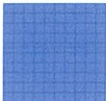




# Making Our Base-Ten System Concrete & Comprehensible

- . Crafting Affordable Manipulatives
  - . RTI Techniques
- . Common Core Responsibilities

Flats	Rods	Units
		

**Dr. Stacy (Keller) Boote**  
University of North Florida  
[s.boote@unf.edu](mailto:s.boote@unf.edu)  
NCTM Annual Conference  
Denver, CO  
April 18, 2013

## Table of Contents

- Common Core Base-Ten Standards (K-5) pg. 3-4
- CCSS Mathematics Grade 3 pg. 5
- CCSS Mathematics Grade 4 pg. 6
- CCSS Mathematics Grade 5 pg. 7
- Hundreds Chart pg. 8
- Place Value Mats pg. 9-11
- Base-Ten Pieces (for Base-Baggies) pg. 12-13
- Area Model of Multiplication (cm paper) pg. 14
- Area Model of Multiplication (Base-Ten pieces) pg. 15
- Area Model of Multiplication Practice Sheet pg. 16
- Centimeter Paper pg. 17
- Division with Craft Sticks Worked Example pg. 18

## Kindergarten

### **Number and Operations in Base Ten (K.NBT)**

#### **Work with numbers 11–19 to gain foundations for place value.**

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, [e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g.,  $18 = 10 + 8$ )]; understand that these numbers (i.e., 11 to 19) are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

## Grade 1

### **Number and Operations in Base Ten (1.NBT)**

#### **Extend the counting sequence.**

1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

#### **Understand place value.**

2. Understand that the two digits of a two-digit number represent amounts of tens and ones.

Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones — called a “ten.”

b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

## Grade 2

### **Number and Operations in Base Ten (2.NBT)**

#### **Understand place value.**

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones (e.g., 706 equals 7 hundreds, 0 tens, and 6 ones).

Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens — called a “hundred.”

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2. Count within 1000; skip-count by 5s, 10s, and 100s.

3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons.

#### **Use place value understanding and properties of operations to add and subtract.**

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

6. Add up to four two-digit numbers using strategies based on place value and properties of operations.

7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

9. Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)

### **Grade 3**

#### **Number and Operations in Base Ten (3.NBT)**

**Use place value understanding and properties of operations to perform multi-digit arithmetic.**

(A range of algorithms may be used.)

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g.,  $9 \times 80$ ,  $5 \times 60$ ) using strategies based on place value and properties of operations.

### **Grade 4**

#### **Number and Operations in Base Ten (4.NBT)**

(Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)

**Generalize place value understanding for multi-digit whole numbers.**

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that  $700 \div 70 = 10$  by applying concepts of place value and division.*
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons.
3. Use place value understanding to round multi-digit whole numbers to any place.

**Use place value understanding and properties of operations to perform multi-digit arithmetic.**

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

### **Grade 5**

#### **Number and Operations in Base Ten (5.NBT)**

**Understand the place value system.**

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and  $1/10$  of what it represents in the place to its left.
2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
3. Read, write, and compare decimals to thousandths.
  - a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, [e.g.,  $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$ .]
  - b. Compare two decimals to thousandths based on meanings of the digits in each place, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons.
4. Use place value understanding to round decimals to any place.

**Perform operations with multi-digit whole numbers and with decimals to hundredths.**

5. Fluently multiply multi-digit whole numbers using the standard algorithm.
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100;

(2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and

(4) describing and analyzing two-dimensional shapes.

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By

comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a

fractional part is relative to the size of the whole. For example,  $\frac{1}{2}$  of the paint in a small bucket could be less paint than  $\frac{1}{3}$  of the paint in a larger bucket, but  $\frac{1}{3}$  of a ribbon is longer than  $\frac{1}{5}$  of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

(4) Students describe, analyze, and compare properties of two dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

### **Mathematical Practices**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

In **Grade 4**, instructional time should focus on **three critical areas**: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g.,  $15/9 = 5/3$ ), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

### Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

In **Grade 5**, instructional time should focus on **three critical areas**: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

### Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100





<b>hundreds</b>	<b>tens</b>	<b>ones</b>

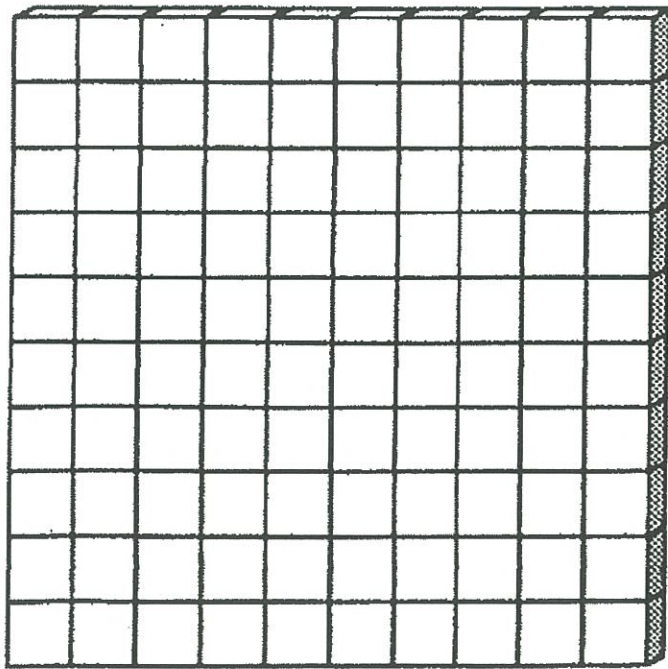
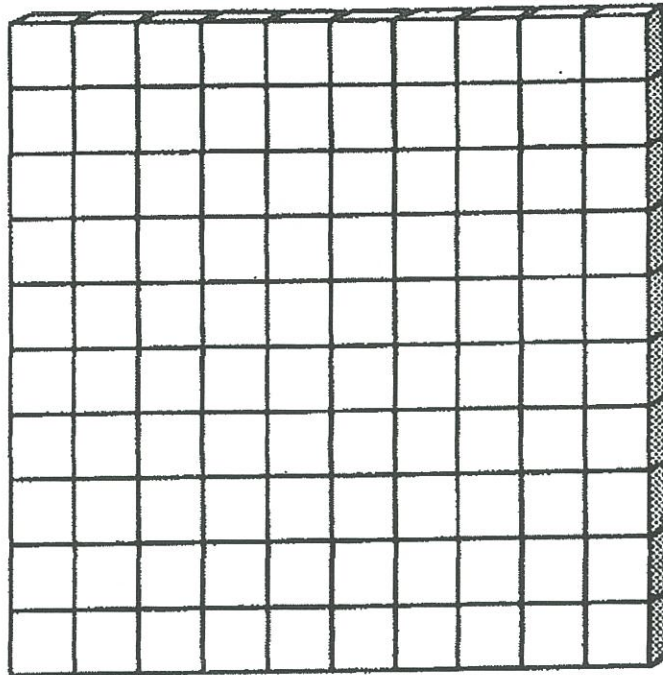
# Thousand

