

Reasoning and Sense Making with At-Risk Students: It's Possible!

Christine Thomas, Georgia State University
Jenny Salls, Washoe County School District

Session #346

Tile Patterns

Use tile patterns to introduce the concept of functions

Marcus drew the first three figures in the pattern below and challenges you to determine the pattern.

Figure 1

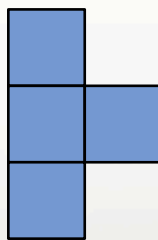


Figure 2

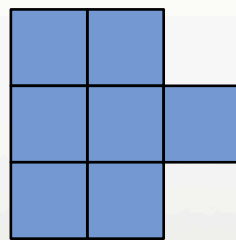
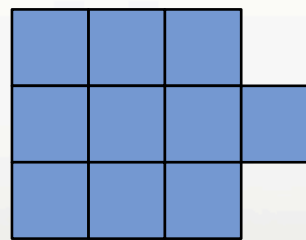


Figure 3



Tile Patterns

Draw the next two figures in the pattern.

Describe what the 10th figure looks like

Figure 1

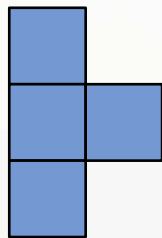


Figure 2

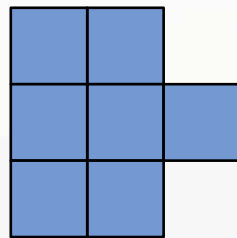
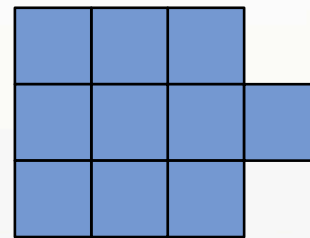


Figure 3



How Do You Describe It?

Describe what the 50th figure looks like

Figure 1

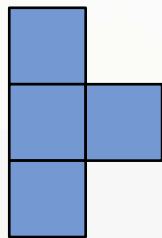


Figure 2

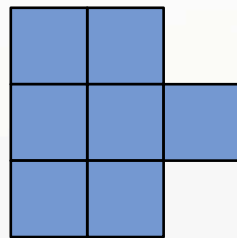
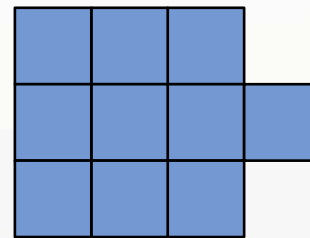


Figure 3



One Way to See This

The 50th figure

Figure 1

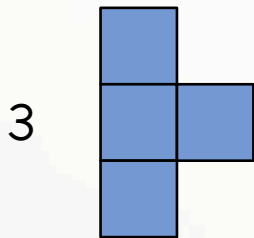


Figure 2

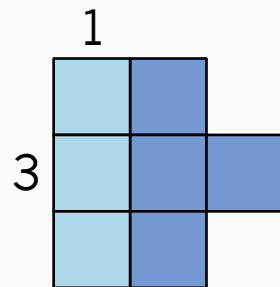


Figure 3

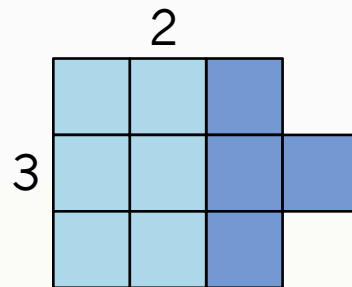
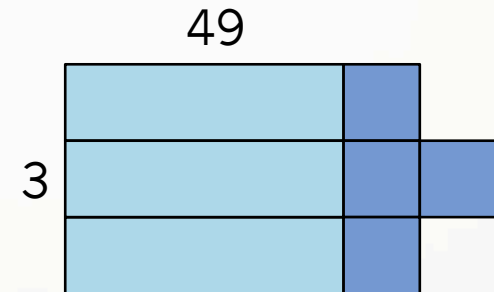


