NEW PATHWAYS FOR STUDENTS NEEDING FOUR YEARS OF MATHEMATICS

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What is the General Idea?

- **To offer an alternative two-year program of rigorous, sophisticated, age-level interest mathematics to replace the traditional Algebra 2 and Precalculus, which may not be for every HS junior and senior.**

  Hands-On Statistics

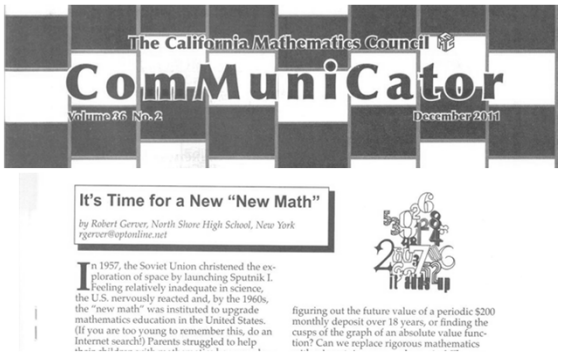
  Advanced Algebra with Financial Applications
- **Why? High school is our last chance to get kids to appreciate (like?!) and feel they can use mathematics.**
- **Appropriate placement has always been crucial for students and teachers; with APPR it is again brought to the forefront.**
- **How can we justify a "one-sequence fits all" curriculum and then claim that we want to differentiate instruction?**

Another Initiative??!!

- **Algebra for all**
- **Calculators for all**
- **Smart boards**
- **Differentiated instruction**
- **APPR**
- **Common Core State Standards**

Any initiative built upon incorrect course placement is doomed to failure. Does every kid need Algebra 2 and Precalculus in high school?

December 2011: CA Journal Article


The California Mathematics Council

ComMuniCator

Volume 36 No. 2 December 2011

It's Time for a New "New Math"

by Robert Gerver, North Shore High School, New York
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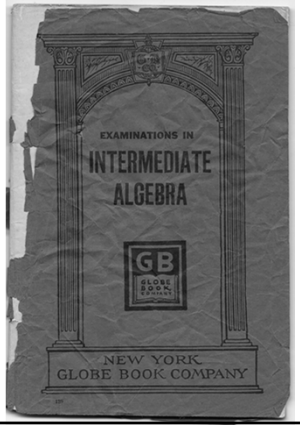
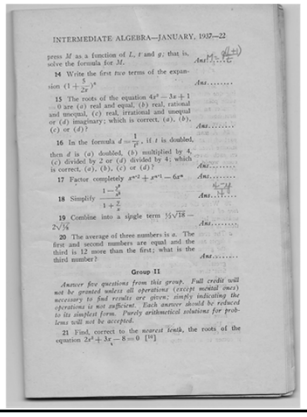
In 1957, the Soviet Union christened the exploration of space by launching Sputnik I. Feeling relatively inadequate in science, the U.S. nervously reacted and, by the 1960s, the "new math" was instituted to upgrade mathematics education in the United States. (If you are too young to remember this, do an Internet search!) Parents struggled to help

figuring out the future value of a periodic $200 monthly deposit over 18 years, or finding the cusps of the graph of an absolute value function? Can we replace rigorous mathematics



Are we "married" to certain Algebra 2 topics just out of habit and tradition?


INTERMEDIATE ALGEBRA—JANUARY, 1937—22

press M as a function of L, t and g; that is, solve the formula for M.

14 Write the first *five* terms of the expansion $(1+\frac{1}{2}x)^8$.

15 The roots of the equation $4x^2-3x+1=0$ are (a) real and equal, (b) real, rational and unequal, (c) real, irrational and unequal or (d) imaginary; which is correct, (a), (b), (c) or (d)?

16 In the formula $d=\frac{1}{2}gt^2$, if t is doubled, then d is (a) doubled, (b) multiplied by 4, (c) divided by 2 or (d) divided by 4; which is correct, (a), (b), (c) or (d)?

17 Factor completely $x^{n+2}+x^{n+1}-6x^n$.

18 Simplify $\frac{1-\frac{x}{y}}{1+\frac{x}{y}}$.

19 Combine into a single term $5\sqrt{18}-2\sqrt{72}$.

20 The average of three numbers is *a*. The first and second numbers are equal and the third is 12 more than the first; what is the third number?

Group II

Answer five questions from this group. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

21 Find, correct to the nearest tenth, the roots of the equation $2x^2+3x-8=0$. [10]

The January 1937 Intermediate Algebra Regents. Notice how much has not changed in 75 years! Does everyone need this material *at the same juncture* in their education?

HOW WELL DO YOUR STUDENTS RELATE TO CONCEPTS LIKE THESE?
HOW NECESSARY ARE THEY?

- **Dividing imaginary numbers**
- **Long polynomial division**
- **Graphing reciprocal trig functions**
- **Complex numbers in trigonometric form**
- **Polar Coordinates**
- **Complex Conjugates**

CAN MORE RELEVANT AND MORE USEFUL RIGOROUS MATHEMATICS BE USED TO ENGAGE AND INSPIRE STUDENTS?

Do You Like Your Cell Phone? TV? GPS? Car? iPod? Laptop?

"When am I ever going to use this?!"

Traditional higher-level mathematics is important; it is key to the creation of items we can't live without! It just might not be relevant for *everyone at the same point* in their schooling.

