SOUND off!

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Least Common Denominators and the Common Core

Under the Common Core State Standards (CCSS), students in fifth grade learn to find a common denominator by multiplying the denominators. In sixth grade, students learn to find the least common multiple of pairs of integers (through 12) but never apply the concept to adding fractions; nor do they see the connection between the greatest common factor and the least common multiple. When these students get to high school, will they be ready to add ratios of polynomials or trigonometric functions? Even more basic, will students come to high school with a shallow understanding of integers and their relationships?

Here's what the Common Core says students should learn to do:

> Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.) (CCSSI 2010, 5.NF)

> Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. (CCSSI 2010, 6.NS)

The CCSS text does not prohibit teaching students to use the least common denominator, but its accompanying "Progressions" publication warns against doing so:

> It is not necessary to find a least common denominator to calculate sums of fractions, and in fact the effort of finding a least common denominator is a distraction from understanding algorithms for adding fractions. (CCSWT, n.d., p. 10)

The North Carolina Department of Public Instruction's official commentary on how to implement the CCSS is explicit about which denominator to use:

> The example provided in the standard has students find a common denominator by finding the product of both denominators. For 1/3 + 1/6, a common denominator is 18, which is the product of 3 and 6. (NCDPI, n.d., "Fifth Grade Mathematics: Unpacked Contents," Number and Operations: Fractions)

Later versions of this document add another example using the least common denominator, saying that students should be aware that there may be a lesser common denominator, but retain this 1/3 + 1/6 example. Whether the student finds $1/3 + 1/6 = 2/6 + 1/6 = 3/6$ or $1/3 + 1/6 = 6/18 + 3/18 = 9/18$, "the

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Standards do not require simplifying fractions into lowest terms, since it is not a mathematically important topic," according to the CCSS mathematics team coordinator (McCallum 2012).

The connection between the greatest common factor (GCF) and the least common multiple (LCM) is one of the beautiful properties of integers. For all positive integers a and b, if we let g = GCF (a, b), then there are relatively prime integers c and d such that $a = cg$ and $b = dg$, and LCM $(a, b) = cdg = cgdg/g = ab/$ GCF (a, b). If students can "find the greatest common factor of two whole numbers less than or equal to 100" (CCSSI 2010, 6.NS), "find all factor pairs for a whole number in the range 1–100" (4.OA), "fluently multiply multi-digit whole numbers" (5.NBT), and "fluently divide multi-digit numbers" (6.NS), why should they be able to find only "the least common multiple of two whole numbers less than or equal to 12" (6.NS)?

The North Carolina commentary suggests the reason why—namely, that students might not learn efficient ways to find least common multiples. One of its endorsed methods is to list the multiples of each number and then take "the least in common from the list" (NCDPI, n.d., "6th Grade Mathematics: Unpacked Contents," 6.NS.4). If students had only this method available to them, I concede that finding the least common multiple of 14 and 18 would be unreasonably tedious. This brute-force method for finding the least common multiple has two additional flaws, however: It does not promote an understanding of the relationships between factors and multiples, and it does not extend to finding the least common multiple of two polynomials.

North Carolina does give a second method, that of using prime factorization:

Step 1: Find the prime factors of 6 and 8.
$6 = 2 \cdot 3$
$8 = 2 \cdot 2 \cdot 2$

Step 2: Find the common factors between 6 and 8. In this example, the common factor is 2.

Step 3: Multiply the common factors and any extra factors: $2 \cdot 2 \cdot 2 \cdot 3$ or 24 (one of the twos is in common; the other twos and the three are the extra factors). (NCDPI, n.d., "6th Grade Mathematics: Unpacked Contents," 6.NS.4)

Note that these instructions find the greatest common factor, 2, but without either identifying it as the GCF or making a connection between the two topics. When students apply these instructions to 8 and 12—

$$8 = 2 \cdot 2 \cdot 2$$
$$12 = 2 \cdot 2 \cdot 3$$

—is it clear to them that "the common factors" are 2 twice, not just once?

If we limit students' practice of

the least common multiple to pairs of positive integers up to 12, how much practice can students get? How many of those pairs have a proper common factor? (Nine: 4 • 6, 4 • 10, 6 • 8, 6 • 9, 6 • 10, 8 • 10, 8 • 12, 9 • 12, 10 • 12.) How many have a proper common composite factor? (Only one: 8 • 12.) Is that enough practice?

Why am I picking on North Carolina when I don't live or teach there? First, the state's education system has a good reputation, and I had high hopes for its explication of the Common Core. Second, the state's "Unpacked" documents are some of the first hits in a Google search and, therefore, the most likely to be widely read and influential. Lowering the expectation that students will master arithmetic of integers is not, however, happening just in North Carolina.

In 2011, NCTM revised its statement on technology, removing this sentence: "The use of technology cannot replace conceptual understanding, computational fluency, or problem-solving skills." The new statement retains the language about "balanced" and "strategic" use of technology and refers to examples of appropriate uses but no longer says what uses are inappropriate. A high school teacher with national influence recently told me that he is not troubled that some of his honors algebra 1 students don't know how to add fractions—they have calculators. He used the NCTM statement to support this pedagogical stance.

What happened to review, remediation, practice, and connections? Fraction addition is ripe with practice in factoring, multiplication, addition, more factoring, and opportunities for discussion about efficient algorithms and thinking before one acts, but not if 1/3 + 1/6 = 9/18. I'm worried that as teacher evaluations are more closely tied to student test scores, teachers will—rightly—argue that they should not be evaluated on topics that students may or may not have mastered the previous year. Yet how can the mathematics curriculum be made noncumulative? Start with fraction addition, by not expecting students to be able to factor.

The Common Core does not say that teachers may not teach students to use the least common denominator, but if the concept disappears from state standardized tests and practice exercises disappear from textbooks, what can we reasonably expect? If the standards and their interpretation by states such as North Carolina do not expect students to have a firm grasp of integers, what can we hope for in our incoming high school students? I already spend a significant amount of time in my high school courses remediating skills that students were supposed to have learned in elementary school. Under the Common Core, I will be teaching basic integer skills that they were never supposed to learn.

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The least common denominator is not necessarily doomed to the rubbish bin. One of the authors of the "Progressions" publication says that the "skill of making use of the lowest common denominator for adding fractions should be taught as an enrichment topic if time is available" (Wu, n.d., p. 37). I am fortunate that the people in charge of the implementation of the Common Core in my state think of the standards as floors, not as ceilings. I hope that states adopting the Common Core will add this clause to each year's goals: "Review, remediate, practice, and deepen students' understandings of topics from previous years and connect them to current topics." Such a clause would complicate teacher evaluation, which is already hard to do well, but that concern should not dictate curriculum design. Our task is to help students develop basic skills, organization, and clear communication and then connect those basic skills to complex algorithms that will solve real-world problems.

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