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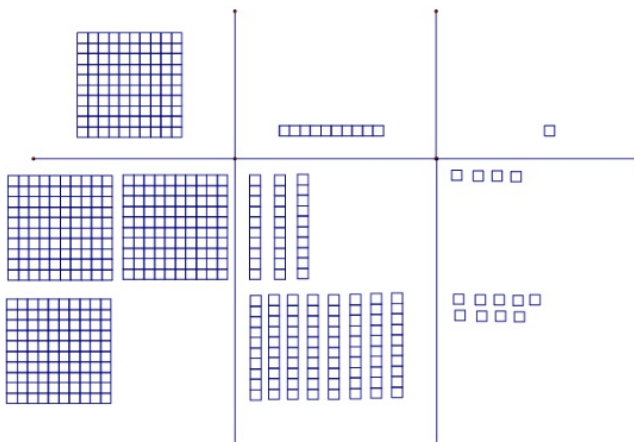
# Base-ten Blocks and Fractions to Beginning Algebra: Unifying Computational Algorithms

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Emporia, KS

# Whole Number Addition

## Base-ten Blocks

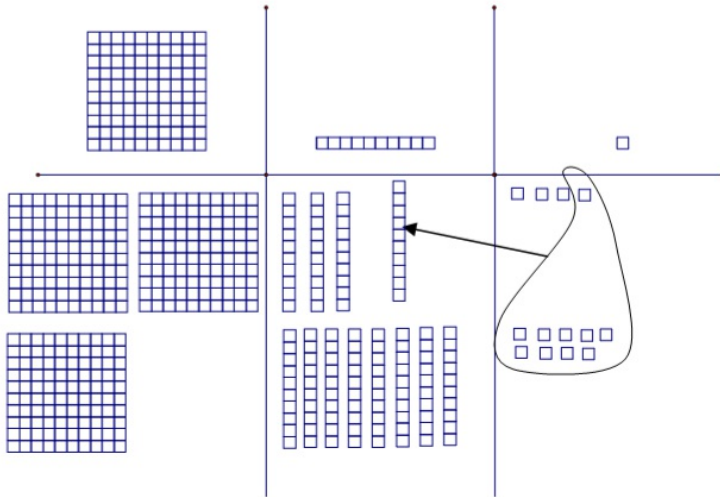
$$\begin{array}{r} 234 \\ + 189 \\ \hline \end{array}$$
 means to combine 234 and 189.



Put the blocks out that represent the two numbers as shown to the left.

## Whole Number Addition (cont.)

### Base-ten Blocks (cont.)



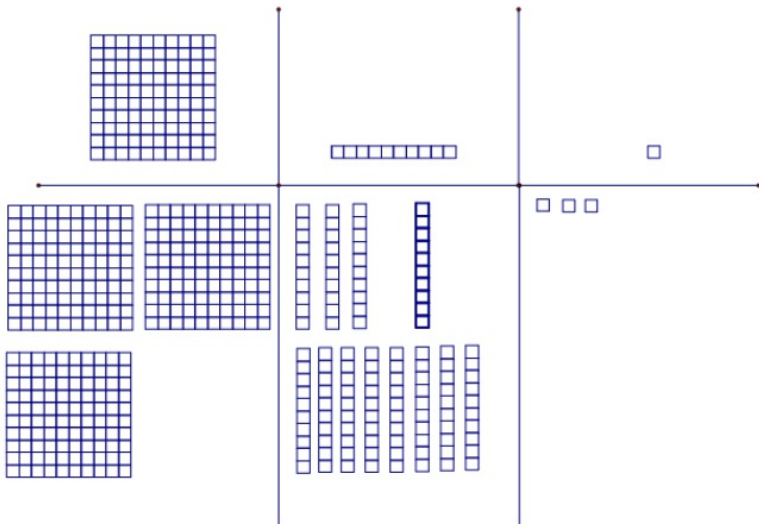
Combine the units to get 11 in total so we trade a long for 10 units.

Represents the long/ten traded for the 10 units.

$$\begin{array}{r} 1 \\ 234 \\ + 189 \\ \hline \end{array}$$

## Whole Number Addition (cont.)

### Base-ten Blocks (cont.)



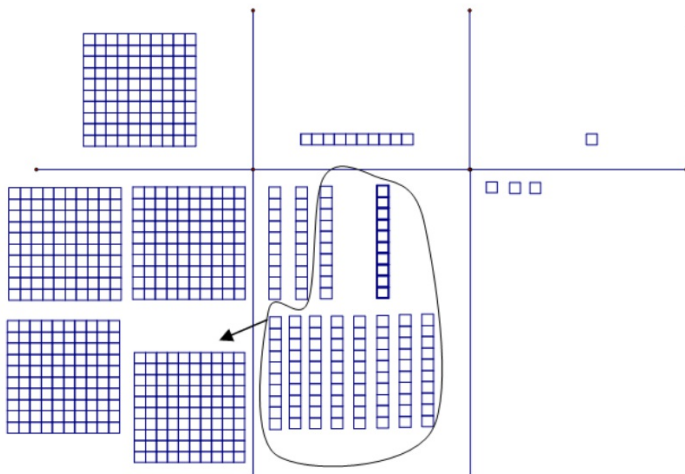
$$\begin{array}{r} 1 \\ 234 \\ + 189 \\ \hline \end{array}$$

3

The number of units that remains.