As a precursor to differentiating *instruction*, differentiating course *placement* is necessary.

MAKING THE CASE FOR NEW PATHWAYS

"Mathematics Education: A Way Forward"

by David Wees August 15, 2011

www.edutopia.org

"I don't see how it's doing society any good to have its members walking around with vague memories of algebraic formulas and geometric diagrams, and clear memories of hating them. We have many people who have learned school math, but don't apply it to their lives and hated learning it. Why do we think this is useful?"

"How to Fix Our Math Education"

By SOL GARFUNKEL and DAVID MUMFORD
NY Times, August 24, 2011

"Today, American high schools offer a sequence of algebra, geometry, more algebra, pre-calculus and calculus.

This highly abstract curriculum is simply not the best way to prepare a vast majority of high school students for life. … For instance, how often do most adults encounter a situation in which they need to solve a quadratic equation? Do they need to know what constitutes a "complex number"?… *Imagine replacing the sequence of algebra, geometry and calculus with a sequence of finance, data and basic engineering.* … Parents, state education boards and colleges have a real choice. …*The traditional high school math sequence is not the only road to mathematical competence.*"

From the Common Core State Standards for Mathematics
Appendix A:
Designing HS Math Courses Based on the CCSS

"A menu of challenging options should be available for students after their third year of math—and all students should be strongly encouraged to take math in all years of high school. Traditionally… students are expected to take precalc. This is a good and worthy goal, but should not be the only option…An array of challenging options will keep math relevant for students, and give them a new set of tools for their futures…"

Common Core State Standards for Mathematical Practices (MP1 – MP8)

The problems we encounter in the "real world"—our work life, family life, and personal health—don't ask us what chapter we've just studied and don' t tell us which parts of our prior knowledge to recall and use. In fact, they rarely even tell us exactly what question we need to answer, and they almost never tell us where to begin. They just happen. To survive and succeed, we must figure out the right question to be asking, what relevant experience we have, what additional information we might need, and where to start.

"A Radical Idea To Transform What Kids Learn In School"

by Marion Brady in the *Washington Post*, May 15, 2012

"America needs good mathematicians. How many? Take math teachers out of the mix, and the number of mathematicians America needs is tiny. If one kid in each high school in the country became a professional mathematician, it would glut the market.

…Running every kid in America through the math gauntlet to get a handful of mathematicians is like buying a bakery to get a loaf of bread."

"Is Algebra Necessary?"

Andrew Hacker, Ph.D., Queens College
NY Times—July 28, 2012

"…Ours is fast becoming a statistical age, which raises the bar for informed citizenship.

I propose that we start thinking of alternatives. Thus mathematics teachers at every level could create exciting courses in what I call "citizen statistics." The proposed course would familiarize students with the kinds of numbers that describe and delineate our personal and public lives…"

From "*Algebra II*"
By Professor James A. Sellers of The Pennsylvania State University

Algebra II is the fork in the road.

How does it make you feel?

64 vs 65?

How does it make kids feel?

"Only 14% of our kids failed Algebra 2. That's great!" LOL

Failure rates are mere statistics to guidance departments, but these "failures" are somebody's children. Setting up someone for failure is, at least, unacceptable, and, more likely, educational malpractice.

We Seriously Need to Re-Evaluate Our Outlook

- **If a student needs weekly AIS sessions, trips to the math lab, frequent extra help, daily homework help, and a tutor, to get a 65 in Algebra 2, is that really the correct placement?!**
- **Do we expect success in Geometry or Algebra 2 from someone who "squeaked through" Algebra 1, or do we need constructive intervention?**
- **To raise the level of math education in the United States, and to compete better with China and Japan, must *EVERY* kid have to have the same cookie-cutter curriculum? And at what cost to students who don't thrive in it?**

The subject we love is reputationally challenged!

"Working Financial Literacy in With the Three R's" by Tara Siegel Bernard
The New York Times April 9, 2010

"Most Americans aren't fluent in the language of money. Yet we're expected to make big financial decisions as early as our teens … even though most of us received no formal instruction on financial matters until it is too late. All of this raises the question: What's happening inside our classrooms? And how many schools even broach the topic? As it turns out, for a country that prizes personal responsibility, we're doing very little."

HOW DO THE COURSES FIT?

What Topics Comprise the Hands-On Statistics Program?

- **Descriptive statistics—Frequency distributions, measures of central tendency, dispersion, correlation, regression, transformations. Standard (z) scores. Normal curve calculations.**
- **Experimental Design.**
- **Probability-the basis for statistical inference.**
- **Inferential statistics—sampling distributions, confidence intervals and hypothesis tests.**

What Financial Topics Comprise the Advanced Algebra with Financial Applications Program?

- **Investing**
- **Starting Your Own Business**
- **Banking**
- **Credit**
- **Owning an Automobile**
- **Employment Basics**
- **Income Taxes**
- **Independent Living**
- **Retirement Planning**
- **Budgeting**

Covers these units using selected topics from Algebra 2, Precalculus, Statistics, Probability and Geometry that are taught at an ability-appropriate level for the Algebra 1-prerequisite audience.

What Mathematical Topics Comprise the Advanced Algebra with Financial Applications Program?

