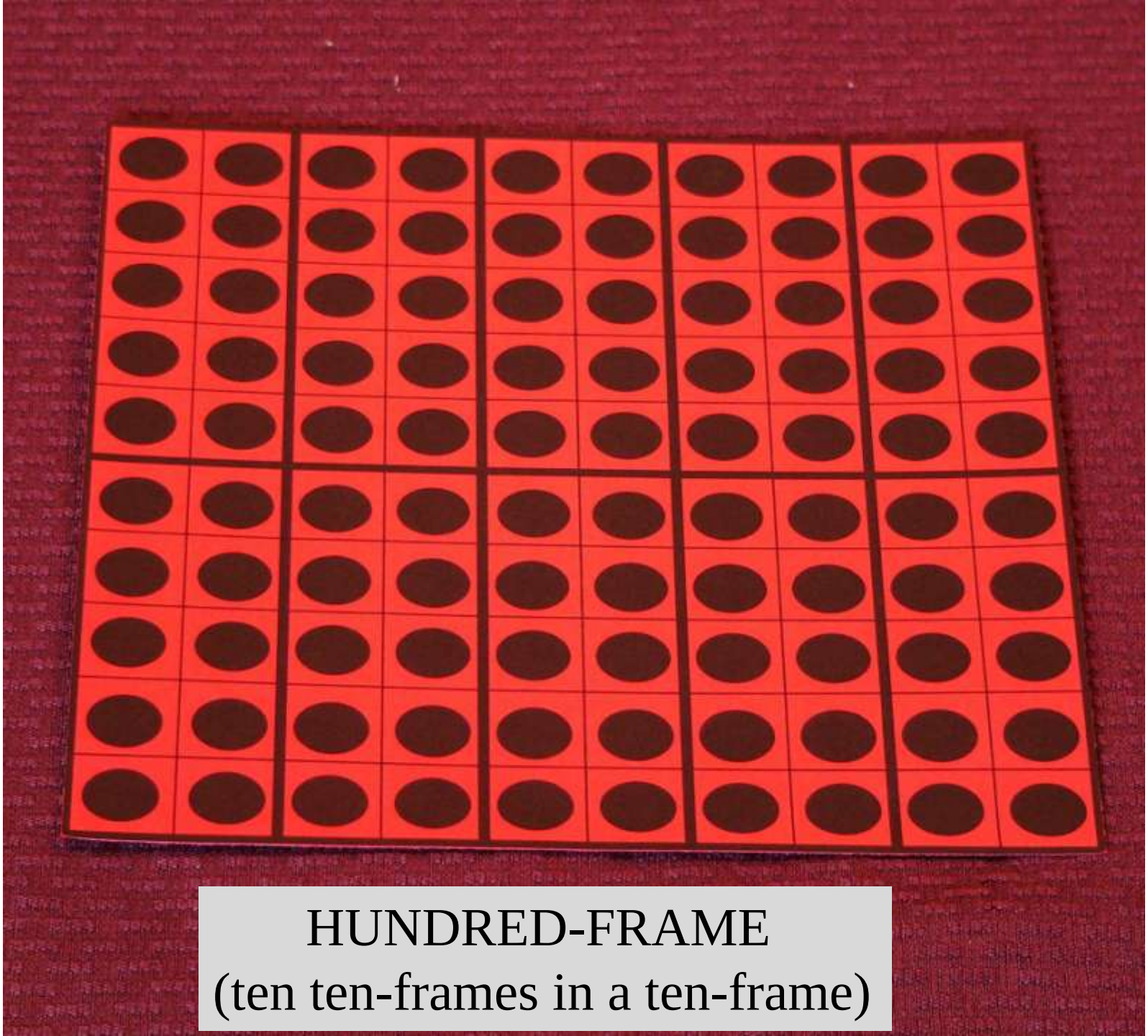
Ten-Thousand Bar



Digi-Blocks (one and ten)



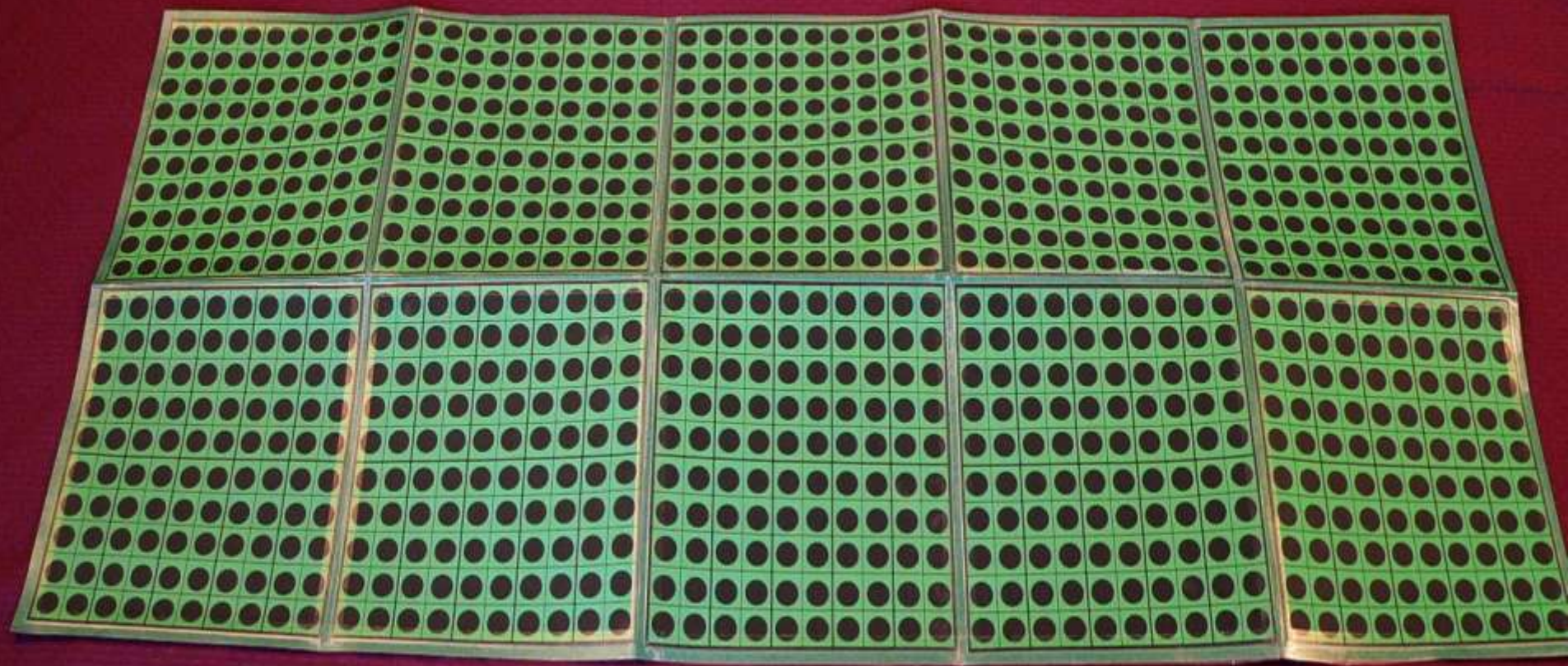
Digi-Blocks
(ten ones filling the ten)



HUNDRED-FRAME
(ten ten-frames in a ten-frame)

Thousand-Frame

(ten hundred-frames in a ten-frame)