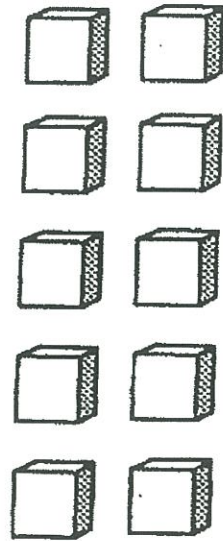
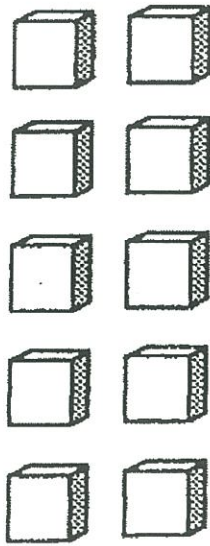
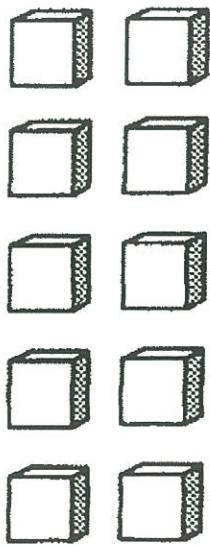
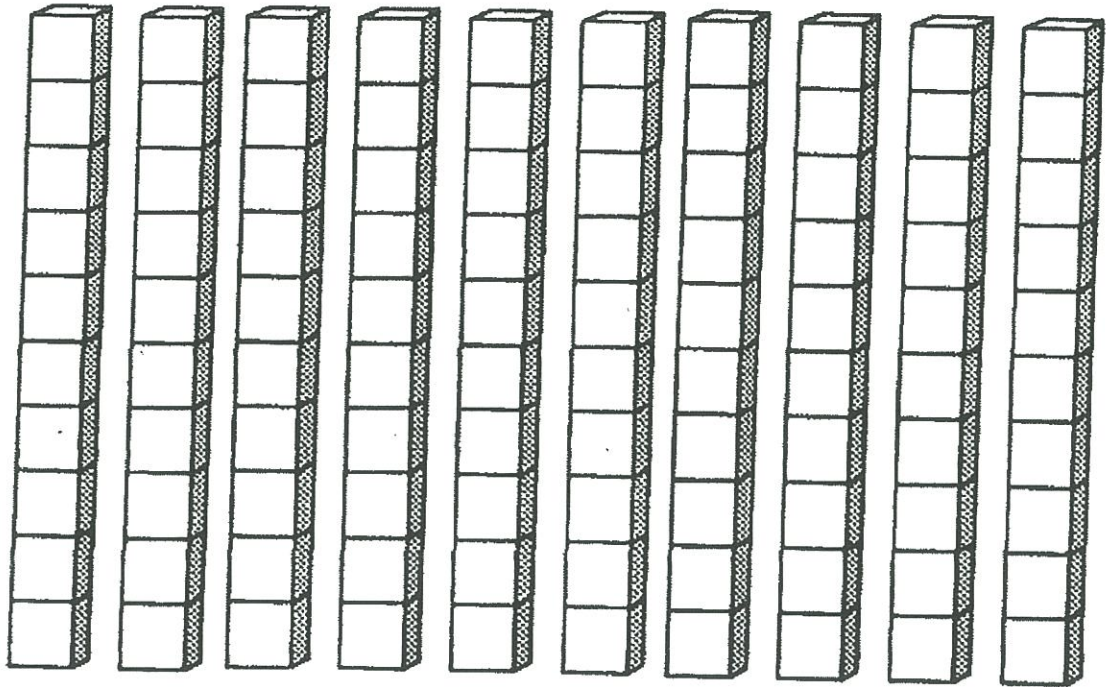
**hundred-  
thousands**