- Piecewise functions
- Linear and curvilinear regression
- Quadratic/linear systems
- Slopes and intercepts
- Inequalities
- Limits
- Maximization
- Exponential functions
- Greatest integer function
- Modified box & whisker plots
- Expected value
- Outliers
- Probability
- Graphing
- Solving equations
- Apothem, area, perimeter
- Rational functions
- Irrational functions
- Spreadsheets
- Literal equations
- Modeling

WHO IS THE TARGET AUDIENCE?

- **Students in need of a third or fourth-year math credit.**
- ***Any* students looking to take a math elective.**
- **Students who may have experienced difficulty in Algebra 1 and/or Geometry and may not be ready for Algebra 2 or Precalculus.**
- **Students who failed Algebra 2, and need another math course.**
- **Mathematically "disenfranchised" students.**

WHAT ARE SOME DIFFERENT SAMPLE PATHWAYS TO GRADUATION THAT INCLUDE HANDS-ON STATISTICS and/or ADVANCED ALGEBRA WITH FINANCIAL APPLICATIONS?

Freshman	Sophomore	Junior	Senior
Algebra 1	Geometry	**HOS**	**AAWFA**
Algebra 1	Geometry	**AAWFA**	**HOS**
Algebra 1	**AAWFA**	Geometry	**HOS**
Algebra 1	**AAWFA**	Geometry	Algebra 2
Two Year Algebra 1		**HOS or AAWFA**	Geometry
Algebra 1	Two-Year Geometry		**AAWFA**
Algebra 1	**HOS**	**AAWFA**	Geometry
Algebra 1	Geometry	Algebra 2	**AAWFA**
Geometry	Algebra 2	**AAWFA**	**HOS**
Geometry	**HOS**	**AAWFA**	Algebra 2
Geometry	Two-Year Algebra 2		**HOS or AAWFA**

Customize a One-Year Senior Course

•A fall semester of matrices, polar coordinates, limits, etc., and then a spring semester of five chapters of Advanced Algebra with Financial Applications—Automobiles, Employment, Income Taxes, Credit and Banking.

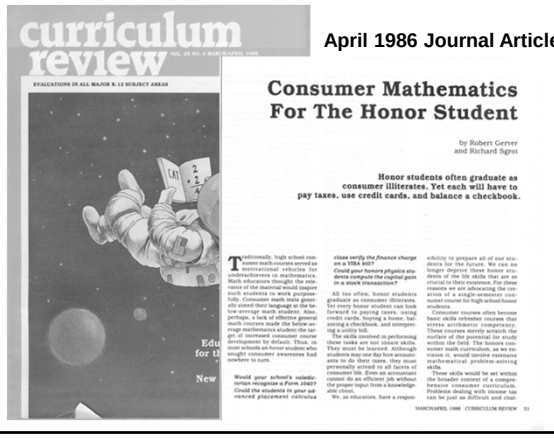
•A fall semester of introductory descriptive statistics and probability, and a spring semester of five chapters of Advanced Algebra with Financial Applications—Automobiles, Employment, Income Taxes, Credit and Banking.

WHY SHOULD SCHOOLS OFFER THE ALTERNATIVE PATHWAY?

- **The Algebra 1-only prerequisite allows students who did or would struggle with geometry proofs, trig, etc., to get a fresh start mathematically in an alternative course.**
- **Incorrect placement is unfair to the student.**
- **Incorrect placement is unfair to the teacher (APPR).**
- **Incorrect placement is unfair to classmates.**
- **Students who succeed at AAWFA and/or HOS may acquire the confidence and ability to then tackle Geometry and/or Algebra 2.**
- **It is a chance for students to gain confidence in, and an appreciation for, mathematics.**
- **All students *need* this material.**
- **It offers a mathematics course that addresses a current "hot topic" in education.**
- **It allows departments to graduate all students with 3 and 4 years of mathematics, and as a result could increase math enrollment.**

A CLOSER LOOK AT ADVANCED ALGEBRA WITH FINANCIAL APPLICATIONS

April 1986 Journal Article


curriculum review

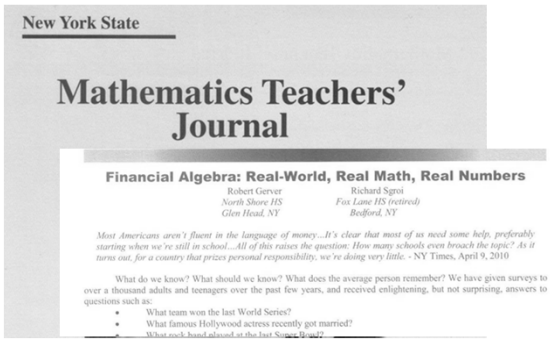
EVALUATIONS IN ALL MAJOR K-12 SUBJECT AREAS

Consumer Mathematics For The Honor Student

by Robert Gerver and Richard Sgroi

Honor students often graduate as consumer illiterates. Yet each will have to pay taxes, use credit cards, and balance a checkbook.

Fall 2011: NYS Journal Article on Financial Algebra


New York State

Mathematics Teachers' Journal

Financial Algebra: Real-World, Real Math, Real Numbers

Robert Gerver, *North Shore HS, Glen Head, NY*
Richard Sgroi, *Fox Lane HS (retired), Bedford, NY*

Most Americans aren't fluent in the language of money…It's clear that most of us need some help, preferably starting when we're still in school…All of this raises the question: How many schools even broach the topic? As it turns out, for a country that prizes personal responsibility, we're doing very little. - NY Times, April 9, 2010

What do we know? What should we know? What does the average person remember? We have given surveys to over a thousand adults and teenagers over the past few years, and received enlightening, but not surprising, answers to questions such as:

- What team won the last World Series?
- What famous Hollywood actress recently got married?
- What rock band played at the last Super Bowl?

