A Conceptual Approach to Standard Deviation

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Agenda

- Why this activity?
- Measures of Center
- Review – calculating standard deviation
- Standard Deviation: A Conceptual Approach
- Feedback
- Conclusion

Measures of Center

Quick discussion: What is the

- Mode
- Median
- Mean

Interesting…

- We tend to define the mode and median with what they actually *are*.
- We tend to define the mean in terms of *how to find it*.

What does the Mean Mean?

Use the color tiles as follows:

- Use the red tiles to form groups of size 11, 3, 5, 14, and 7 (no need to put these in any sort of order)
- Push all the red tiles together
- Redistribute the red tiles to that each of the five groups has the same number of tiles.

- Compare the steps in this task with the steps used to calculate the mean.

What is the Mean?

- Equal sharing definition – (in this case) the mean is the number of tiles that each group has if the tiles are evenly distributed.

- Another interpretation of the mean is the **balance point**, but we leave that for another day and write it in smaller font.

What is the Standard Deviation?

- Review: Calculate the population standard deviation for this data set:

  11, 3, 5, 14, and 7.

Calculate the Mean

$$\bar{x} = \frac{\sum x_i}{n}$$

$$\bar{x} = \frac{11 + 3 + 5 + 14 + 7}{5} = 8$$

We did this in Activity 1.

Find the Deviations from the mean.

x_i	$(x_i - \bar{x})$
11	11 - 8 = 3
3	3 – 8 = -5
5	5 – 8 = -3
14	14 – 8 = 6
7	7 – 8 = -1

Find the Squared Deviations from the mean.

$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
3	9
-5	25
-3	9
6	36
1	1

Calculate the variance: the mean of the squared deviations.

$$v = \frac{\sum(\boldsymbol{x_i} - \overline{\boldsymbol{x}})^{\,2}}{n}$$

$$v = \frac{9 + 25 + 9 + 36 + 1}{5} = \frac{80}{5} = 16$$

Find the standard deviation.

$$S = \sqrt{v}$$

$$S = \sqrt{16} = 4$$

Discuss:

- Why did we do any of that?

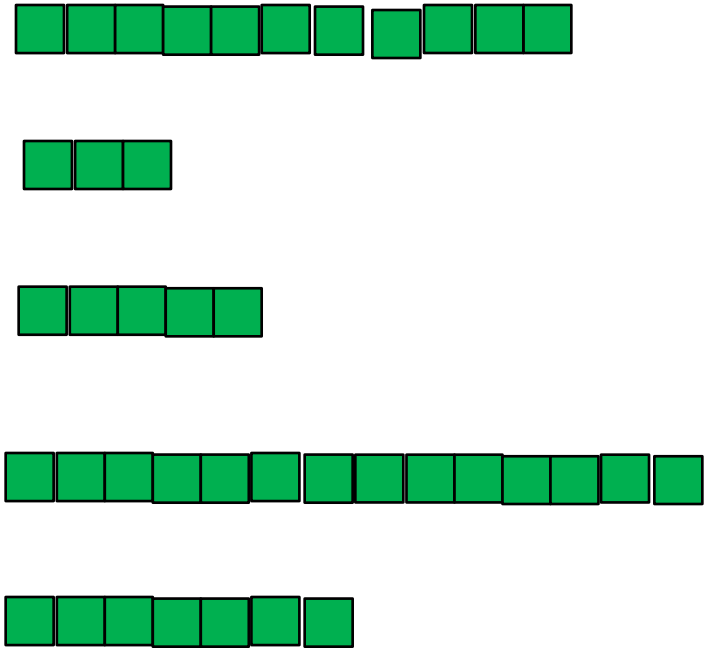
Student comments:

- *Courtney: "The 4 tells us how far away each data point is from the mean, but it isn't even for each data point. So 4 isn't for each one… Hhmmm. I have no clue how that works."*