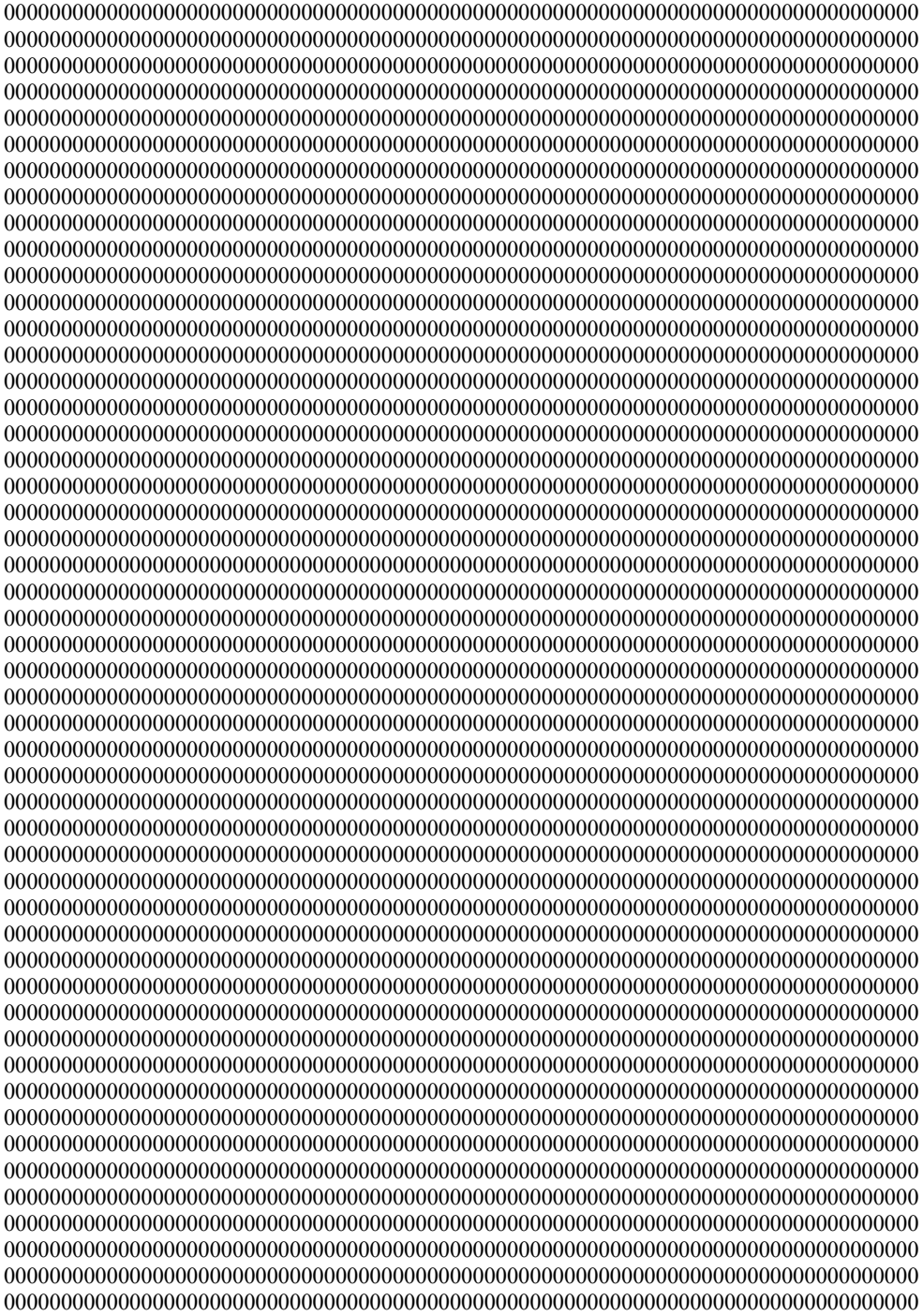


HUNDRED-FRAME

(ten ten-fingers cards in a ten-frame)

THOUSAND-FRAME

(ten hundred-frames in a ten-frame)



4,000 zeroes
(one page of
Book of One Million –
which will contain
250 of these pages!)

Books Dealing With Big Numbers

Carter (2007). *600 Black Spots.*

Friedman. (1994). *The King's Commissioners.*

Hertzberg (1993). *One Million*.

Michelson (2000). *Ten Times Better.*

Milbourne & Riglietti (2007). *How Big is a Million?*

Morrison & Morrison (1982). *Powers of Ten: About the Relative Size of Things in the Universe***.**

Neuschwander (2009). *Sir Cumference and All the King's Tens.*

Nolan (1995). *How Much, How Many, How Far, How Heavy, How Long, How Tall is 1000?*

Pilegard (2001). *The Warlord's Beads*.

Pollotta (2003). *Count to a Million*.

Sayre & Sayre (2003). *One is a Snail, Ten is a Crab.*

Schwartz (1985). *How Much is a Million?*

Schwartz (1989). *If You Made a Million.*

Schwartz (1999). *On Beyond a Million: An Amazing Math Journey.*

Schwartz (2003). *Millions to Measure.*

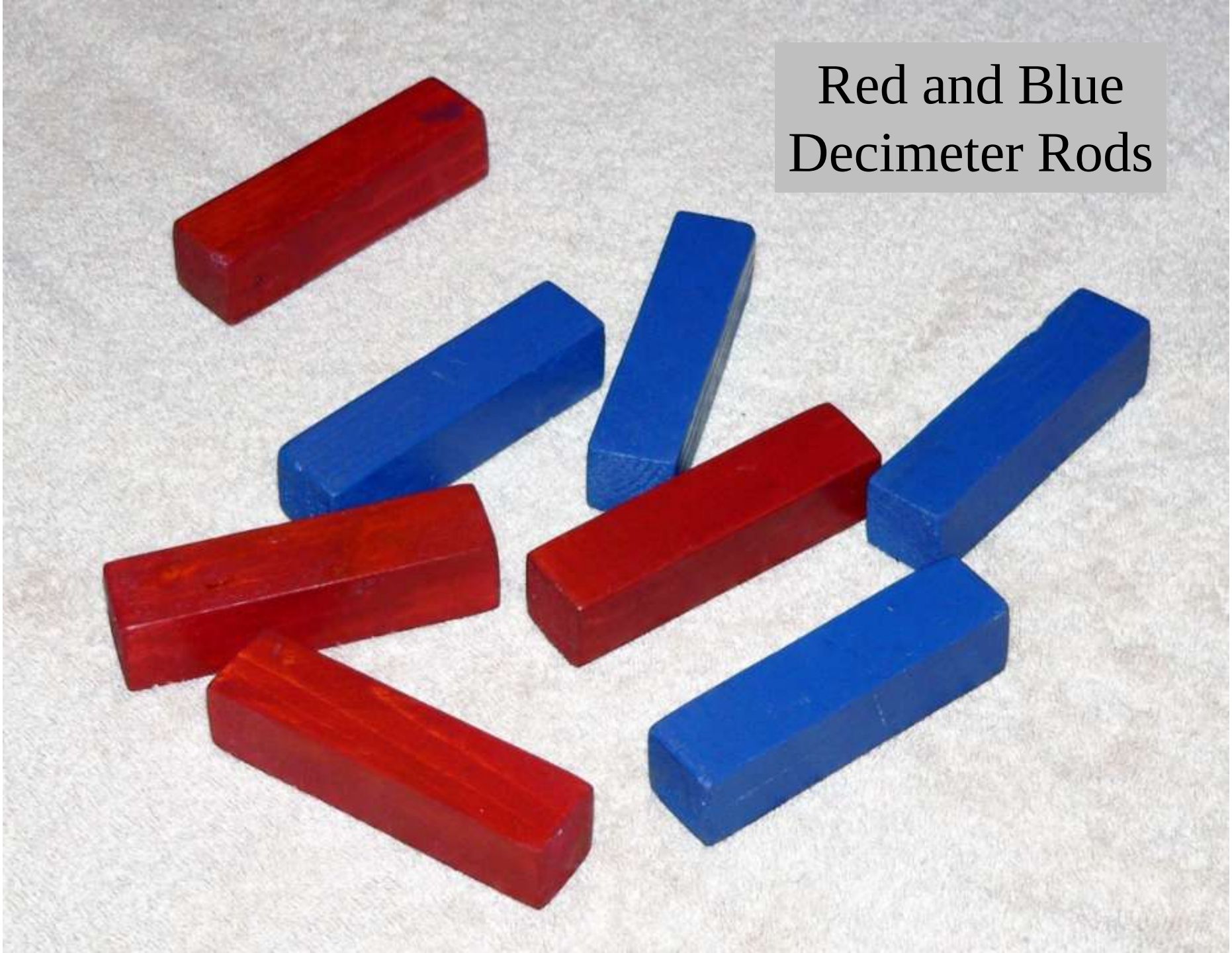
Wells (2000). *Can You Count to a Googol?*