ADVANCED ALGEBRA WITH FINANCIAL APPLICATIONS
FINANCIAL ALGEBRA **is aligned with the**
NATIONAL COMMON CORE STATE STANDARDS

NATIONAL COMMON CORE STATE STANDARDS

Financial Algebra by Gerver & Sgroi	Common Core Standard
In Financial Algebra, the mathematics necessary for daily living is embedded in content that directly relates to financial decisions adults make in their daily lives. The mathematical formulas, functions, and pictorial representations used in Financial Algebra assist students in making sense of the financial world around them through mathematical modeling and, equip them with the ability to make sound financial decisions based on data.	Mathematics' High School Modeling★ Modeling Standards *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).*

Financial Algebra Chapter & Section	Financial Algebra Page Numbers	Common Core Standard
		CHAPTER 1
C1 1-1	Pages 5-9	Algebra - Creating Equations★ A-CED **Creating equations that describe numbers or relationships** 1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* Algebra - Reasoning with Equations and Inequalities A-REI. **Solve equations and inequalities in one variable** 3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
C1 1-2 (continued on next page)	Pages 10-15	Number and Quantity - Quantities★ N-Q **Reason quantitatively and use units to solve problems** 1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
		Number and Quantity - Quantities★ N-Q **Reason quantitatively and use units to solve problems** 2. Define appropriate quantities for the purpose of descriptive modeling.

What are the Essential Elements of the Advanced Algebra with Financial Applications Classroom?

How is it the same as a "typical" math class?
Do now, motivation, development, model problems, practice, and applications problems.

How does it differ?
Discussion, passion due to age-level interest, reading, interpreting quotes, using outside resources, projects.

A TEACHER OCCASIONALLY ADMITTING
"I don't know—let's find out!"

WHY DO STUDENTS LIKE
ADVANCED ALGEBRA WITH FINANCIAL APPLICATIONS?

- **It treats them like an adult with age-level interest material.**
- **It finally gives them a place to see where they NEED mathematics.**
- **It gives them a chance to use their mathematical skills to save them money.**
- **The motivational topics are of current interest to them.**
- **They have a chance to discuss, comment, and argue in a mathematics class.**
- **They can succeed at it.**

HOW DOES
ADVANCED ALGEBRA WITH FINANCIAL APPLICATIONS
DIFFERENTIATE INSTRUCTION?

- **The problem sets generally graduate in difficulty level, making developing appropriate assignments a teacher-friendly process.**
- **Projects allow students to demonstrate knowledge in many alternative ways.**
- **Projects can be completed at many different skill levels.**
- **Sections and chapters can be skipped without loss of continuity.**
- **The course offering allows students to demonstrate mastery of rigorous math concepts in a format alternative to the traditional course path.**
- **Order of presentation of chapters can be changed.**

Two Key Approvals!

- **NCAA:** ***Financial Algebra*** **has received NCAA approval as a "core" mathematics course, and can be used in a college-preparatory mathematics sequence by potential NCAA college applicants.**

- **UC a - g:** ***Financial Algebra*** **has received University of California "a – g" approval as a 'c' level, core mathematics course,** ***Advanced Algebra with Financial Applications*****.**

It's of immediate interest to most high school students…

AUTOMOBILE INSURANCE

Mollie has 100/300/50 liability insurance, and $50,000 PIP insurance. She runs a stop sign and hits a telephone pole and bounces into a minivan with 8 people inside. Some are seriously hurt and sue her. Others have minor injuries. Three passengers in Mollie's car are also hurt.

a. The pole will cost $7,000 to replace. Mollie also did $6,700 worth of damage to the minivan. What insurance will cover this, and how much will the company pay?

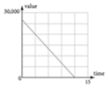
b. The minivan's driver was a concert violinist. The injury to his hand means he can never work again. He sues for $4,000,000 and is awarded that money in court. What type of insurance covers this, and how much will the insurance company pay?

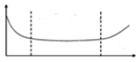
c. The minivan's driver (from part b) had medical bills totaling $60,000 from his hospital trip and physical therapy after the accident. What type of insurance covers this, and how much will the insurance company pay?

d. The three passengers in Mollie's car are hurt and each requires $12,000 worth of medical attention. What insurance covers this, and how much will the company pay?

How does your car appreciate or depreciate; linearly, exponentially, or like a historical "bath tub"?

AUTOMOBILE DEPRECIATION

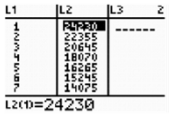




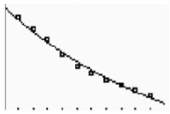
Celine bought a new car for $33,600. She made a $4000 down payment and pays $560 each month for 5 years to pay off her loan. She knows from her research that the make and model of the car she purchased is straight-line depreciated over 10 years.