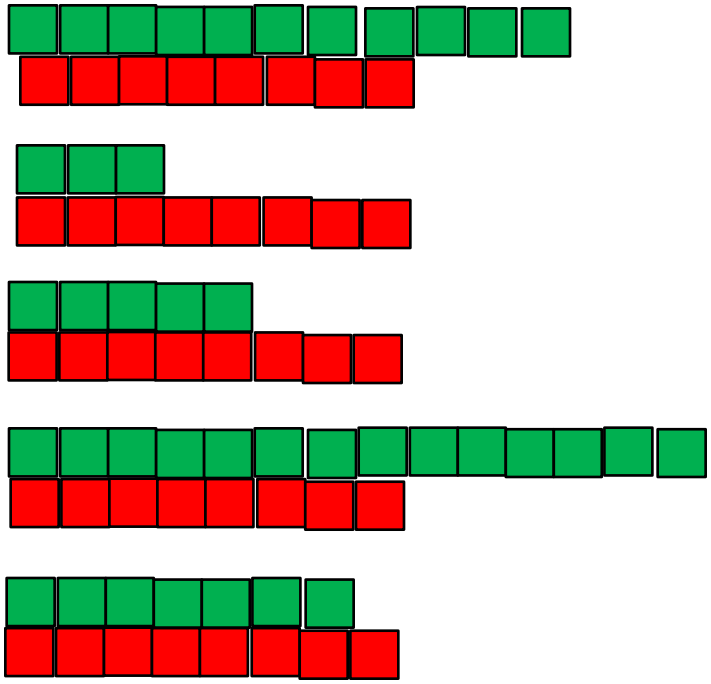
- *Bob: "We subtract the mean to find how far away it is but I don't know why we do the rest of those steps."*

Conceptual Approach Activity (Example 1)

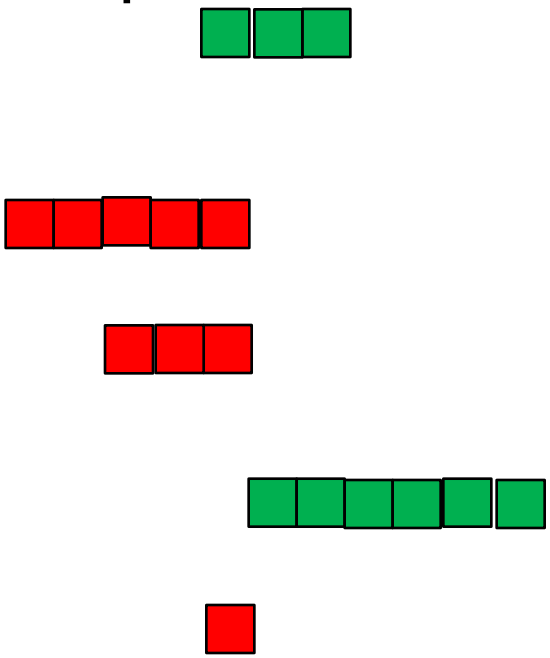
- Task 1. Use green tiles to form rows of lengths 11, 3, 5, 14, and 7



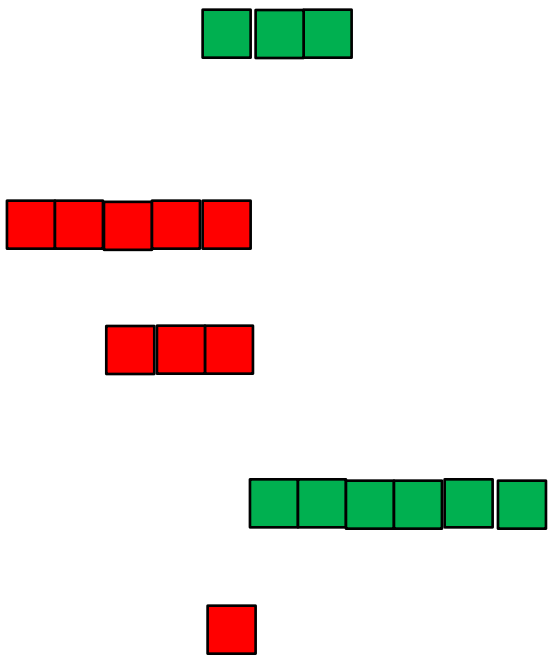
- Task 2. Align the red tiles from *the end of* Activity 1 with these green tiles. Notice that, by setting up this alignment, we are comparing the mean to the original group of tiles.



- Task 3. Remove the ‘matched up’ tiles, leaving five segments. These remaining tiles represent each difference from the mean for the data points.