Other Opportunities to Practice Base-Ten Awareness:

- graphing
- metric measurement

Ten Frame Trays

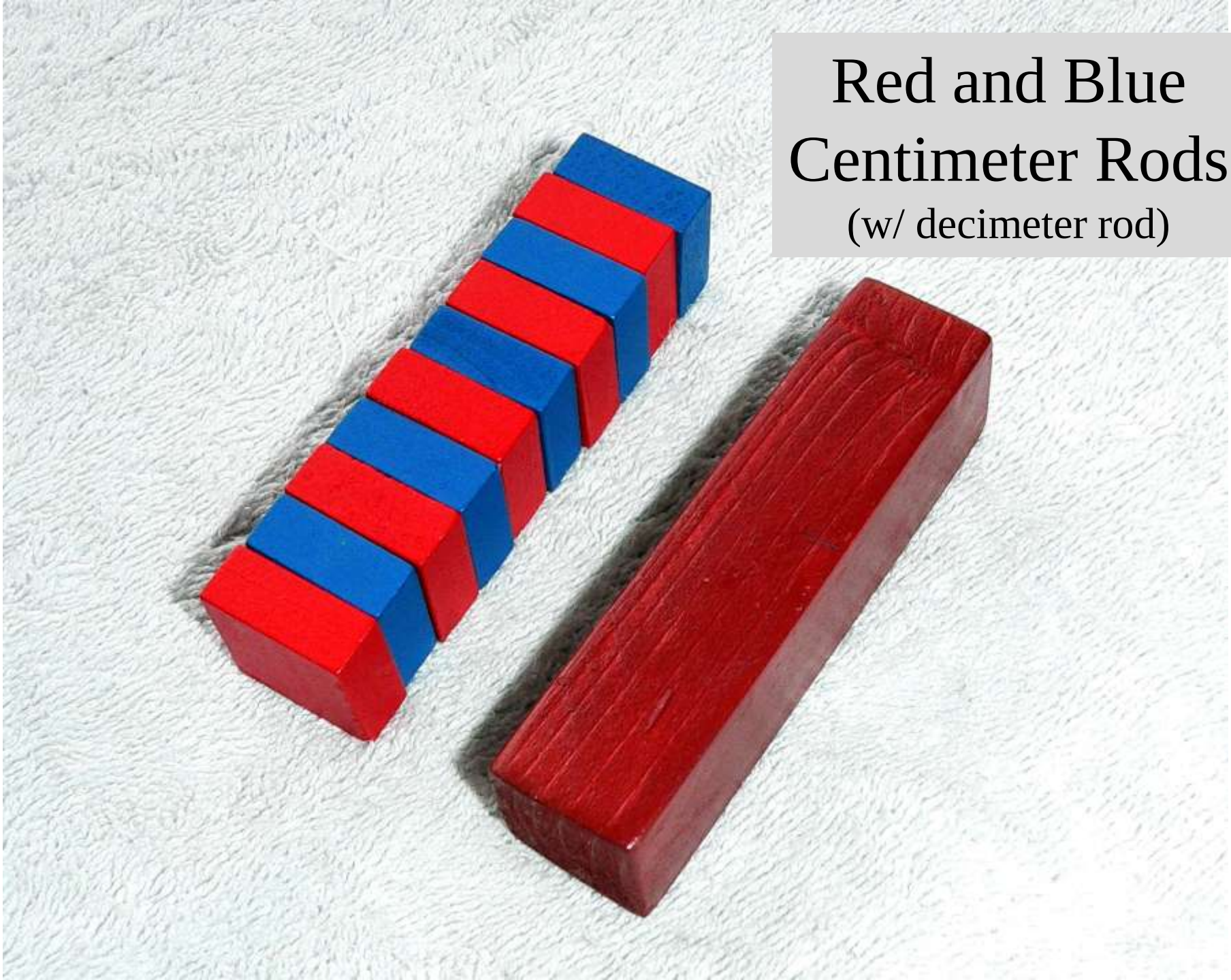
used for
sorting and
graphing


Red and Blue
Decimeter Rods

Red and Blue
Meter Rods

Red and Blue Centimeter Rods

(w/ decimeter rod)



Measuring Railing:

2 meters

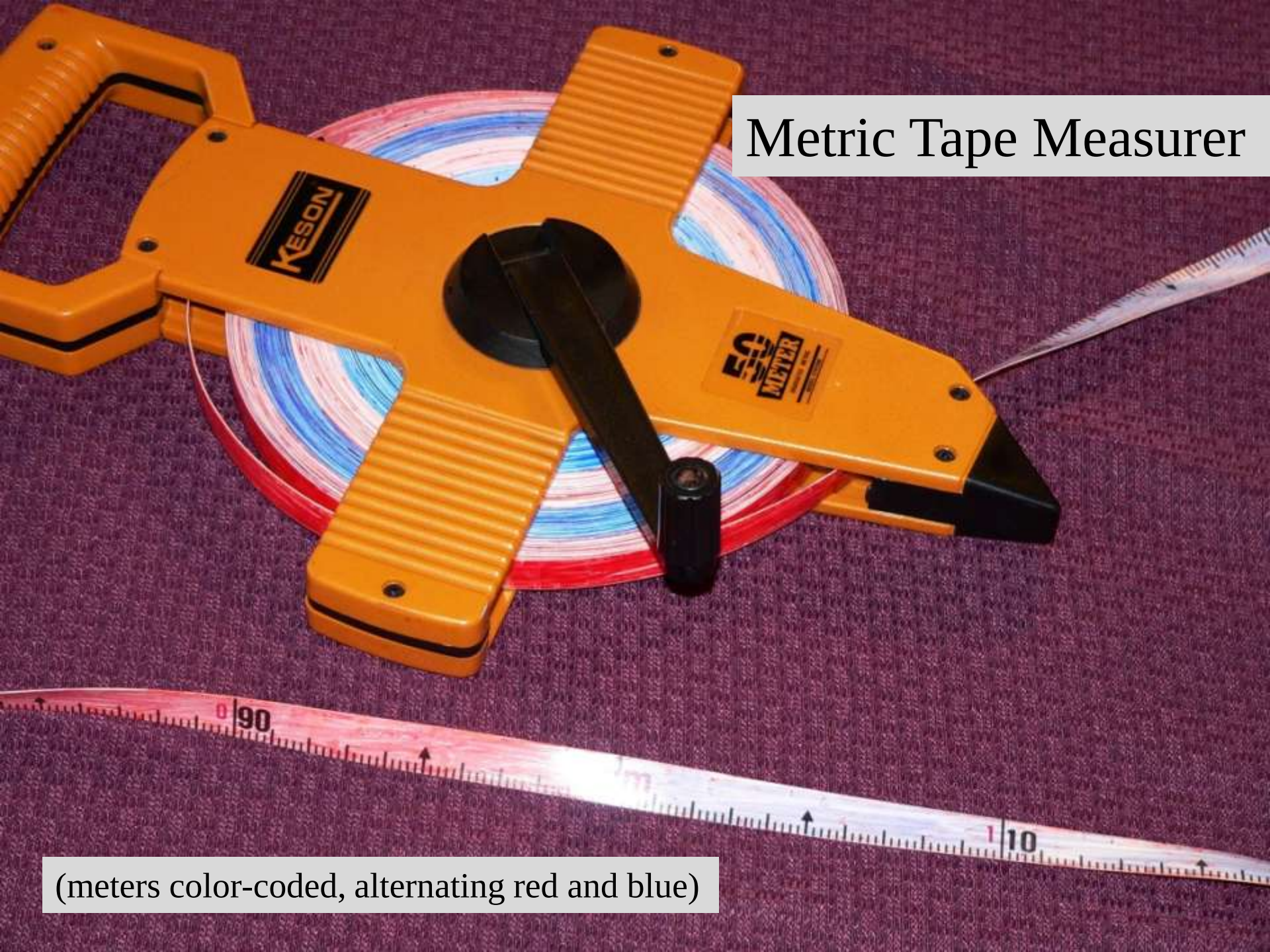
+

8 decimeters

+

5 centimeters

(= 2.85 meters!)



Metric Tape Measurer

(meters color-coded, alternating red and blue)

Keep Place-Value Sense Alive as Move into Multi-digit Operations

Strategy Prompts

$142 + 499$

$199 + 442$

$449 + 197$

$397 + 205$

TARGET

– Addition and Subtraction Estimation –

Task: Can you quickly estimate within 20 of the actual answer? Within 10?

176 + 78

(actual answer: 254)

516 + 235

(actual answer: 751)

868 - 89

(actual answer: 779)

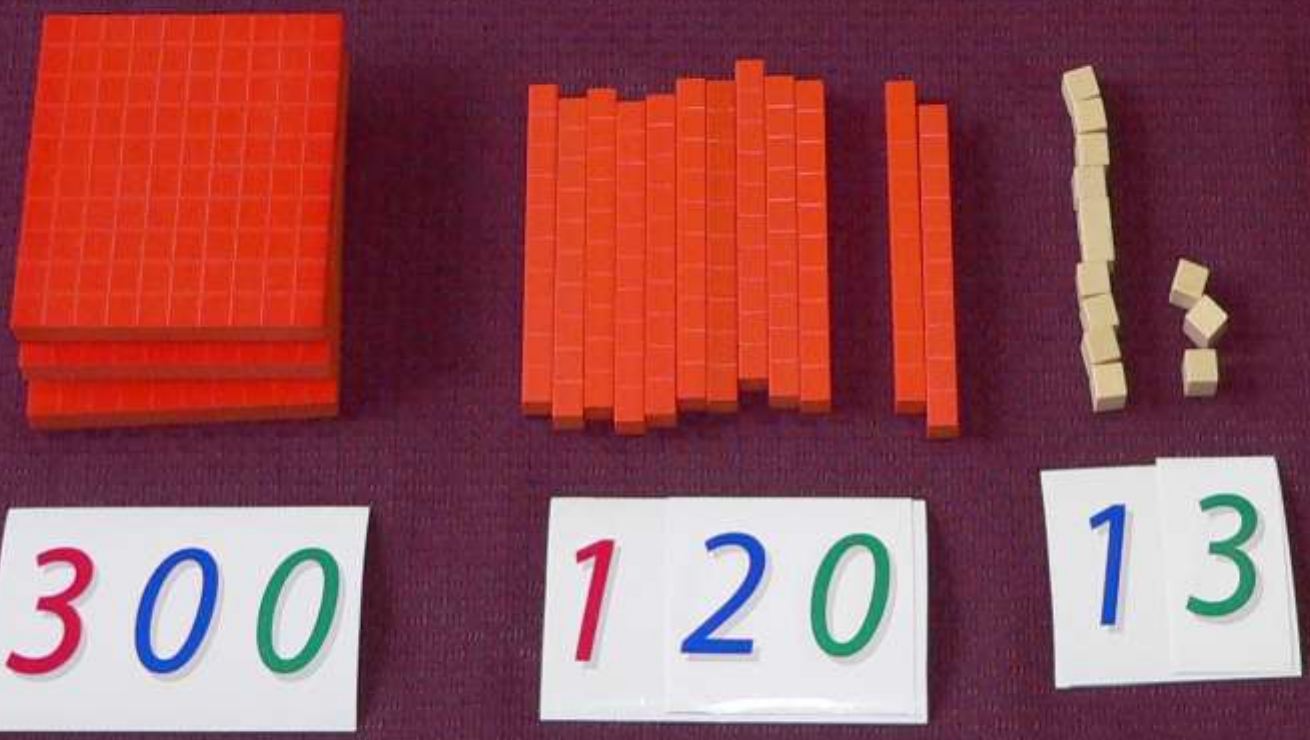
1048 - 769

(actual answer: 279)

10,293 - 996

(actual answer: 9,297)