EXPONENTIAL DEPRECIATION:

Mathematically modeling the loss of a constant ***percent*** **of value each year.**



AGE	VALUE		AGE	VALUE
1	24230		6	15245
2	22355		7	14075
3	20645		8	13100
4	18070		9	12325
5	16265		10	11525



Using Quadratic and Irrational Functions to Model Auto Accident Deconstruction

Simple arithmetic:

A car traveling 55 miles per hour covers 4840 feet per minute, or about 80 feet in one second. ***It covers 60 feet in the reaction time of ¾ second!***

A quadratic function:

Braking Distance = $5(.1s)^2$, where s = speed

A square root function:

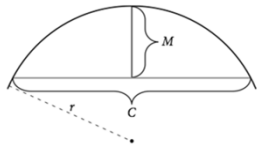
Skid speed S = $\sqrt{30Dfn}$

S = speed entering skid; D = skid distance; f= drag factor (an index); n = braking efficiency (an index).

YAW MARKS: Geometry and Irrational Functions

C: length of yaw mark chord
M: length of middle ordinate
S: minimum speed
***f*: drag factor**
***r*: radius of yaw mark arc**

$S = \sqrt{15fr}$ $\quad r = \frac{C^2}{8M} + \frac{M}{2}$



Getting a "Feel" for Compound Interest

Jennifer has a bank account that compounds interest daily at a rate of 3.2%.

On the morning of Feb 10 the principal is $1,234.98. That day she withdraws $200.

Later that day she is mailed a $34 check, and she deposits that in the bank.

On Feb 11, she deposits her $345.77 paycheck.

What is her balance at the end of the day on Feb 11?

Date→	Feb 10	Feb 11
Opening Balance	$1,234.98	$1,069.07
Deposit (+)	$34.00	$345.77
Withdrawal (-)	$200.00	---
Principal Used to Compute Interest	$1,068.98	$1,414.84
Day's Interest rounded to the nearest cent	$0.09	$0.12
Ending Balance- (also tomorrow's opening balance)	$1,069.07	$1,414.96

Derivation of Formulas

After the calendar introduction, students use examples of semi-annual and quarterly compounding to find patterns and derive the compound interest formula

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

They use tables on a calculator to look for a pattern and evaluate

$$e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$$

They use $B = Pe^{rt}$ to compute continuous compounding.

Promissory note terms, loan interest, lending institutions, loans, credit ratings, computing average daily balances and finance charges on a credit card, credit worthiness.

CONSUMER CREDIT

What is the monthly payment on a $50,000,10-year loan with an APR of 4.1% ?

The monthly loan payment formula must be carefully entered into a calculator—understanding the placement of the parentheses is crucial!

$$M = \frac{\left(P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{12t}\right)}{\left(\left(1+\frac{r}{12}\right)^{12t}-1\right)}$$

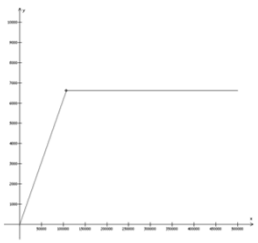
MORTGAGES

The mathematics is taught alongside the vocabulary.

adjustable rate mortgage
assessed value **closing costs**
back-end ratio **balloon mortgage**
debt-to-income ratio **escrow**
foreclose **front-end ratio**
homeowner's insurance
interest only **market value**
mortgage **property taxes**

What is that "FICA" box on your paystub?
SOCIAL SECURITY & MEDICARE PAYROLL TAXES

For 2012, the Social Security Tax maximum salary was $110,100. The tax rate was 4.2% of all gross earnings up to this maximum. (A piecewise function with a cusp).



How Can You Model Federal Tax Schedules?

Schedule X—If your filing status is Single

If your taxable income is: Over—	But not over—	The tax is:	of the amount over—
$0	$8,500	..... 10%	$0
8,500	34,500	$850.00 + 15%	8,500
34,500	83,600	4,750.00 + 25%	34,500
83,600	174,400	17,025.00 + 28%	83,600
174,400	379,150	42,449.00 + 33%	174,400
379,150	.....	110,016.50 + 35%	379,150

$$t(x) = \begin{cases} .10x \text{ if } 0 < x \le 8500 \\ .15(x-8500)+850 \text{ if } 8500 < x \le 34500 \\ .25(x-34500)+4750 \text{ if } 34500 < x \le 83600 \\ .28(x-83600)+17025 \text{ if } 83600 < x \le 174400 \\ .33(x-174400)+42449 \text{ if } 174400 < x \le 379150 \\ .35(x-379150)+110016.50 \text{ if } x > 379150 \end{cases}$$

Using Algebra to Model Tax Worksheets

Schedule X—If your filing status is Single

If your taxable income is: Over—	But not over—	The tax is:	of the amount over—
$0	$8,500	..... 10%	$0
8,500	34,500	$850.00 + 15%	8,500
34,500	83,600	4,750.00 + 25%	34,500
83,600	174,400	17,025.00 + 28%	83,600
174,400	379,150	42,449.00 + 33%	174,400
379,150	.....	110,016.50 + 35%	379,150

Express algebraically: *t(x)* = .33(x – 174400) + 42449
Get into y = mx + b form: *t(x)* = 0.33*x* – 15103
This is what the IRS uses on the tax worksheet:

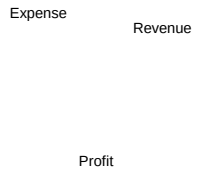
Section A—Use if your filing status is **Single.** Complete the row below that applies to you.