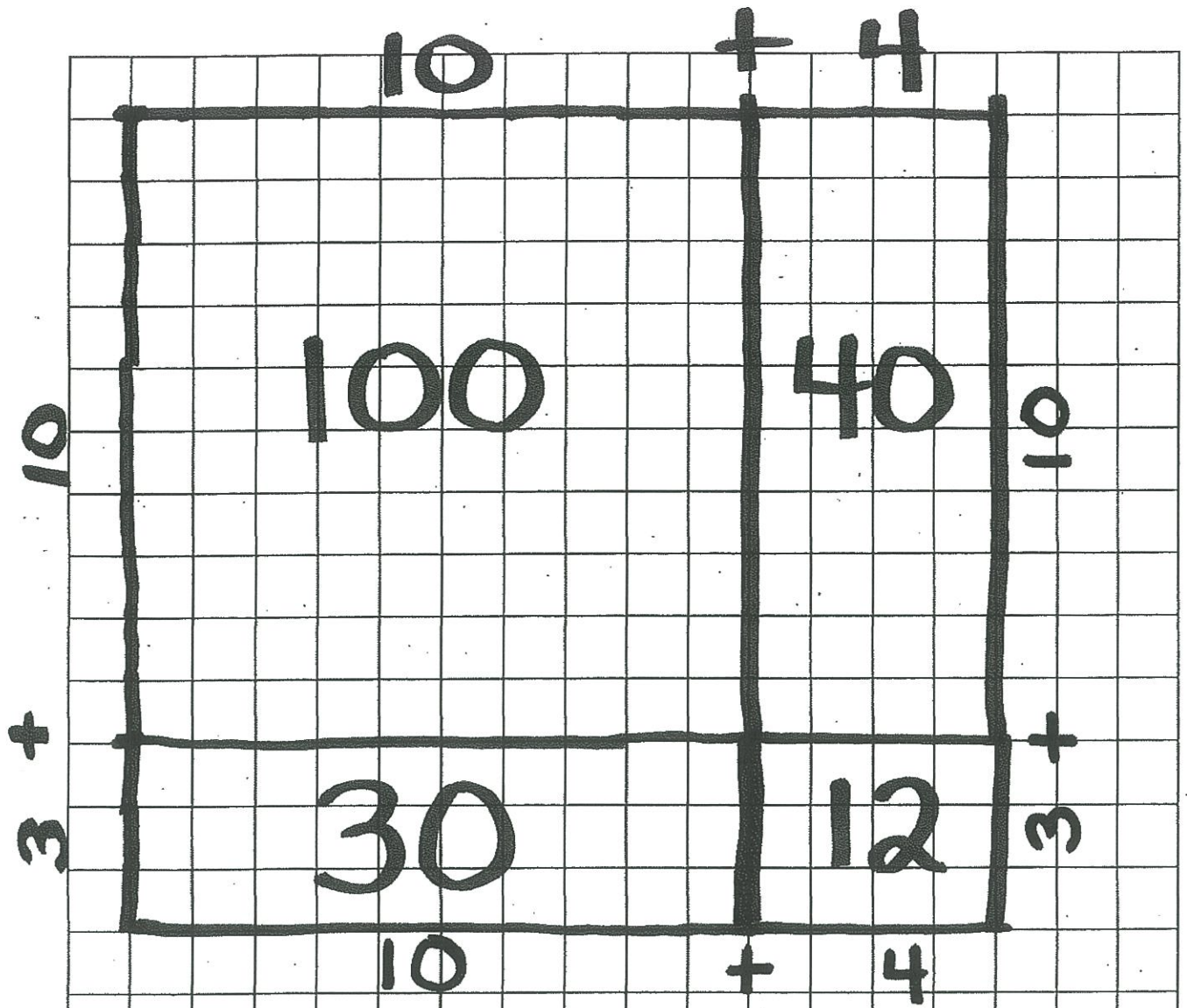
**ten-  
thousands**

**thousands**

<b>and</b> 	<b>tenths</b>	<b>hundredths</b>	<b>thousandths</b>
			

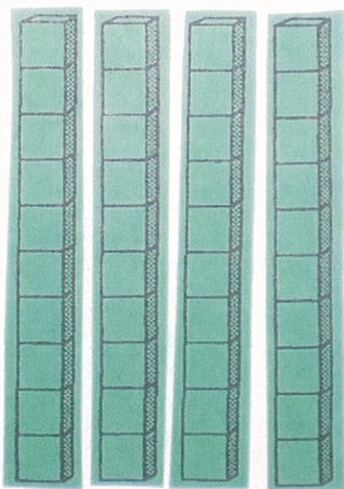
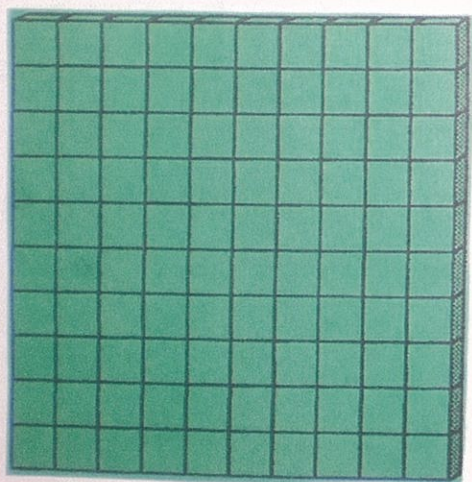
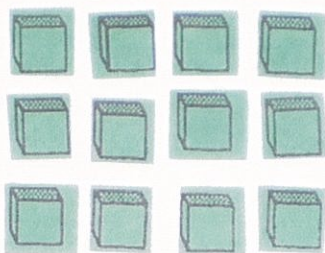
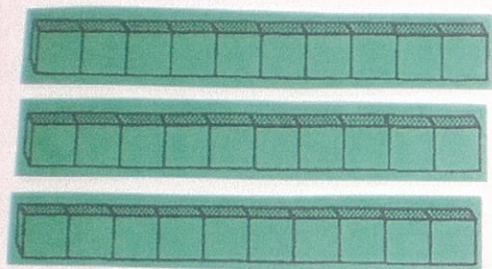