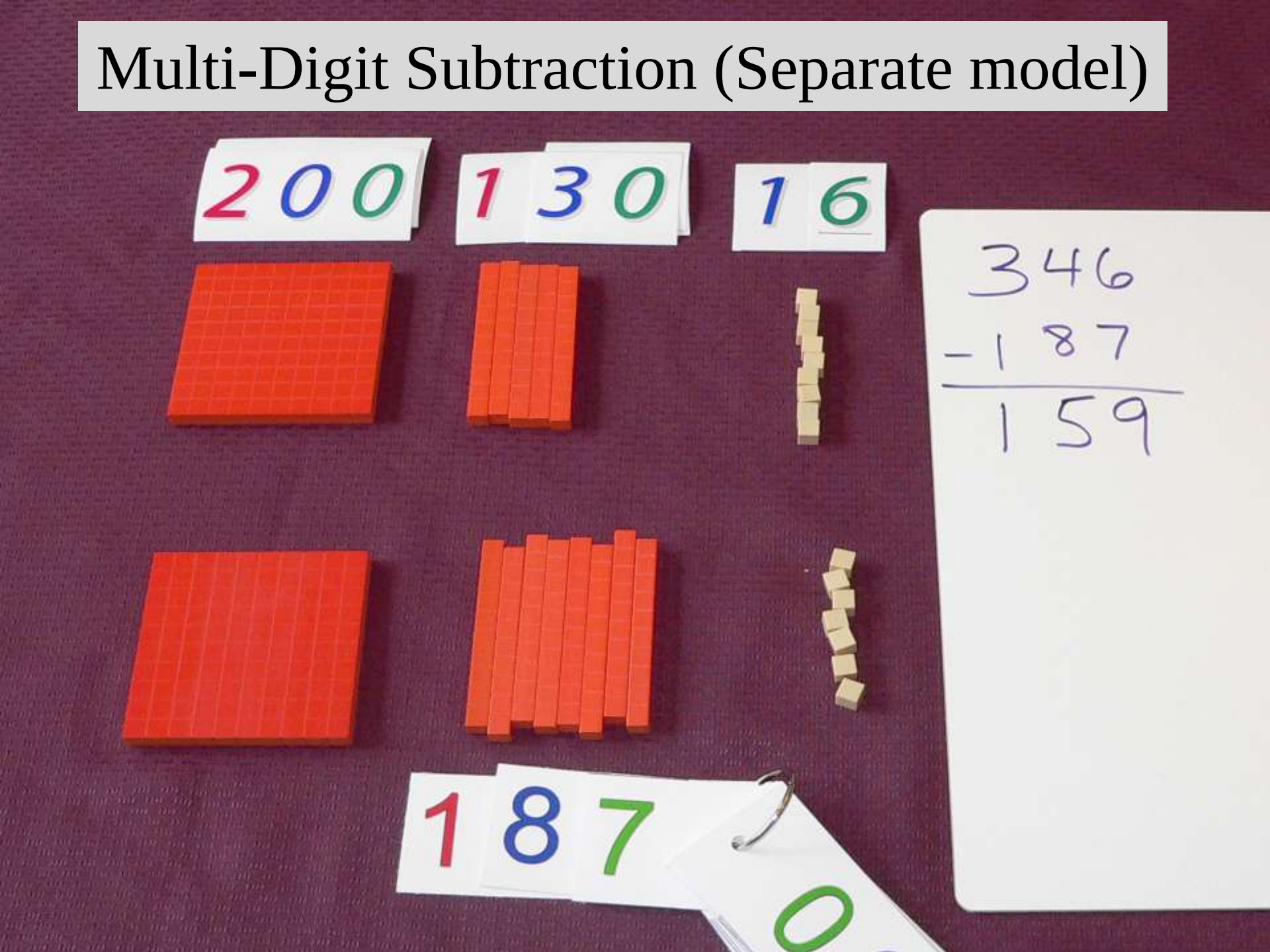
Multi-Digit Addition (Combine model)



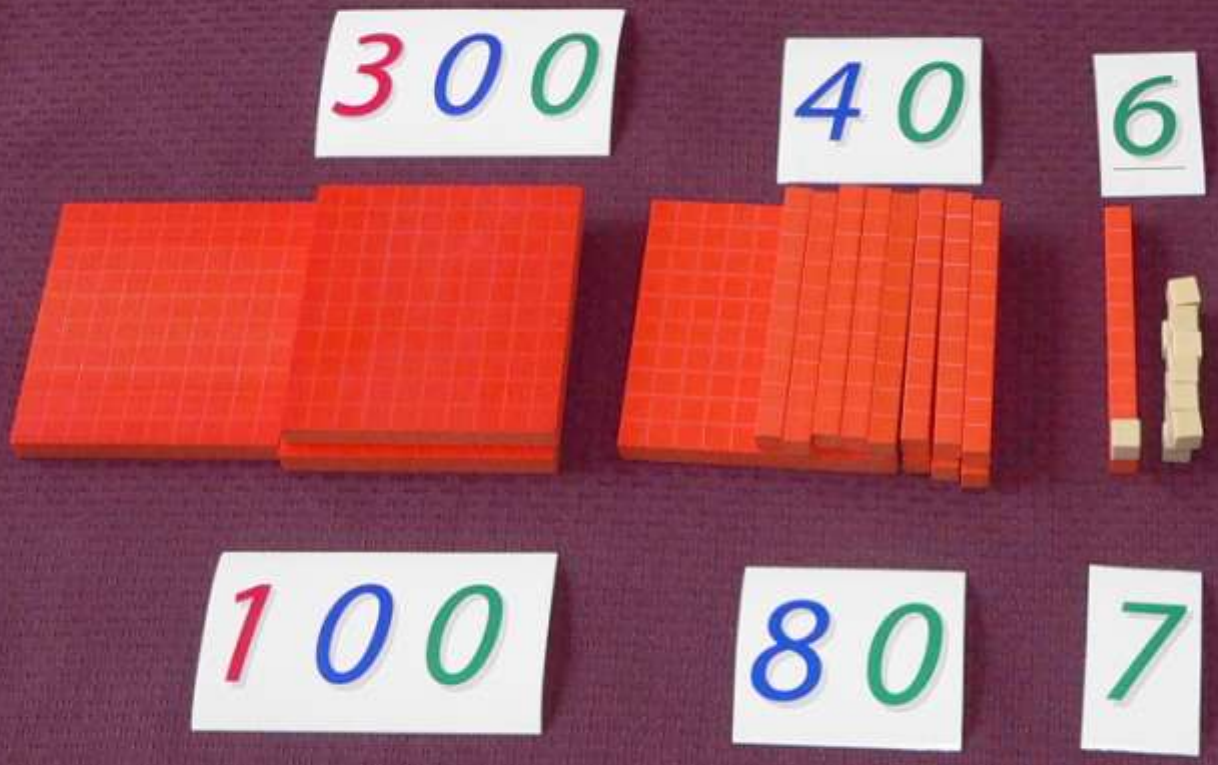
287
+146

13
120
+300

Multi-Digit Subtraction (Separate model)



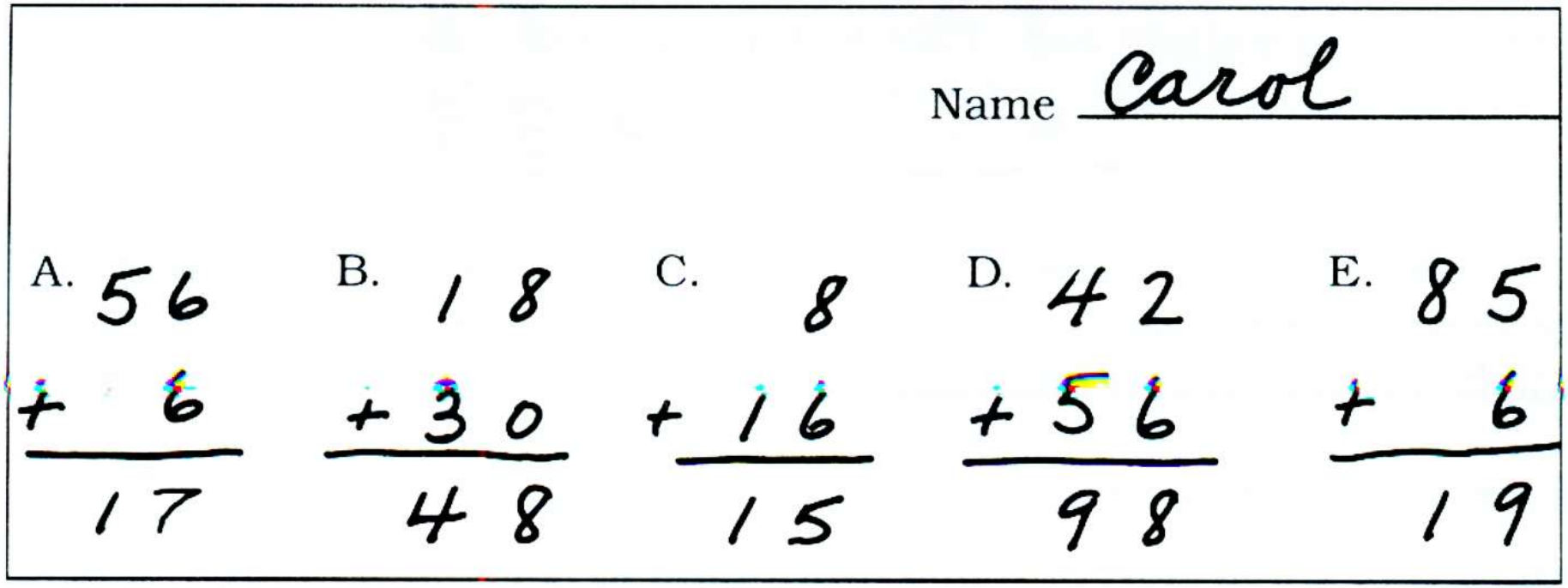
Multi-Digit Subtraction (Compare model)