Taxable income. If line 43 is—	(a) Enter the amount from line 43	(b) Multiplication amount	(c) Multiply (a) by (b)	(d) Subtraction amount	Tax. Subtract (d) from (c). Enter the result here and on Form 1040, line 44
At least $100,000 but not over $174,400	$	× 28% (.28)	$	$ 8,543.00	$
Over $174,400 but not over $379,150	$	× 33% (.33)	$	$ 15,103.00	$
Over $379,150	$	× 35% (.35)	$	$22,589.00	$

How can profit be modeled as the difference between a quadratic and linear function?

PROFIT = REVENUE - EXPENSES

Students get q in terms of p from the demand function, combine like terms, and view the profit parabola algebraically and graphically as the difference between revenue and profit.



How many BTU's do I need?
HOME OWNERSHIP

Mike's bedroom measures 16 feet by 14 feet, and has a 9-foot ceiling. It is well-insulated, and is on the west side of his house. He wants to purchase an air conditioner. How large an air conditioner should he purchase?

$$\text{BTU rating} \approx \frac{while}{60}$$

l, w, h = length, width, height
i = insulation (an index)
e = exposure (an index)

Combining piecewise functions and the greatest integer function to model

CELL PHONE EXPENSES

A cell phone calling plan has a basic charge per month, which includes a certain amount of free minutes. There is a charge for each additional minute. The split function below gives the price f(x) of an x-minute phone call. Fractions of a minute are charged as if they were a full minute.

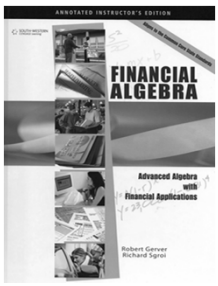
Describe the cost of the plan by interpreting the split function.

$$f(x) = \begin{cases} 40 \text{ if } x \le 750 \\ 40 + 0.35(x - 750) \text{ if } x > 750 \text{ and } x \text{ is an integer} \\ 40 + 0.35(\lceil x - 750 \rceil + 1) \text{ if } x > 750 \text{ and } x \text{ is not an integer} \end{cases}$$

Combining the Dozens of Expenses Addressed in Advanced Algebra With Financial Applications:
HOUSEHOLD BUDGET SPREADSHEETS

Students learn how to use algebra to set up cell formulas when creating spreadsheets for BUDGET, CASH FLOW, and DEBT REDUCTION analysis.

The scope and sequence of Advanced Algebra with Financial Applications, as covered in this presentation, is contained in the book



Financial Algebra:
Advanced Algebra with Financial Applications
by Gerver/Sgroi

A *brand-new* text--not an adaptation of an arithmetic-based personal finance book. For more information, check out this link:

www.cengage.com/community/financialalgebra

The Jump Start Coalition for Personal Financial Literacy- ***Get on their mailing list!***

919 Eighteenth Street NW
Suite 300
Washington, DC 20006-5517
www.jumpstart.org

A CLOSER LOOK AT

HANDS-ON STATISTICS

AP Statistics or Hands-On Statistics?

- You must inform parents and guidance counselors about AP Stat vs non-AP Stat.
- Hands-on Statistics is a subset of AP Stat, with activities and projects added that may or may not be in a selected school's AP Stat course.
- Kids like to "dabble" in AP and they can in history, English and science—they never could in math because they had to have certain prerequisites to take AP Calc.
- Guide students to pick the correct course to optimize their success.
- Schedule AP Stat and non-AP Stat the same period to facilitate easy program changes.

Pirating Statistics Activities

- Every stat book has a teacher's manual with hands-on activities.
- Some textbooks have hands-on activities.
- Internet searches will turn up tons of activities!
- You can make up your own activities.
- Probability activities abound.
- Plan a curriculum project to have all activities ready before the school year begins.

Die-Within-a-Die Activity

Students run Chi-Square Goodness of Fit tests and Chi-Square Test of Association and Independence to see if the inside die is truly moving freely.

Chi-Square Goodness of Fit Activity: Checking the Distribution of m&m's in a Given Bag

Statistical Abstract of the United States

Get old copies from the library or go online- hundreds of tables of data!

WHERE IS THE MEAN?

MATERIALS: For this activity, you will need a pair of dice, a calculator, and this sheet.

PROCEDURE:

1. Roll the pair of dice ten times. Find the sum of the numbers on the two dice. We will call these sums x_1 through x_{10}. Record the ten sums in this table:

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}

2. Plot the ten sums on the number line below.

3. Find the mean of the ten sums. $\bar{x}$ = _____. Do not round.

4. Plot the mean on the number line above.

5. Find the differences between the score and the mean for each score. Enter on the table below. Do not round.

$x_1-\bar{x}$	$x_2-\bar{x}$	$x_3-\bar{x}$	$x_4-\bar{x}$	$x_5-\bar{x}$	$x_6-\bar{x}$	$x_7-\bar{x}$	$x_8-\bar{x}$	$x_9-\bar{x}$	$x_{10}-\bar{x}$

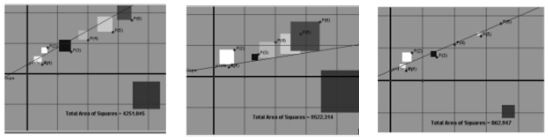
6. Find the sum of the differences._____

7. Compare your difference to the differences found by all other students in the class.

This die-roll activity helps students discover that the sum of the directed distances from the mean is always zero.