Figure 50



The 50th figure is the base figure with a 3 by 49 rectangle

One Way to See This

The n^{th} figure

Figure 1

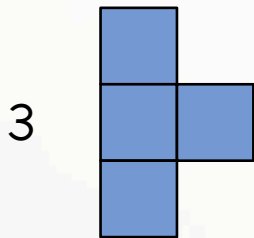


Figure 2

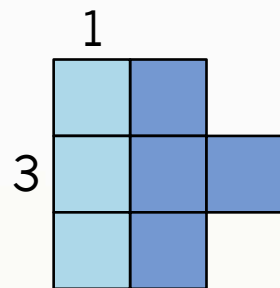


Figure 3

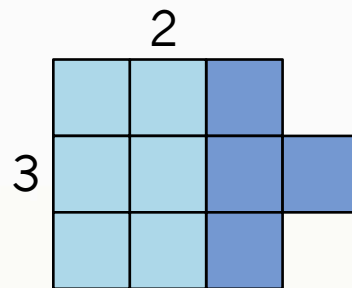
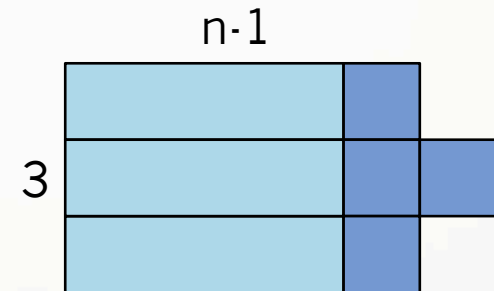


Figure n

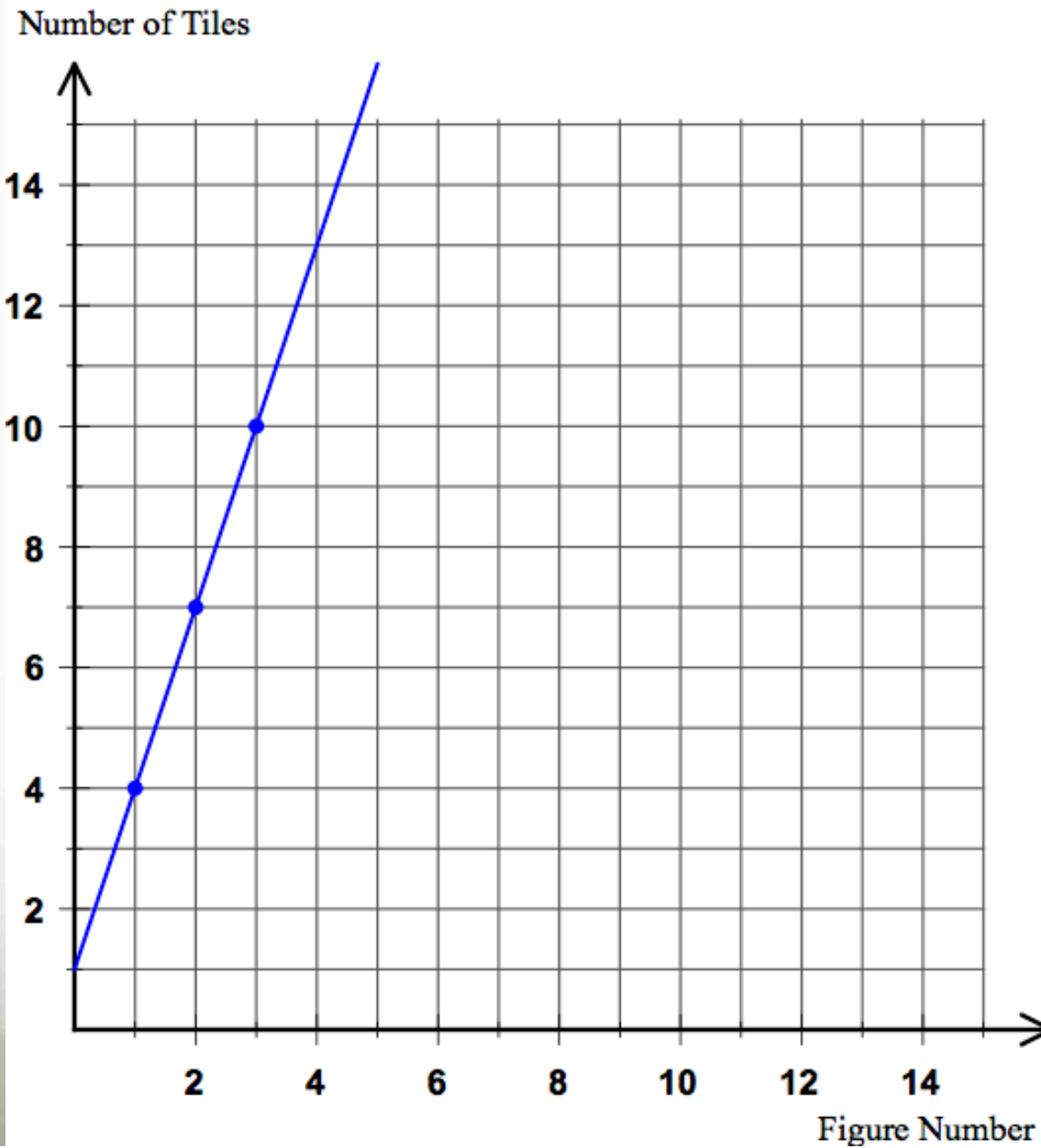


The n^{th} figure is the base figure with a 3 by $n-1$ rectangle

Table

| Figure Number | Number of Tiles |
|---------------|-----------------|
| 1 | 4 |
| 2 | 7 |
| 3 | 10 |
| ⋮ | ⋮ |
| 50 | $3(49)+4$ |
| ⋮ | ⋮ |
| n | $3(n-1)+4$ |