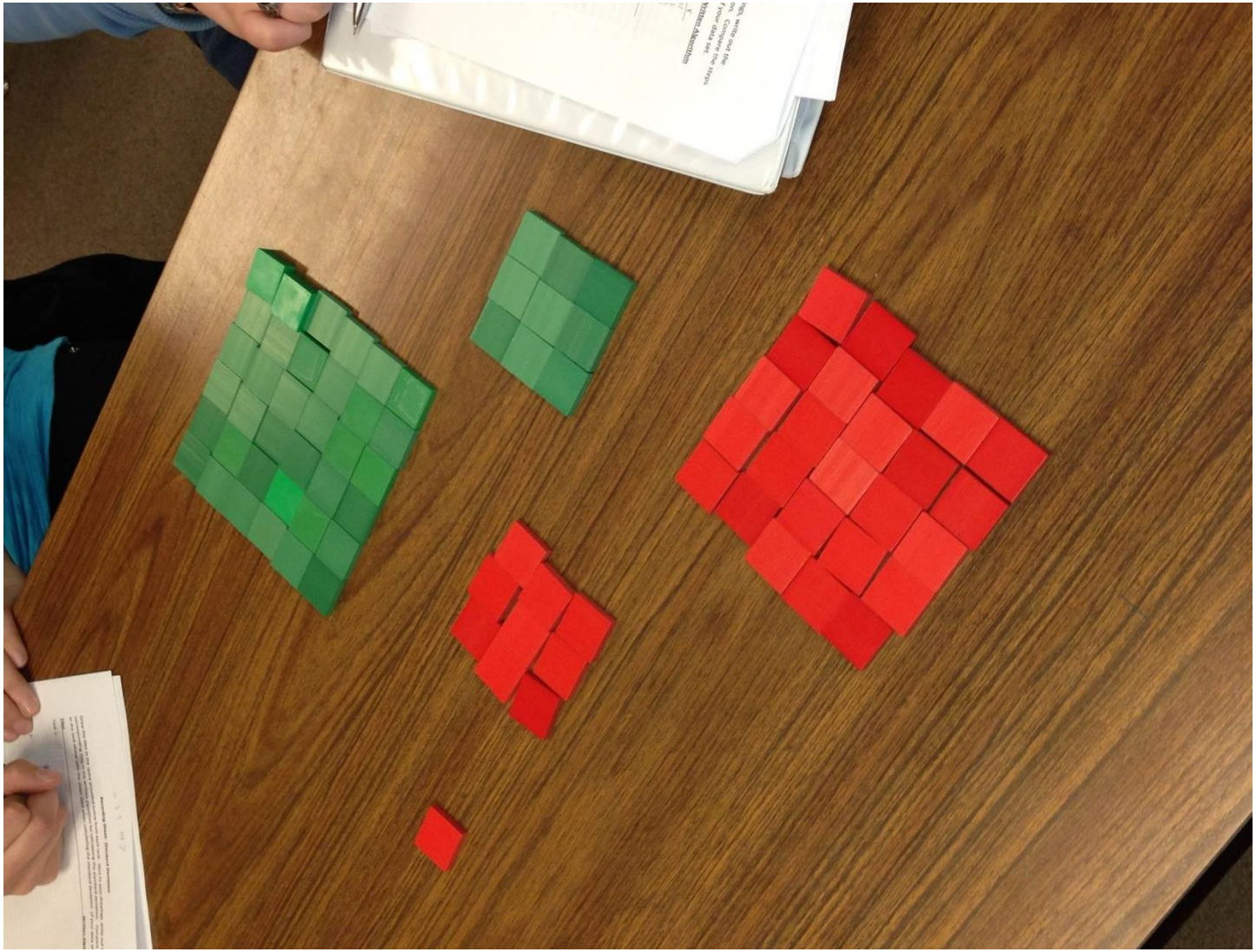
Deviations from the mean.