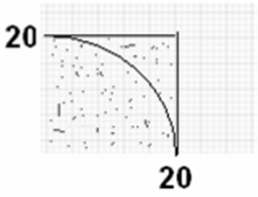
Internet Activity: Finding Least Squares

Google "Least Squares Geometer's Sketchpad" (there are other similar sites)



Students move line until the sums of the squares are as small as possible.

Activity: Monte Carlo π




```
rand20
      3.461873027
      12.29366859
      1.137952347
      15.60947377
       12.6951618
      17.18407631
```

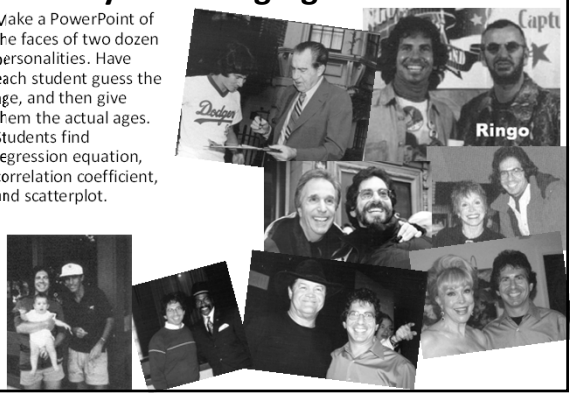
Gives one point

Gives another point

Gives a third point

Activity: Guessing Ages from Photos

Make a PowerPoint of the faces of two dozen personalities. Have each student guess the age, and then give them the actual ages. Students find regression equation, correlation coefficient, and scatterplot.



Activity: "Make the Lurk Work"

Go to *Statistical Abstract of the United States*. Find two sets of two variables each. Make sure the explanatory variable, x, is the same for both sets of data. Pick sets of data that are highly correlated.

1. Find the correlation coefficient for each set of data.

2. Find the coefficient of determination for each set of data. Explain what it means in a sentence.

3. Make sure your two explanatory variable data sets have the exact same x-values. Leave out some data if you need to to make this so.

4. Put the two response variables into a 2-column table. Find the correlation between the two sets of y-values. Must this correlation be high since the original data sets have high r, and they have the same explanatory variables? Explain.

5. Give a reasonable, fictional theory on how your two y-values could actually have a cause-and-effect relationship. Attempt to legitimize this theory no matter how ridiculous it may sound. Have fun with it!

6. Identify and explain the role of a lurking variable in making your response variable associate so highly. Try to be realistic.

Activity: Sports Name Recognition

Generating Data to Investigate Measures of Spread
Range, MD, MAD, Variance and Standard Deviation

	x	(x – M)	Σ(x-M)	Σ(x-M)²
1. Phil Mickelson				
2. Eli Manning				
3. Albert Pujols				
4. Serena Williams				
5. John Tortorella				
6. Lance Armstrong				
7. Nathan Adrian				
8. Robinson Cano				
9. Novak Djokovic				
10. LeBron James				

Sports Arbitration Case Project

In certain sports, when a player's contract is up, he can go to arbitration. The team and the player each get a lawyer to argue their case. The player's lawyer compiles reasons for a high salary, and the team's lawyer compiles reasons for a lower salary. They present their cases to an arbitrator who then makes a final, binding decision which cannot be appealed. Obviously both lawyers are stuck using the same set of numbers that the player has compiled.

How can the same numbers be twisted to back up two opposing cases?!

You are going to give an argument for the two sides of a hypothetical arbitration case for a ***real*** player, using the ***real*** statistics. You can get the data online or in the library.

You can use subsets of the data, more recent data, and any statistical analyses to prove your point.

Use graphs—side by side boxplots, scatterplots, histograms, stem plots, etc

Use confidence intervals and/or hypothesis tests.

Activity: Auto Sales and Purchases

Using Regression and Modified Boxplots to Gain an Advantage

Students use newspapers and the Internet to find several prices for the same model.

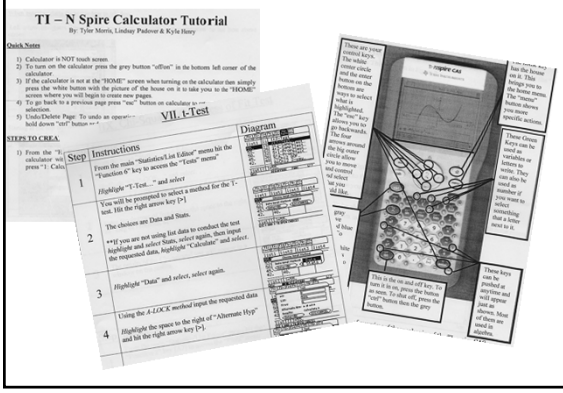
Price
$22,000
$19,000
$18,000
$16,700
$15,900

Mileage, x	Price, y
21,000	$22,000
30,000	$19,000
40,000	$18,000
51,000	$16,700
55,000	$15,900

Prices can be turned into a modified boxplot, and regression analysis can be done on the mileage and the price.

$8,500 $8,500 $8,500 $9,900 $10,800 $10,800 $11,000
$12,500 $12,500 $13,000 $13,000 $14,500 $23,000

PROJECT: Writing a Calculator Manual for Stat Functions



Activity: The Birthday Problem

How many randomly-selected people would you have to have in a room to have the probability of a matching birthday be at least 50%?