## Whole Number Addition (cont.)

### Base-ten Blocks (cont.)



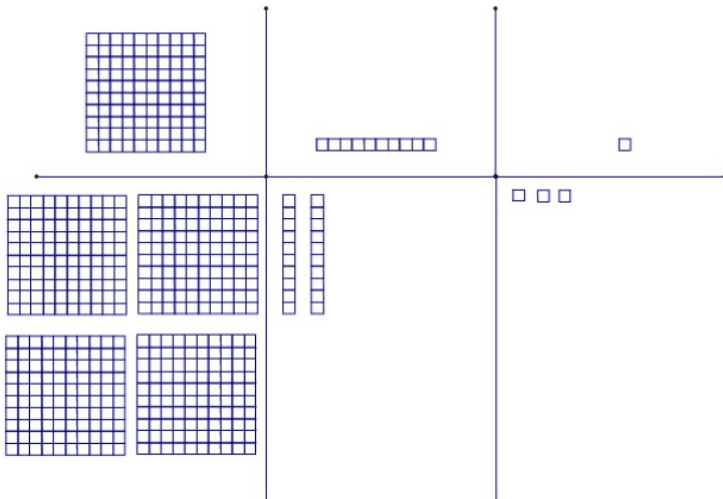
Combine the longs to get 12 longs and trade 10 for a flat.

Represents the flat from the trade of the 10 longs.

$$\begin{array}{r} 11 \\ 234 \\ + 189 \\ \hline 3 \end{array}$$

## Whole Number Addition (cont.)

### Base-ten Blocks (cont.)



$$\begin{array}{r} 1 \ 1 \\ 2 \ 3 \ 4 \\ + 1 \ 8 \ 9 \\ \hline \end{array}$$

Number of longs that remain.

Combine the flats.

$$\begin{array}{r} 1 \ 1 \\ 2 \ 3 \ 4 \\ + 1 \ 8 \ 9 \\ \hline 4 \ 2 \ 3 \end{array}$$

Number of flats that are combined.

## Whole Number Addition (cont.)

### Traditional Algorithm

$$\begin{array}{r} 11 \\ 234 \\ + 189 \\ \hline 423 \end{array}$$

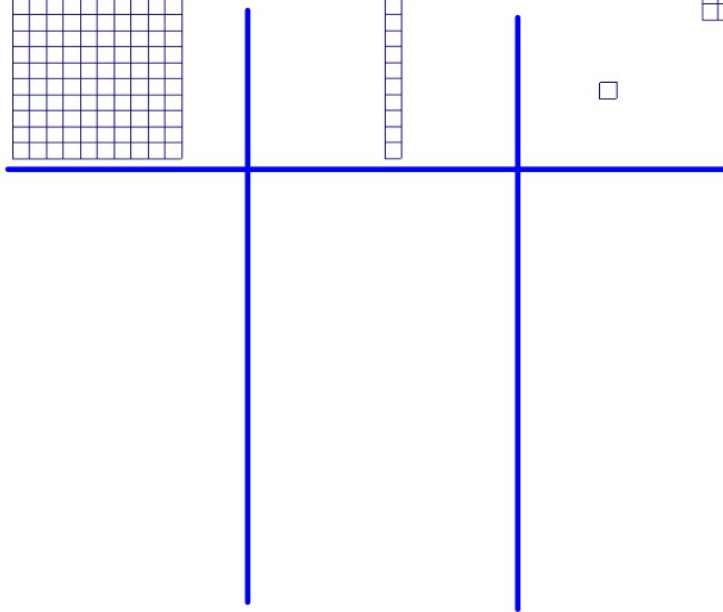
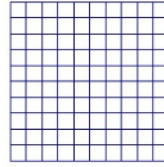
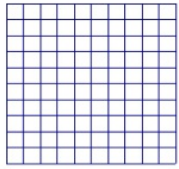
### Alternative Algorithm

$$\begin{array}{r} 234 \\ + 189 \\ \hline 13 \\ 110 \\ 300 \\ \hline 423 \end{array}$$



Now let's try:

$$\begin{array}{r} 175 \\ + 189 \\ \hline \end{array}$$



## Integer Addition

### Chip Model

Yellow chips represent positive integers and red chips represent negative integers. Each pair of yellow/red chips neutralize each other, that is, they represent a sum of zero.



$+3 + +4$  means *Combine 3 yellow chips and 4 yellow chips.*



## Integer Addition (cont.)

### Chip Model

$+3 + (-4)$  means *Combine 3 yellow chips and 4 red chips.*



]

## Integer Addition (cont.)

### Chip Model

$-3 + (-4)$  means *Combine 3 red chips and 4 red chips.*



## **Integer Addition (cont.)**

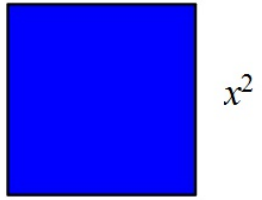
**When adding two integers with the **same signs**:**

**Ignore the signs and add the two numbers; then, append the sign of the two numbers.**

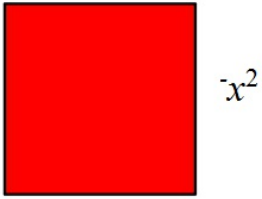
**When adding two integers with **different signs**:**

**Ignore the signs and find the difference; then attach the sign of the number with the greatest absolute value.**

# Algebra Tiles



$x^2$




$-x^2$




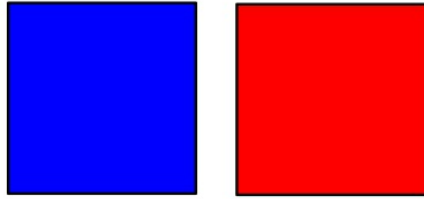
$x$



$-x$

 1 or  $y$

  $-1$  or  $-y$



0



0

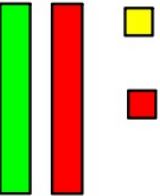
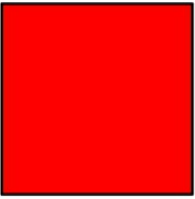
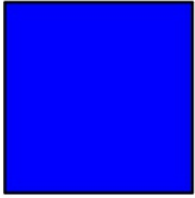