$$13 \times 14 =$$

(R) x (C)



$$13 \times 14$$

Also,

$$13$$

$$\times 14$$

$$\hline 3 \times 4 = 12$$

$$3 \times 10 = 30$$

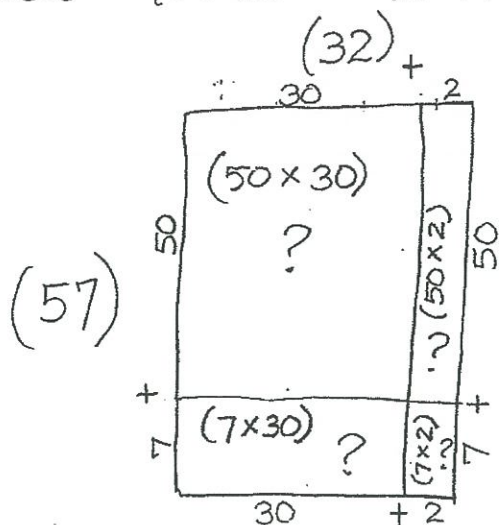
$$10 \times 4 = 40$$

$$10 \times 10 = 100$$

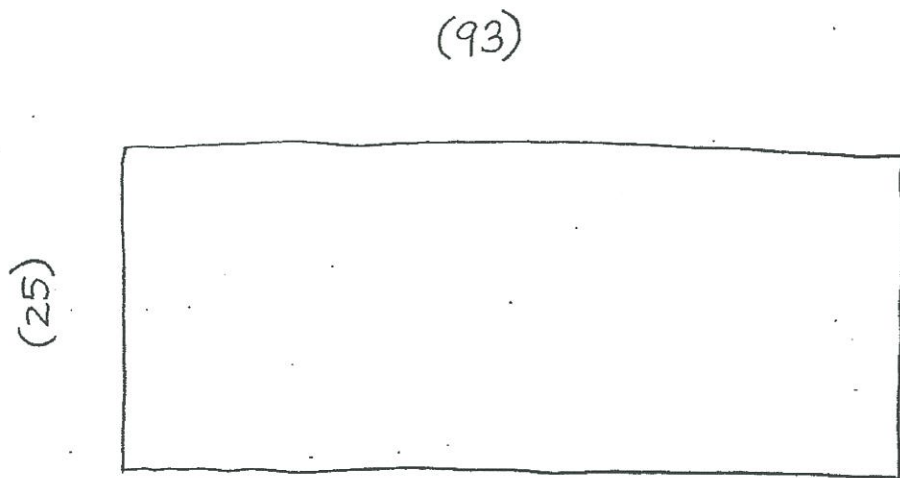
$$\hline + 182$$

Use an Area Model to solve these multiplication problems:

①  $57 \times 32$

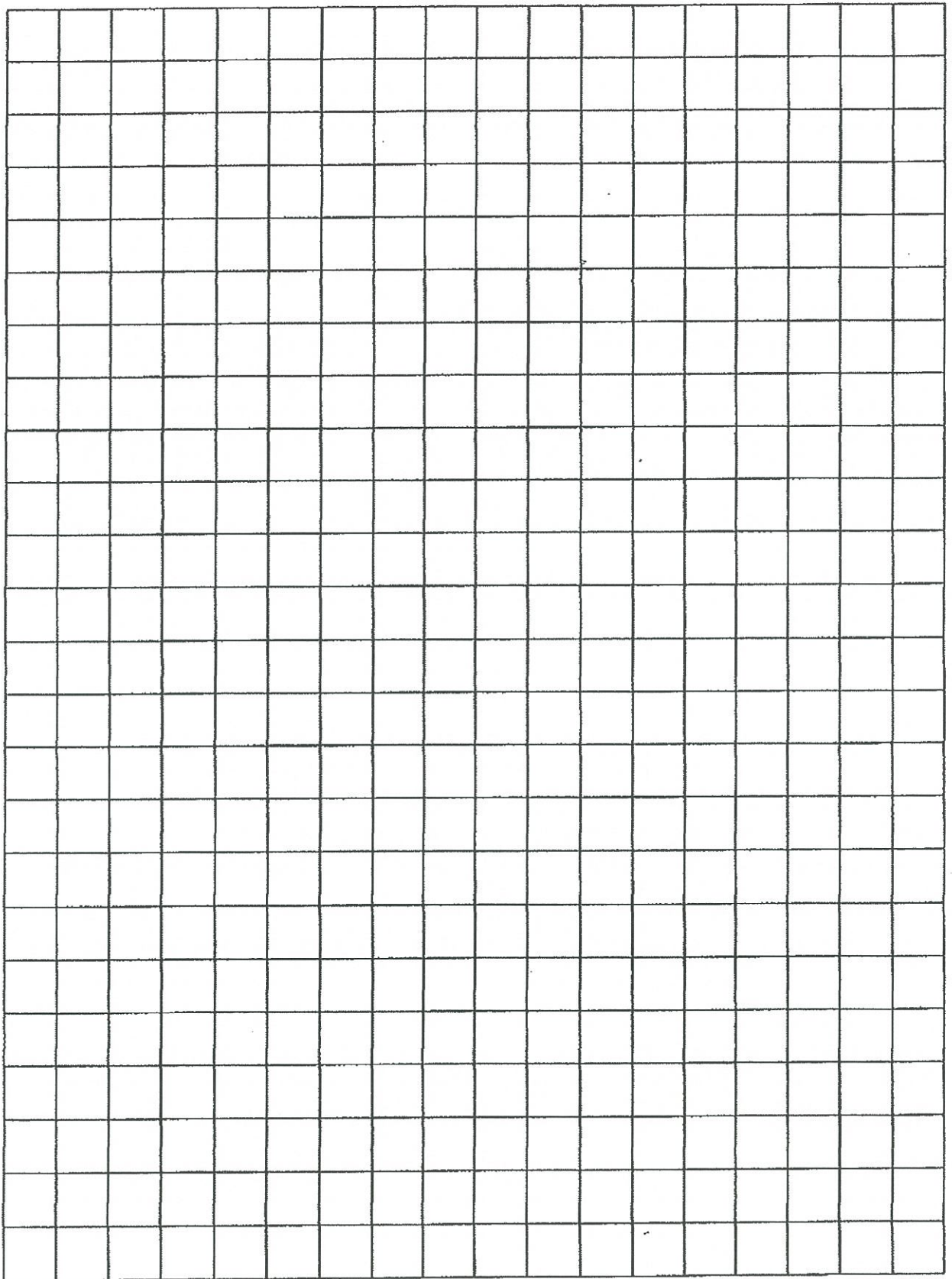


②  $25 \times 93$



③  $678 \times 345$





## Division with Craft Sticks Worked Example

	baggies	bundles	sticks
$a_3$	$e_x$	$f_7$	$g_9$
$b_2$		$c_3$	$d_8$
$-g_2$		$1$	$i_{\downarrow}$
		$h_2$	$8$
		$-k_2$	$7$
			$l_1$
			$m_{\frac{1}{3}}$

- Number of people you are sharing the sticks with
- "2 baggies" equivalent to 200
- "3 bundles" equivalent to 30
- "8 sticks" equivalent to 8
- (Always begin with your largest place value.) Question 1: "I have 2 baggies. Is this enough to share fairly with 3 people?" No. Place an "x" in the quotient above the "2" to prevent accidentally placing another number there later on.
- Comment and Question 2: "Since I can't share 2 baggies fairly, I need to exchange them for 20 bundles. I already have 3 bundles, so I will combine the 20 and 3 bundles and share 23 bundles (notice the 23 in the dividend) among 3 people. Is that enough? Yes. Each of the 3 people get 7 bundles."
- "21" is the total number of bundles fairly shared among 3 people.
- "2" is the number of bundles left over. I can't share them fairly with 3 people, so I need to exchange them for 20 sticks.
- Comment and Question 3: "After exchanging the leftover 2 bundles for 20 sticks and combining them with my original 8 sticks, I have 28 sticks to share fairly with 3 people. Do I have enough? Yes."
- "9" is the number of sticks each of the 3 people get
- "27" is the total number of sticks passed out
- "1" is the number of sticks left over. It's not enough to share fairly with 3 people, so it becomes a remainder.
- " $\frac{1}{3}$ " is the fraction of the leftover stick each person gets

$\frac{1}{3}$