1. How many people would guarantee a match?

2. An educated guess—how many people would make the probability 50% or better for a match?

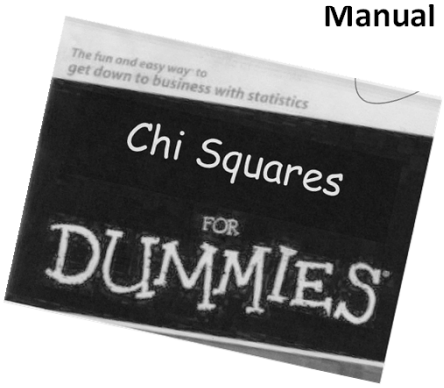
3. If you were forced to bet one way or another, how many people would have to be in the room before you'd bet there was a match?

4. Get in a group of two. Plan a simulation with 15 repetitions, each with a group of 23 people, just to get a gauge on the percent of matches. *Use the random integer and Sort features to make this easy.* How many of your 15 simulations had matches?

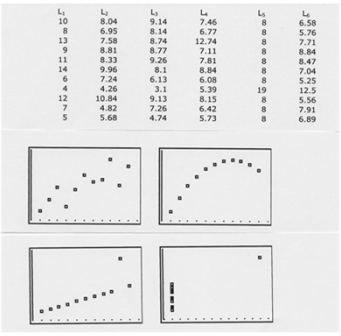
5. Let's pool the class' data and get a percent of groups of 23 that had matches.

6. Let's do some math. Hint—to find the probability of one or more matches, find the probability of the complement—no matches.

Writing a "Dummies" Manual

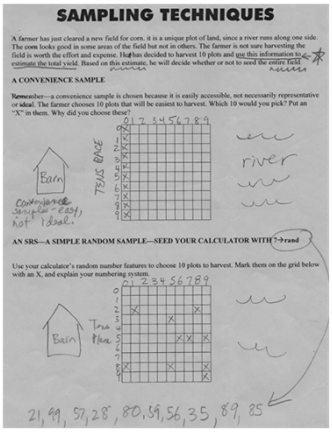


ACTIVITY: WHY DO WE NEED TO LOOK AT SCATTERPLOTS AFTER WE HAVE ALL OF THE REGRESSION LINE AND CORRELATION COEFFICIENT INFORMATION?



L_1	L_2	L_3	L_4	L_5	L_6
10	8.04	9.14	7.46	8	6.58
8	6.95	8.14	6.77	8	5.76
13	7.58	8.74	12.74	8	7.71
9	8.81	8.77	7.11	8	8.84
11	8.33	9.26	7.81	8	8.47
14	9.96	8.1	8.84	8	7.04
6	7.24	6.13	6.08	8	5.25
4	4.26	3.1	5.39	19	12.5
12	10.84	9.13	8.15	8	5.56
7	4.82	7.26	6.42	8	7.91
5	5.68	4.74	5.73	8	6.89

If you compute the regression line and correlation coefficient for the bivariate data from
L1, L2
L1, L3
L1, L4
L5, L6
you will get the same regression equation and correlation coefficient! Students discover that it is necessary to examine the scatterplot!



Activity: Types of Samples

Students need to decide which type of sample- convenience, simple random sample, or stratified sample, gives the most accurate gauge of the typical plot in a parcel of land.

Newspaper Project on Experimental Design

The two-week assignment requires students to find 6 articles (3 newspaper and 3 Internet), and give pertinent information on each, including:
Number of subjects
Who conducted the study
Where study was conducted
When study was conducted
Design
Hypotheses
Findings

What Are Balloon-Help Tutorials?

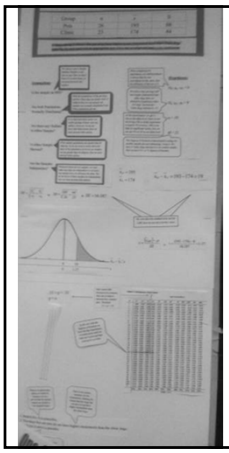
- **Designed to gradually break that old math habit—"boxing" numerical answers devoid of any verbal explanation.**
- **They require students to explain selected (or all) aspects of a solution to a problem; enhancing it with anything they feel helps explain the problem.**

Benefits of Balloon-Help Tutorials

- **Gets students in the habit of writing original, complete sentences more often.**
- **"If you can say it, you don't know it."**
- **Gets the writing practice frequent, consistent, and spaced out through the year.**
- **Writing practice translates to better free response answers.**
- **An alternative form of assessment.**
- **The grade from these projects can be used in many ways.**
- **Can be used for extra credit options, pinpointed on specific student trouble areas.**
- **Makes for a great showcase or bulletin board. Raises profile of the Hands-On Statistics class.**
- **By-product of trying to teach them the writing skills is they learn they math they are working on.**

Bulletin Board Balloons

Add Stat quotes found online!



Creating balloon sentences requires knowledge and expression of that knowledge.

Selected Stat Textbook Titles

Use as primary texts and/or as instructor resources

- **Statistics in Action—Watkins, Scheaffer, Cobb**
- **Statistics Through Applications—Starnes, Yates, Moore**
- **Activity-Based Statistics—Scheaffer, Watkins, Gnanadesikan, Witmer**
- **Workshop Statistics—Rossman, Chance**

Get samples at conferences, look online, and go to used book websites for out-of-print books.

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