0

## Algebra Tiles: Addition

$(2x^2 - 1x + 4) + (1x^2 + 3x - 2)$  means to

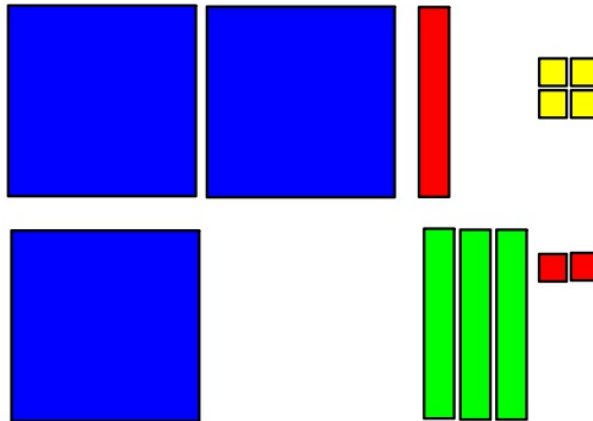
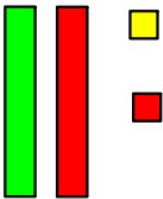
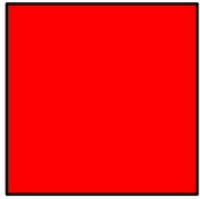
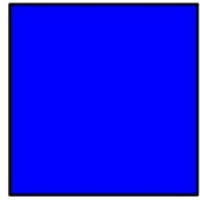
combine  $2x^2 - 1x + 4$  and  $1x^2 + 3x - 2$ .



## Algebra Tiles: Addition (cont.)

$(2x^2 - 1x + 4) + (1x^2 + 3x - 2)$  means to

combine  $2x^2 - 1x + 4$  and  $1x^2 + 3x - 2$ .

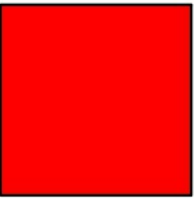
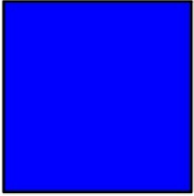


$$\begin{array}{r} 2x^2 - 1x + 4 \\ + 1x^2 + 3x - 2 \\ \hline 3x^2 + 2x + 2 \end{array}$$



Now let's try:

$$\begin{array}{r} 1x^2 + 3x - 4 \\ + -2x^2 - 4x - 2 \\ \hline \end{array}$$

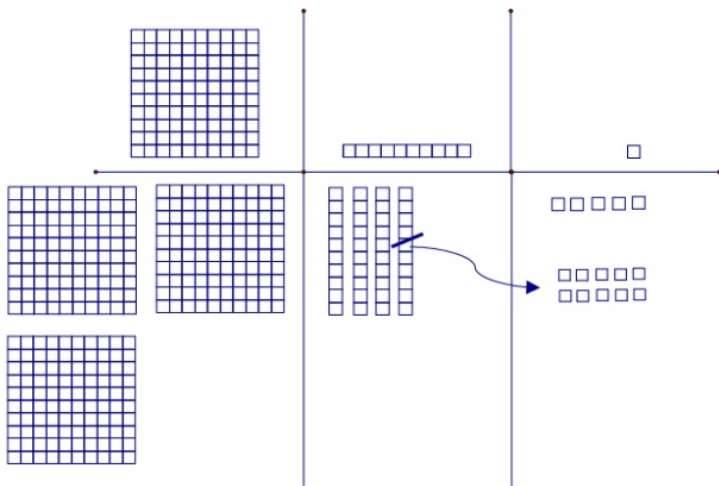


# Whole Number Subtraction

## Base-ten Blocks

$$\begin{array}{r} 345 \\ - 168 \\ \hline \end{array}$$

means to take 168 away from 345.

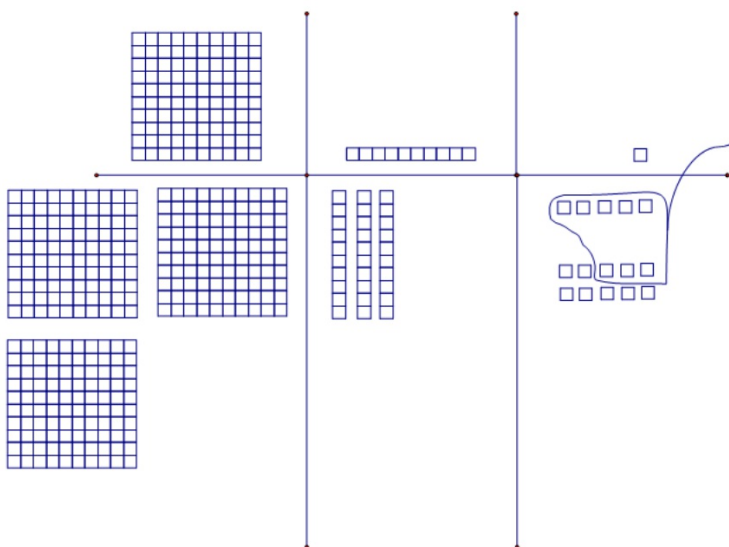


Since we cannot take 8 units from the 5 units, we need to trade a long for 10 units.

$$\begin{array}{r} 345 \\ - 168 \\ \hline \end{array}$$

## Whole Number Subtraction (cont.)

### Base-ten Blocks (cont.)



Represents the long that are left.