Graph



Reasoning and Sense Making

NCTM has advocated for years, and the Common Core State Standards demand a paradigm shift toward teaching for meaning and understanding rather than relying on procedural memorization of mathematical concepts.

NCSM Newsletter (Fall, 2011)

But...

What about our at-risk students?

We cannot afford to avoid mathematics because our students enter our course behind or are at-risk of failure.

All of our students
need opportunities
for reasoning and
sense making

Reasoning and Sense Making

- Reasoning
 - The process of making conclusions based on evidence or stated assumptions
- Sense Making
 - Developing an understanding of a situation, concept, or context by connecting it with existing knowledge

Focus on High School Mathematics (NCTM 2009)

Reasoning Habits

Reasoning is the foundation of mathematical competence

- Conceptual understanding
- Procedural fluency
- Strategic competence
- Adaptive reasoning
- Productive disposition

Adding It Up (Kilpatrick, Swafford and Findell 2001)

Multidimensional Classrooms

- Group-worthy problems
- Allow for multiple representations
- Focus on sense making and reasoning
 - Communicate thinking
 - Justify conclusions
- Promote the belief that all students can learn

Jo Boaler “Stories of success: Changing students’ lives through sense making and reasoning” in *FHSM: Fostering reasoning and sense making for all students* (2011)

Tasks

- Multiple entry points
 - Multiple representations
 - Table, rule, graph, equation
 - Multiple problem-solving strategies
 - Picture, guess and check, act it out, simpler problem
- Connect to the mathematics they're learning
- Encourage critical thinking
 - Collaborative learning, open-ended questions, share strategies

Encourage Critical Thinking

- Opportunities for collaboration
 - Group-worthy tasks, not exercises
 - Expect students to communicate their reasoning
- Share strategies – Peg Smith, Univ. Pittsburgh
 - Identify strategies you want students to see
 - Select a variety of solutions that show a range of complexity
 - Sharing to make reasoning public

A Tile Patterns Task to Introduce Function Concepts

- The task:
 - Draw the next 2 figures
 - Describe the 50th figure
 - Describe the n^{th} figure
 - Write an expression that describes the number of tiles in the n^{th} figure
 - Create an x-y table: x represents the figure number, y represents the number of tiles in the figure
 - Make a scatterplot representing the table
 - Describe the pattern that this table represents
 - Write an equation showing the relationship between x and y

Mathematical Practices

1. Make sense of problems & persevere in solving them
2. Reason abstractly & quantitatively
3. Construct viable arguments; critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Encourage Critical Thinking

- Open-ended questions
 - Who solved this a different way?
 - Why did you do that?
 - Can you prove that statement?
 - What do you think about Jonathan's statement?
 - Would this work every time?
 - Would Nathan's way work with this problem?

The Teacher's Task

- Draw students' attention to the form of the function (linear, quadratic, exponential), shape of graph, and additional features
- Distinguish between recursive and explicit function definitions
- Connect the sequences to arithmetic and geometric sequences

Connecting to Important Mathematics

- Principles and Standards 9-12 Algebra Standards
 - Generalize patterns using explicitly defined and recursively defined functions
 - Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior
 - Understand and compare the properties of classes of functions

Connecting to Important Mathematics

- Common Core
 - Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers (F.IF.3)
 - Graph functions expressed symbolically and show key features of the graph ... (F.IF.7)
 - Determine an explicit expression, a recursive process, or steps for calculation from a context (F.BF.1a)
 - Write arithmetic and geometric sequences both recursively and with an explicit formula ... (F.BF.2)

Our At-Risk Students Can Reason and Make Sense of Mathematics

- Multiple entry points
 - Multiple representations
 - Table, rule, graph, equation
 - Multiple problem-solving strategies
 - Picture, guess and check, act it out, simpler problem
- Encourage critical thinking
 - Collaborative learning, open-ended questions, share strategies

Thank You for
attending and
participating!

Christine Thomas – cthomas11@gsu.edu
Jenny Salls - jsalls@washoeschools.net