346
-187

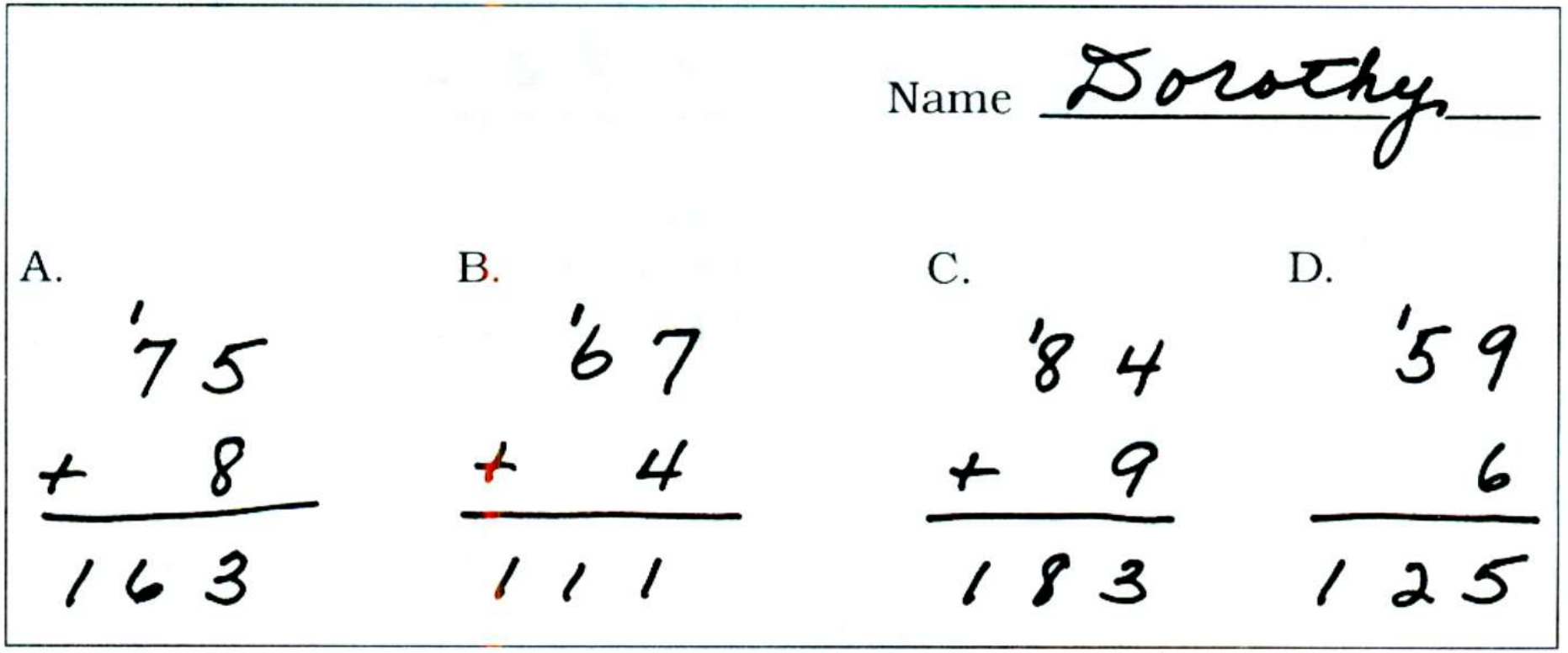
9
50
100

What's wrong with this picture?
Why would a child put down such silly answers?
We need to make sure we don't run classrooms that elicit this type of mathematics.



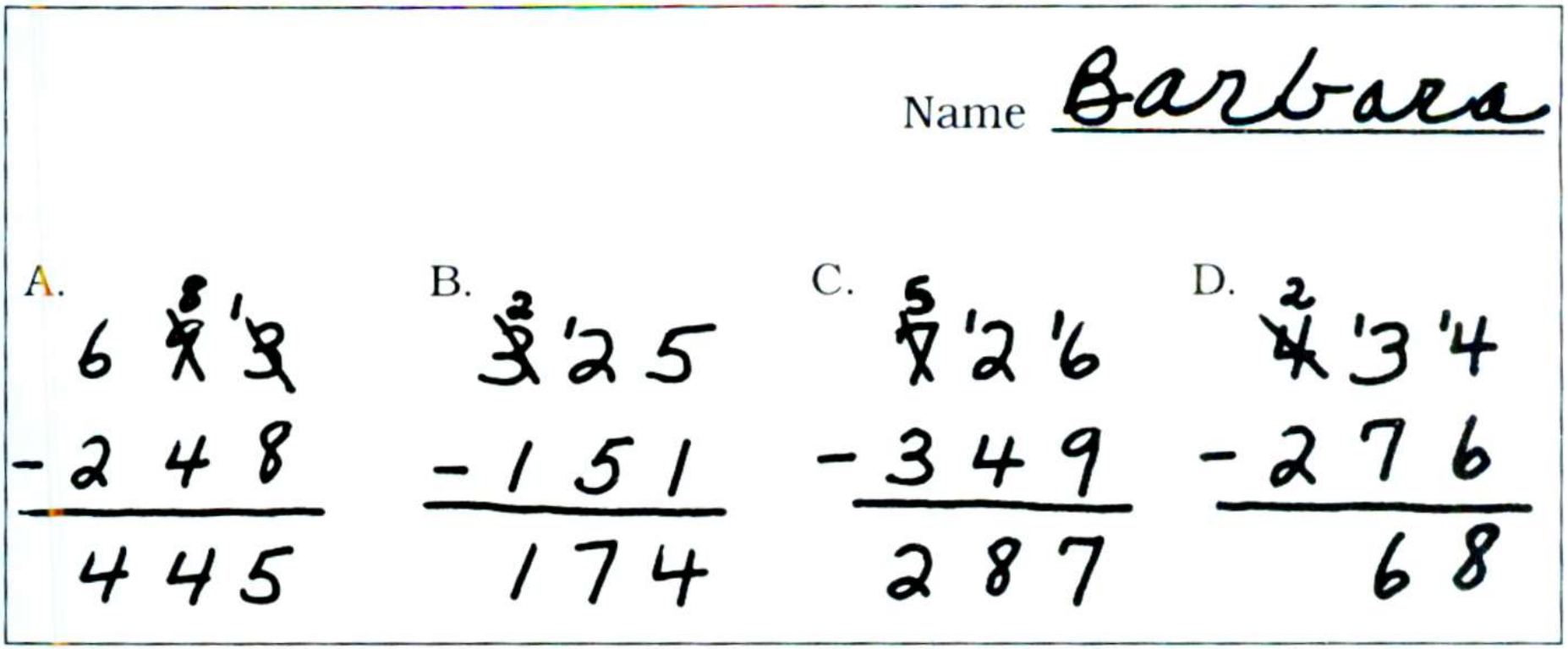
(from Ashlock, *Error Patterns in Computation*, 8th Ed., 1994).