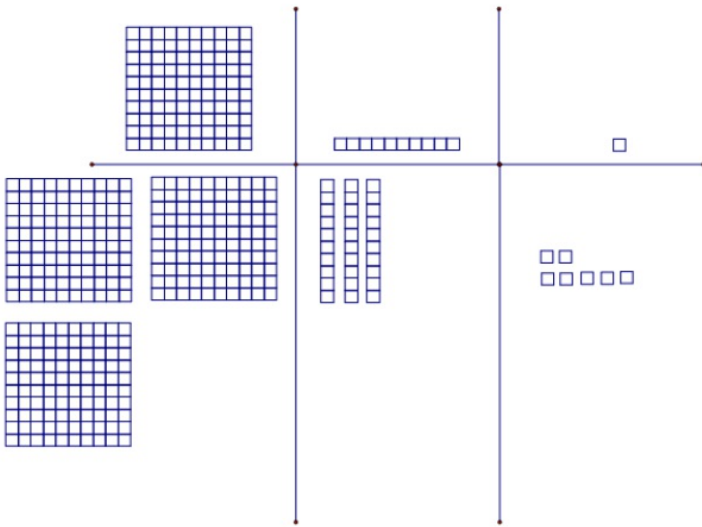
Number of units after trade.

$$\begin{array}{r} 3 \quad 15 \\ 3 \quad \cancel{4} \quad \cancel{8} \\ - 1 \quad 6 \quad 8 \\ \hline \end{array}$$

Now take away the 8 units.

## Whole Number Subtraction (cont.)

### Base-ten Blocks (cont.)

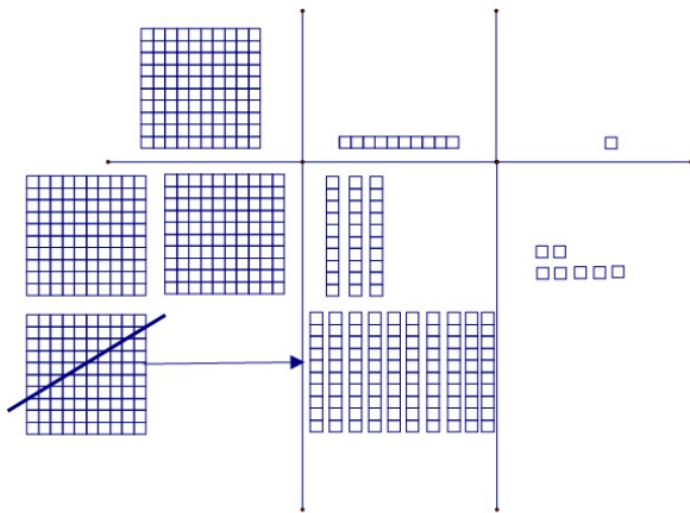


$$\begin{array}{r} 315 \\ - 168 \\ \hline \end{array}$$

Number of units that remain.

## Whole Number Subtraction (cont.)

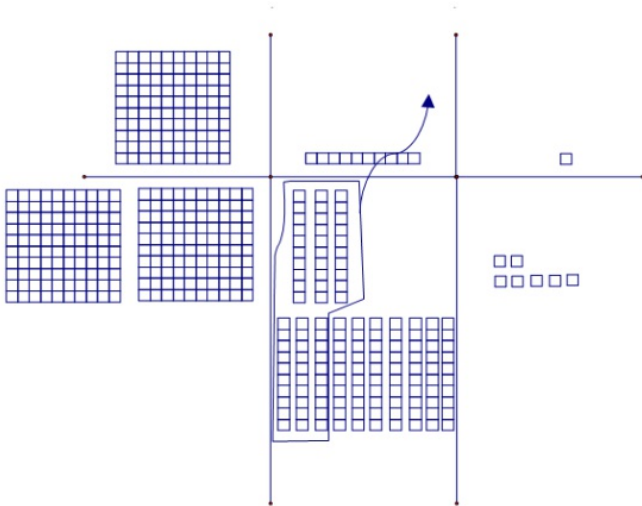
### Base-ten Blocks (cont.)



Since we cannot take 6 longs from the 3 longs we have we need to trade a flat for 10 longs.

## Whole Number Subtraction (cont.)

### Base-ten Blocks (cont.)



Represents the flats that are left.

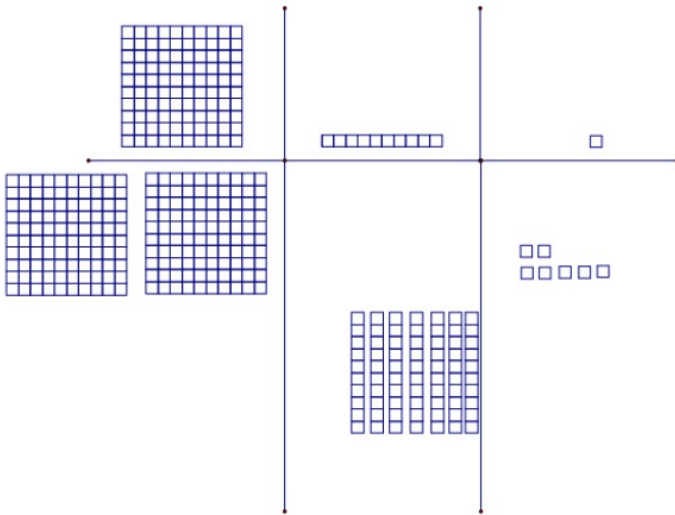
Number of longs after trade.

$$\begin{array}{r}
 13 \\
 2 \cancel{ } 15 \\
 \cancel{ } \cancel{ } \cancel{ } \\
 - 1 \ 6 \ 8 \\
 \hline
 7
 \end{array}$$

Now take away the six longs.

## Whole Number Subtraction (cont.)

Base-ten Blocks (cont.)