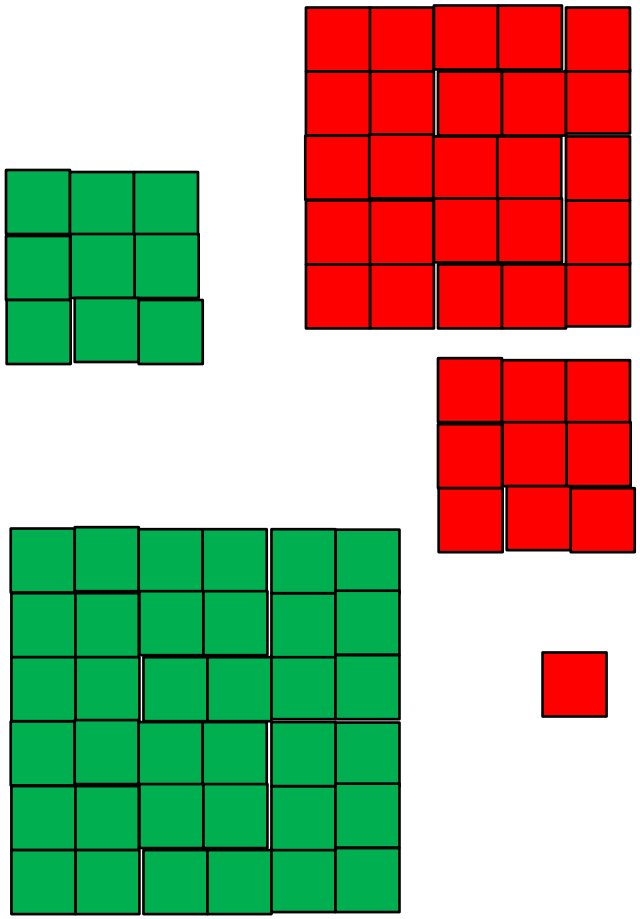
x_i	$(x_i - \bar{x})$
11	11 - 8 = 3
3	3 – 8 = -5
5	5 – 8 = -3
14	14 – 8 = 6
7	7 – 8 = -1

- Task 4. Use each of these segments to represent the length of the side of a square, **and then use more tiles to build each square**. For example, if the difference in the number of matched tiles is 3, add more tiles to build a 3 X 3 square. When completed, you will have five squares - we will call these **difference squares.**

- Note: the color used to build the squares is irrelevant. Feel free to mix colors at this point.



Squared Deviations from the mean.



$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
3	9
-5	25
-3	9
6	36
1	1

- Task 5. Use the *equal sharing method* to find the mean area of the five squares that you formed. That is, redistribute the squares into five piles all with the same number of tiles.

- Rearrange the tiles in the five piles into squares.

- Task 5 (continued). Arrange each of the five groups to form a square.

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Task 5 (continued)

- What is the average size (area) of our difference squares?

- The average area of the difference squares is the **variance** of the data set.

Task 6

- What is the side length of the mean of our difference squares?

- The lengths of the sides of these squares is the **standard deviation** of the data set.

Student comments

- Matt: " The standard deviation is just the side length of one of the squares we just made."

- Cassie: "I can see it now. The variance is the same as the areas of the difference squares."

Example 2

- Use the difference squares to model the process for finding the standard deviation for this data set

  10 1 9 2 3 6 4 5

This data set has one challenging data point…

Comment:

- The first and second data sets, the mean was a whole number and the variance was a perfect square.
- One can create additional data sets by adding, for example, 3 to each data point and having students find the mean and standard deviation. This can lead to a rich discussion about why the mean increases by 3, but the standard deviation stays the same.

Example 3

- Use the difference squares to model the process for finding the standard deviation for this data set

  6 2 13 5 9

This data set poses a different challenge…

Student comments on Example 3:

- Cassie: We can't make the difference squares here. It's impossible. So this one doesn't have a standard deviation."

- *Matt: "No, if we could slice the tiles then we could make the difference squares. It just wouldn't be a whole square. We would have a side length between 3 and 4, so 3 and a little bit if we could slice the tiles."*

- *Cassie: "Oh, so we just can't show this one using the tiles. We can do it on paper."*

Example 4

- Use the difference squares to model the process for finding the standard deviation for this data set:

  12 5 10 3 10 2

- Yet another challenge within this data set …

Advantages/disadvantages

- Representation
- Time
- Materials
- Data sets

Feedback:

Conclusion

THANK YOU!