..here we go again



(from Ashlock, *Error Patterns in Computation*, 8th Ed., 1994).

You get the point – these children are trapped in an endless loop of making what I call *algorithm salad*



(from Ashlock, *Error Patterns in Computation*, 8th Ed., 1994).

An even bigger part of the solution…

How We Talk to Children
(and how we encourage them
to talk to each other)

- BUILDING A COMMUNITY OF MATHEMATICIANS -

The Socio-Mathematical Norms that produce

Academic Press

Valuing:

1. **EXPLANATIONS** – Explanations keep the focus on mathematical arguments, not procedural summaries.
2. **ERRORS** – Errors and differences of opinion are treated as valuable opportunities to explore misconceptions, contradictions, and alternative strategies.
3. **MULTIPLE STRATEGIES** – Value is placed on using multiple strategies and understanding the relationships among them.
4. **COLLABORATION** – Judgments of "correctness" are based on reaching consensus and shared understanding through mathematical argumentation that values each individual's contribution and understanding.

(from Kazemi, "Discourse that promotes conceptual understanding." TCM. 1997).

"...of all the virtues related to intellectual functioning, the most passive is the virtue of knowing the right answer. Knowing the right answer requires no decisions, carries no risks, and makes no demands. It is automatic. It is thoughtless....

The virtues involved in <u>not</u> knowing are the ones that really count in the long run. What you do about what you don't know is, in the final analysis, what determines what you will ultimately know."

Eleanor Duckworth, *"The Having of Wonderful Ideas" and Other Essays on Teaching and Learning*

Other Aspects of Scaffolding Children's Thinking (One-on-One and in Group Discussions)

- Provide wait time (and impose wait time), to allow time for thinking to take place.
- Make the group responsible for judging correctness.
- Get rid of your "tells" (respond the same way whether you think the response you heard is correct or not).
- Encourage alternate solution strategies and multiple ways of representing solutions.
- Share your thinking by beginning with "I thought…" (i.e., keep your own ideas on the same level as that of the children.
- Emphasize listening skills (i.e., children should not just talk to the teacher, they should listen to their peers and respond to what they have to say).
- Keep numbers adjectives as long as possible (e.g., *eight apples plus four apples*), rather than treating them as nouns (e.g., *eight plus four*).
- Avoid rapid topic-switching. Have children fully process one child's idea before moving on to another. Ask follow-up questions.

(Scaffolding Children's Thinking, cont.)

- Don't mix assessing (asking children to share what they know) with teaching (helping them know what you know). Don't correct; instead, change the task or the next question.
- Good teaching as improv (i.e., whatever the child shares, treat it as real to them and deserving of a response that respects that reality).
- Most strategies don't need to be taught; instead, they should be recognized and emphasized when children mention using them.
- Less is more (i.e., give them fewer problems to solve, and leave more time for thinking and sharing).
- The more varied the ways children can represent what they know, the better and deeper they know it.
- Remember: Language is not children's best modality for expressing what they know.

Capturing Children's Place-Value Learning Trajectory:

From Awareness to Fluency

Assessment Rubric
SECURE PLACE-VALUE AWARENESS

Rating Scale:

1 – concept has been introduced (indicate date)

2 – child struggles with concept, hesitating or making occasional errors (indicate date)

3 – child is comfortable with the concept and answers confidently (indicate date)

Child's Name:

Can:	1	2	3
Instantly interpret teen numerals in terms of tens and units			
Instantly name teen quantities represented as one ten plus loose units			
Orally count by tens to 100 and backwards from 100			
Instantly interpret numerals up to 99 in terms of tens and units			
Instantly name quantities up to 99 represented as one tens plus loose units			
Distinguish between 47 and 74 and explain why they are different amounts			
Confidently make greater than – less than judgments for pairs of numerals up to 99 (e.g., 29 and 41) and explain their reasoning in place-value terms			
Explain what each of the digits in the numeral 100 mean			
Count forward and backwards comfortably through the decades (e.g., …38, 39, 40, 41,… and 82, 81, 80, 79, 78…)			
Restate quantities presented in nonstandard groupings (e.g., 3 tens 27 units) in standard notation (i.e., 57) and do the reverse (e.g., name 73 as 4 tens and 33 units)			
Count forward and backwards by tens from a non-decade starting point (e.g., 58, 68, 78, 88,… or 82, 72, 62, 52, …)			
Make reasonable estimates of quantities up to 100 displayed as loose units (e.g., estimate 50 for a display of 66 buttons; estimate 30 for a display of 28 Q-tips)			

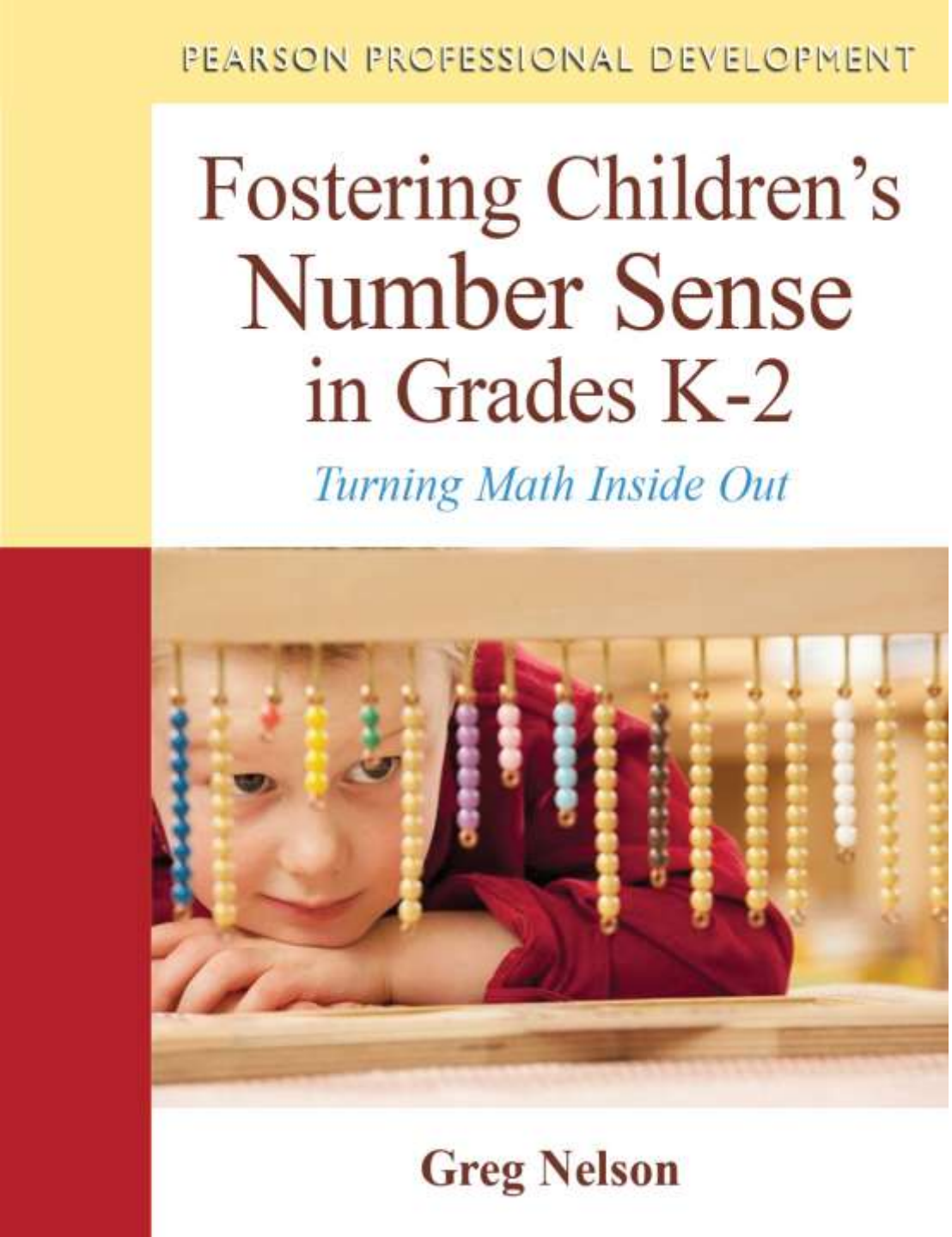
From Greg Nelson, *Fostering Children's Number Sense in Grades K-2* (Pearson, 2014).