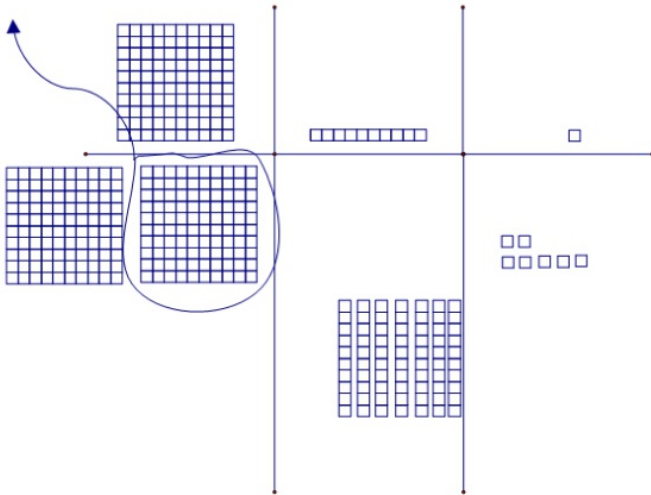


$$\begin{array}{r} 13 \\ 2 \cancel{5} 15 \\ \cancel{1} \cancel{6} \cancel{8} \\ - 168 \\ \hline 77 \end{array}$$

Number of longs that remain.

## Whole Number Subtraction (cont.)

### Base-ten Blocks (cont.)



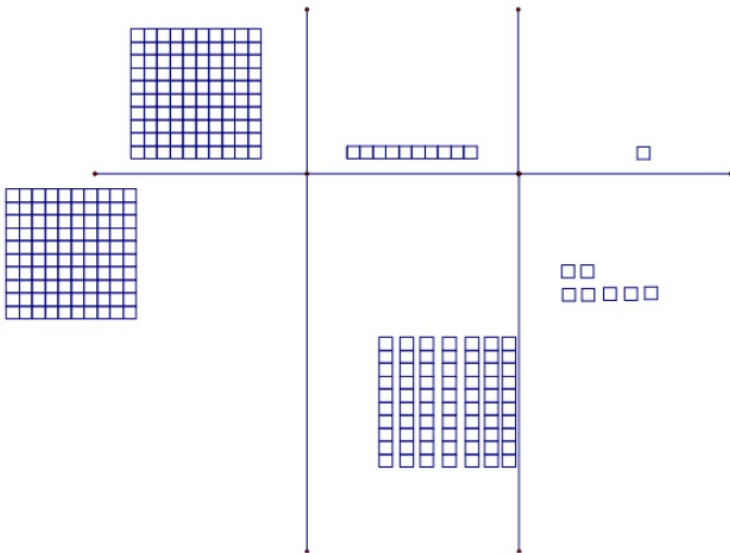
Now take away one flat from the two that are left.

$$\begin{array}{r} 13 \\ 2 \cancel{1} 15 \\ \cancel{1} \cancel{1} \cancel{1} \\ - 1 \quad 6 \quad 8 \\ \hline 7 \quad 7 \end{array}$$



## Whole Number Subtraction (cont.)

Base-ten Blocks (cont.)

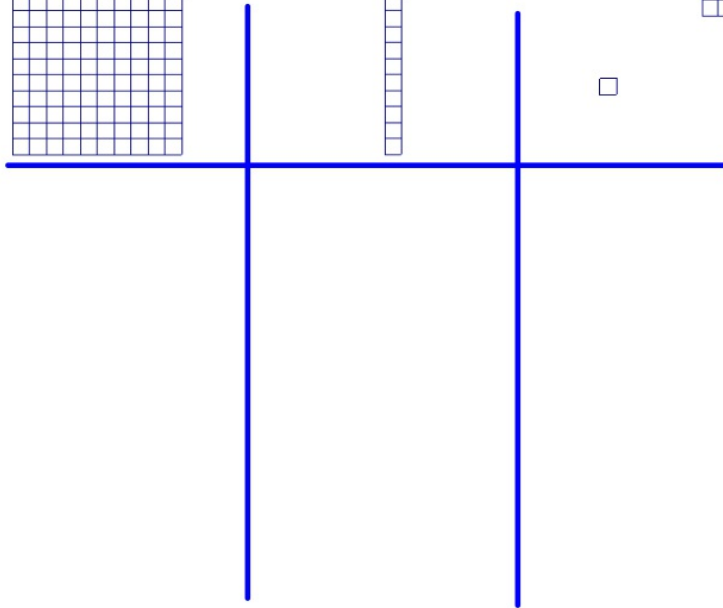
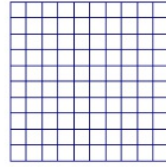
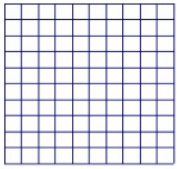


$$\begin{array}{r} 13 \\ 2 \cancel{5} \ 15 \\ \cancel{1} \ \cancel{4} \ \cancel{8} \\ - 1 \ 6 \ 8 \\ \hline 1 \ 7 \ 7 \end{array}$$

Number of flats that remain.

Now let's try:

$$\begin{array}{r} 364 \\ - 189 \\ \hline \end{array}$$



## Integer Subtraction

### Chip Model (take-away model)

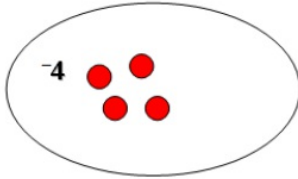
- Create one group. The group should have the color and quantity of chips corresponding to the minuend.
- Take away or move out the quantity and color of chips corresponding to the subtrahend.
- The resultant set is the difference.

- a.  $-4 - (-3)$  means *Start with a set equivalent to 4 red chips.  
Then take away 3 red chips.*

## Integer Subtraction (cont.)

### Chip Model (cont.)

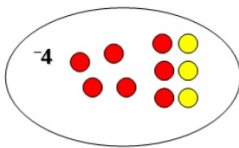
b.  $-4 - +3$  means *Start with 4 red chips. Then take away three yellow chips.*



However, there are not three yellow chips to be removed!

## Integer Subtraction (cont.)

Chip Model (cont.)



Now, three yellow chips can be removed.

## Integer Subtraction (cont.)

### *Subtraction Using the Additive Inverse*

Use your two color counters to find the solutions to each of the following problems.

a.  $2-8$

b.  $2-(-8)$

c.  $-12-(-5)$

d.  $-12-5$



a.  $2+(-8)$

b.  $2+8$

c.  $-12+5$

d.  $-12+(-5)$

**What do you notice about the two columns of problems? That is, how are they similar and how do they differ? What does this tell about the subtractions of integers?**

## Integer Subtraction (cont.)

### *Subtraction Using the Additive Inverse*

For all integers  $a$  and  $b$

$$a - b = a + ^{-}b.$$

Using the fact that  $a - b = a + (^{-}b)$ , solve the following problems.

a.  $5 - 9$

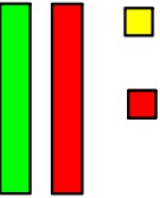
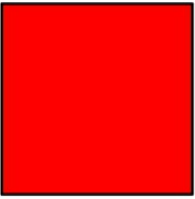
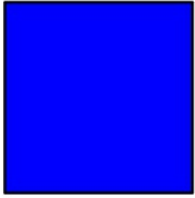
b.  $7 - (^{-}8)$

c.  $^{-}10 - (^{-}3)$

d.  $^{-}9 - 5$

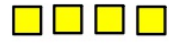
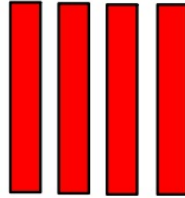
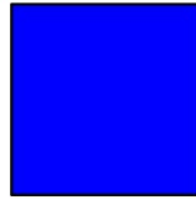
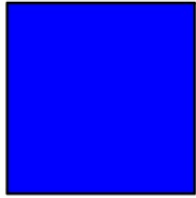
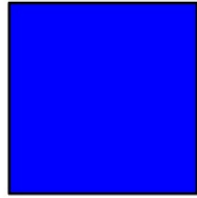
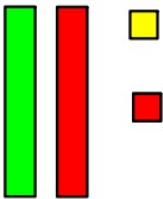
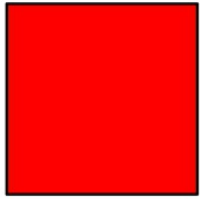
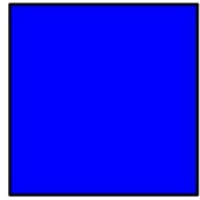
## Algebra Tile Subtraction

$(2x^2 - 1x + 4) - (-1x^2 + 3x - 2)$  means to take away  
 $-1x^2 + 3x - 2$  from  $2x^2 - 1x + 4$ .





## Algebra Tile Subtraction (cont.)



$$\begin{array}{r}
 2x^2 - 1x + 4 \\
 - (-1x^2 + 3x - 2) \\
 \hline
 3x^2 - 4x + 6
 \end{array}$$

## Algebra Tile Subtraction (cont.)

### *Subtraction Using the Additive Inverse*

Remember: For all integers  $a$  and  $b$   
 $a - b = a + ^{-}b.$

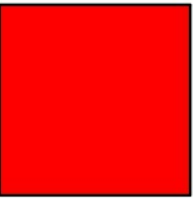
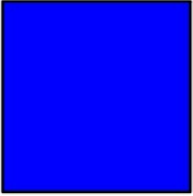
$$\begin{array}{r} 2x^2 - 1x + 4 \\ - (-1x^2 + 3x - 2) \\ \hline 3x^2 - 4x + 6 \end{array} \implies \begin{array}{r} 2x^2 - 1x + 4 \\ + (1x^2 + ^{-}3x + 2) \\ \hline 3x^2 - 4x + 6 \end{array}$$

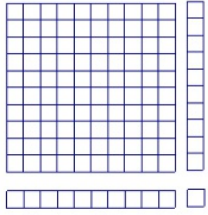
OR

$$\begin{aligned} & (2x^2 - 1x + 4) - (-1x^2 + 3x - 2) \\ &= (2x^2 + ^{-}1x + 4) - (-1x^2 + 3x + ^{-}2) \\ &= (2x^2 + ^{-}1x + 4) + ^{-}(-1x^2 + 3x + ^{-}2) \\ &= (2x^2 + ^{-}1x + 4) + (1x^2 + ^{-}3x + 2) \\ &= (2x^2 + 1x^2) + (^{-}1x + ^{-}3x) + (4 + 2) \\ &= 3x^2 + ^{-}4x + 6 \quad \text{OR} \quad 3x^2 - 4x + 6 \end{aligned}$$

Now let's try:

$$\begin{array}{r} 1x^2 + 3x - 4 \\ - (2x^2 + 4x + 2) \\ \hline \end{array}$$

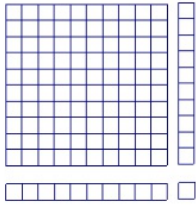




## Whole Number Multiplication

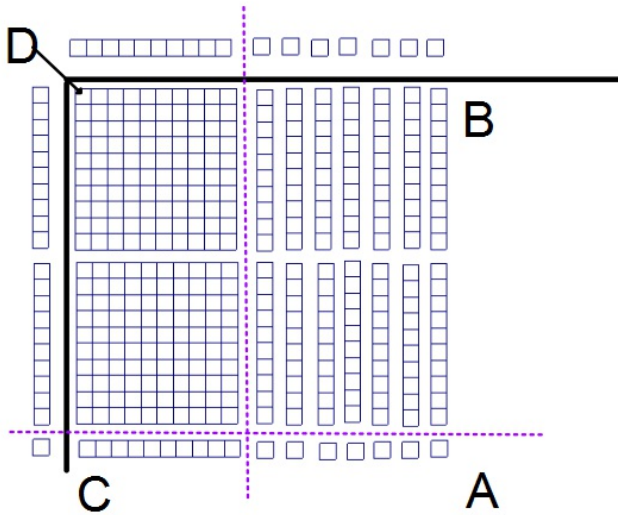
$$\begin{array}{r} 21 \\ \times 17 \\ \hline \end{array} \text{ means 21 rows of 17.}$$





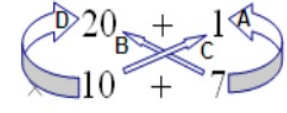
## Whole Number Multiplication (cont.)