Assessment Rubric

EXTENDING PLACE-VALUE AWARENESS AND ADDITION-SUBTRACTION BEYOND THE HUNDREDS

Can:	1	2	3
Name the quantity represented by any of the digits in a four-digit number.			
Represent any four-digit number with place-value materials.			
State any number in expanded notation (e.g., 4,692 is 4 thousands, six hundreds, 9 tens, and 2 units).			
Explain what zeros in a numeral represent (e.g., the zero in 3,607 means there are no tens).			
Explain how much a quantity in one place-value column would be if it were moved to an adjacent place-value column (e.g., how many tens would 2 hundreds be? 30 units would be how many tens?)			
Count forward and backward through place-value transition points (e.g., 803, 802, 801, 800, 799, 798,… or 998, 999, 1000, 1001, 1002,…).			
Skip-count in any place-value column (e.g. 377, 387, 397, 407, 417,…).			
Explain that 1,000 is the units place of thousands, and that the next category would be tens of thousands, then hundreds of thousands.			
Name the missing part of 100 (e.g., given 34 the missing part of 100 is 66) and of 1000 when the units place of the given part is zero (e.g., given 460, the missing part of 1000 is 540).			
Rename place-value quantities in nontraditional arrangements (e.g., 941 is 8 hundreds, 13 tens, and 11 units).			
Explain why, when addition or subtraction problems are written in vertical format, the place-value digits must line up.			
Explain his solution of multi-digit addition or subtraction problems in place-value terms.			
Provide a reasonably accurate estimate of multi-digit sums and differences before computing the exact answer.			
Accurately use the standard algorithm for multi-digit addition and subtraction, both with and without exchanging.			
Explain the steps in the standard algorithm for addition and subtraction in place-value terms.			

From Greg Nelson, *Fostering Children's Number Sense in Grades K-2* (Pearson, 2014).



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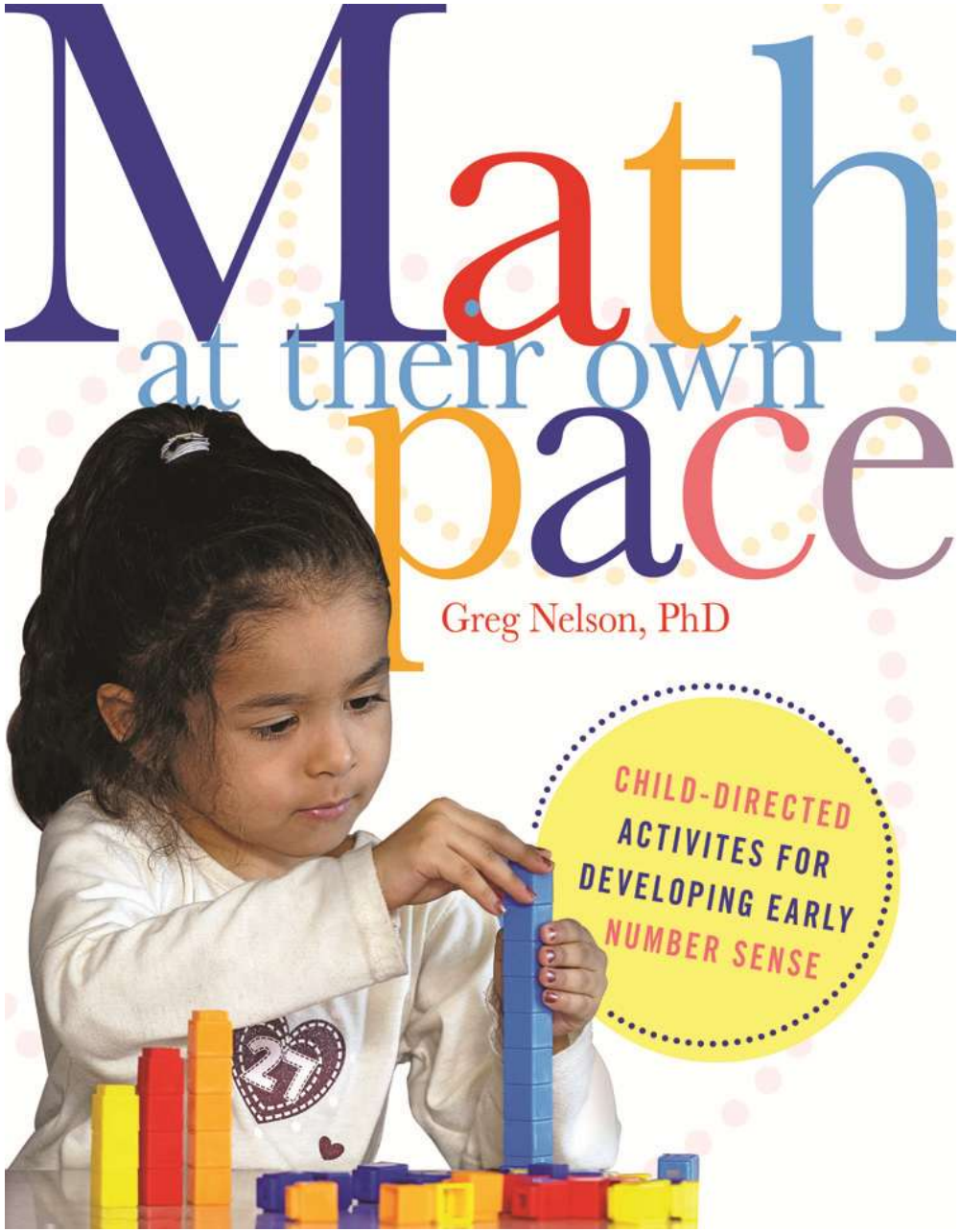

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