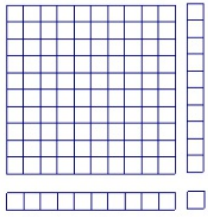
$$\begin{array}{r} 21 \\ \times 17 \\ \hline \end{array} \text{ means 21 rows of 17.}$$



$$\begin{array}{r} 21 \\ \times 17 \\ \hline 7 \\ 140 \\ \hline 357 \end{array} \begin{array}{l} \text{Region A} \\ \text{Region B} \\ \text{Region C} \\ \text{Region D} \end{array}$$

## Whole Number Multiplication (cont.)

$\begin{array}{r} 21 \\ \times 17 \\ \hline \end{array}$	$\Rightarrow$	$\begin{array}{r} 21 \\ \times 17 \\ \hline \end{array}$	$\Rightarrow$	
$\begin{array}{r} 7 \\ 140 \\ 100 \\ 200 \\ \hline 357 \end{array}$		$\begin{array}{r} 7 \\ 140 \\ 100 \\ 200 \\ \hline 357 \end{array}$		$\begin{array}{r} 7 \text{ A} \\ 140 \text{ B} \\ 100 \text{ C} \\ 200 \text{ D} \\ \hline 357 \end{array}$



Now let's try:

$$\begin{array}{r} 13 \\ \times 24 \\ \hline \end{array}$$



## Integer Multiplication

Chip Model:  $5 \times 4$  means to combine 5 groups of 4 yellow chips.

How many in all?



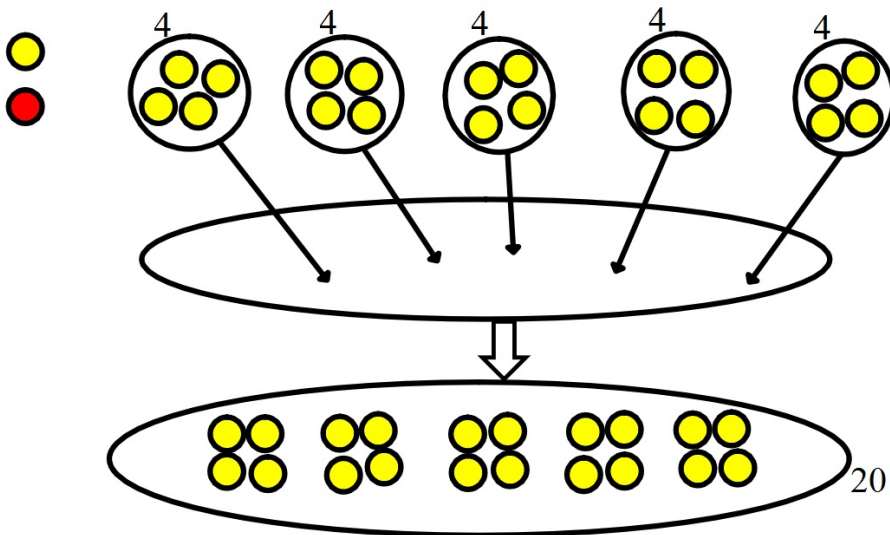


## Integer Multiplication (cont.)

Chip

Model:

$5 \times 4$  means to combine 5 groups of 4 yellow chips. How many in all?



So,  $5 \times 4 = 20$

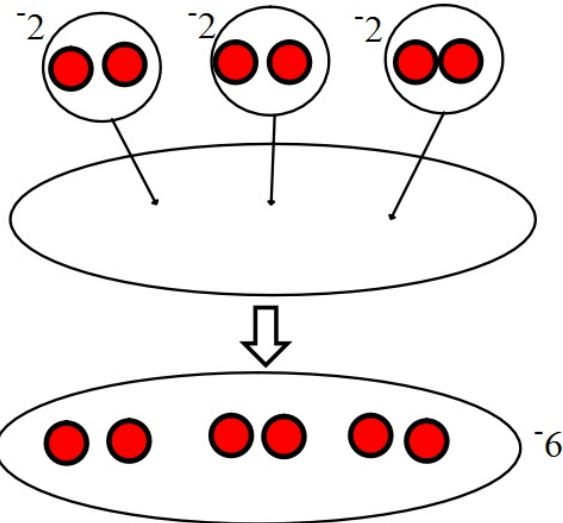
## **Integer Multiplication (cont.)**

$3 \times -2$  means to combine 3 groups of 2 red chips.  
How many in all?



## Integer Multiplication (cont.)

$3 \times^{-} 2$  means to combine 3 groups of 2 red chips.  
How many in all?



So,  $3 \times^{-} 2 =^{-} 6$ .

## Integer Multiplication (cont.)

$-3 \times 2$  means take away 3 groups of 2 yellow chips.  
What is the remaining value?



## Integer Multiplication (cont.)

$-3 \times -2$  means take away 3 groups of 2 red chips.  
What is the remaining value?



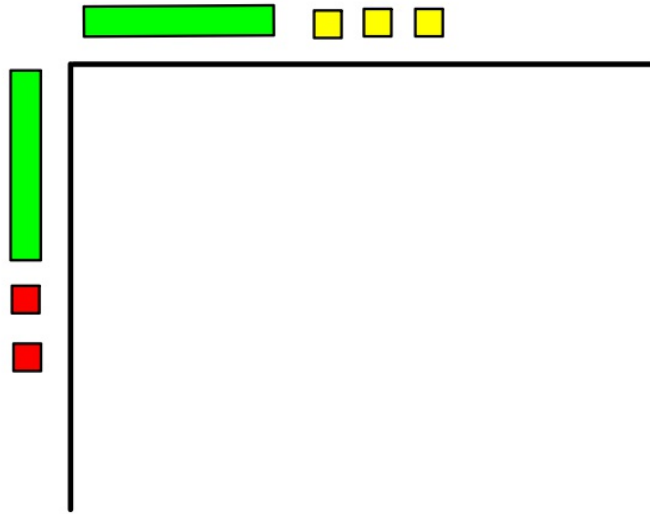
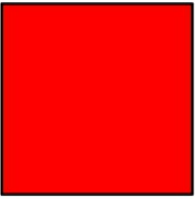
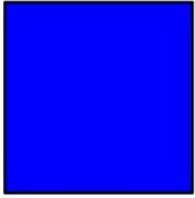
## **Integer Multiplication (cont.)**

When multiplying two non-zero integers with the **same signs (both positive or both negative)**: The product will be positive.

When multiplying two non-zero integers with **different signs (one positive and one negative)**: The product will be negative.

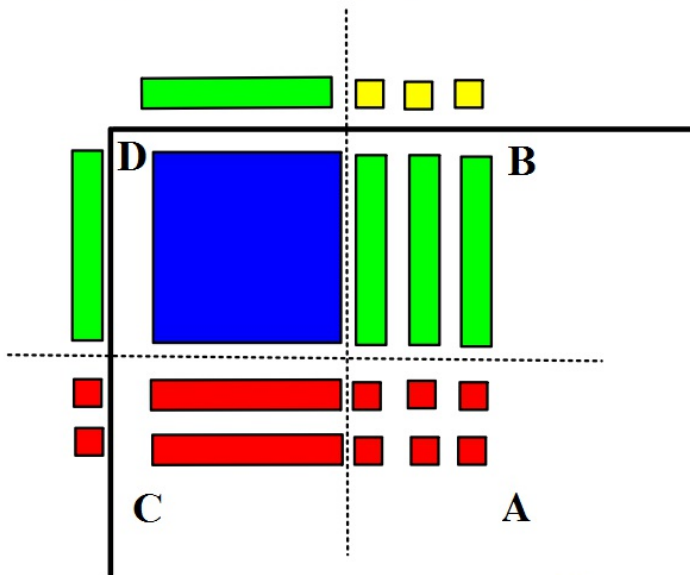
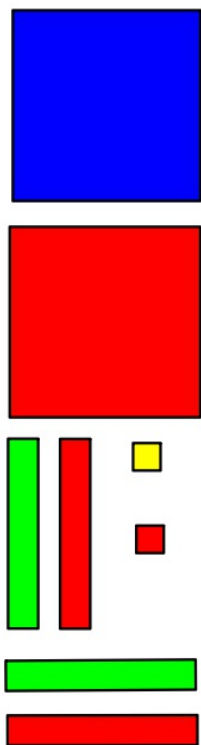
## Algebra Tile Multiplication (cont.)

$(x-2)(x+3)$  means  $(x-2)$  rows of  $(x+3)$



## Algebra Tile Multiplication (cont.)

$(x-2)(x+3)$  means  $(x-2)$  rows of  $(x+3)$

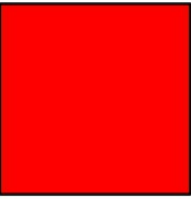
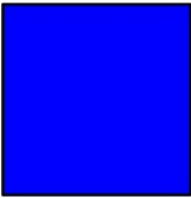


$$\begin{aligned} (x - 2)(x + 3) &= x^2 + 3x - 2x - 6 \\ &= x^2 + 1x - 6 \end{aligned}$$

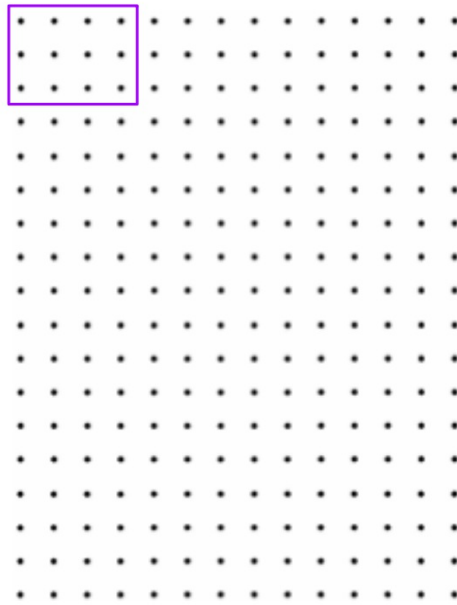
$x$	$-$	$2$	
$x$	$+$	$3$	
$x^2$	$-2x$	$3x$	$-6$
$x^2$	$+x$	$-6$	



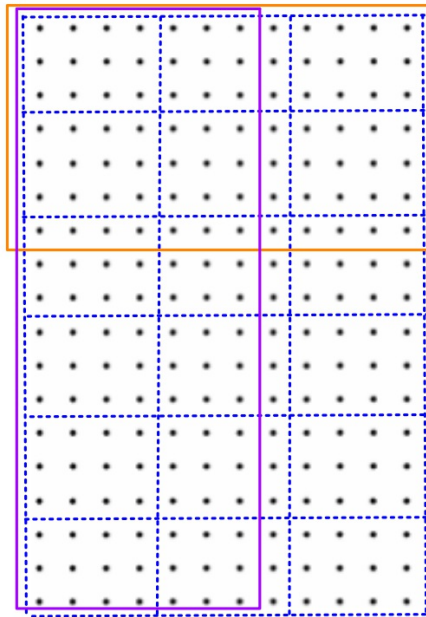
Now let's try:  $(x+4)(2x-1)$



$$\frac{2}{3} \times \frac{3}{4}$$



$$2\frac{1}{3} \times 1\frac{3}{4}$$



$$\begin{array}{r} 2 + \frac{1}{3} \\ \times 1 + \frac{3}{4} \\ \hline \frac{3}{12} \\ \frac{6}{4} \\ \frac{1}{3} \\ \hline 2 \end{array}$$