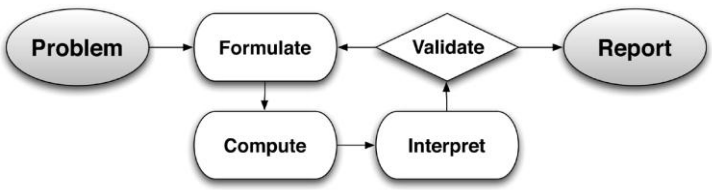
USING LINEAR FUNCTIONS TO MODEL MATH IDEAS



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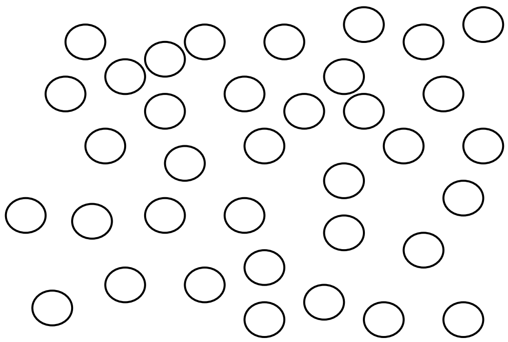
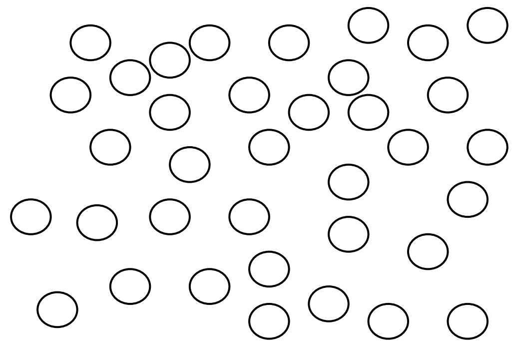
LEFTY-RIGHTY EXPERIMENT
(From *MathLinks*: Linear Functions 1.3)

1. Use your left hand to write as many X's as you can inside the circles. Wait for your teacher's signal to start. Stop when your teacher says time is over.



2. Use your right hand to write as many X's as you can inside the circles. Wait for your teacher's signal to start. Stop when your teacher says time is over.



3. Count the number of circles that have an X on your **lefty** side. _________________

4. Count the number of circles that have an X on your **righty** side. _______________

5. Record class data.

# of circles with X's on the **lefty** side (*x*)												
# of circles with X's on the **righty** side (*y*)												

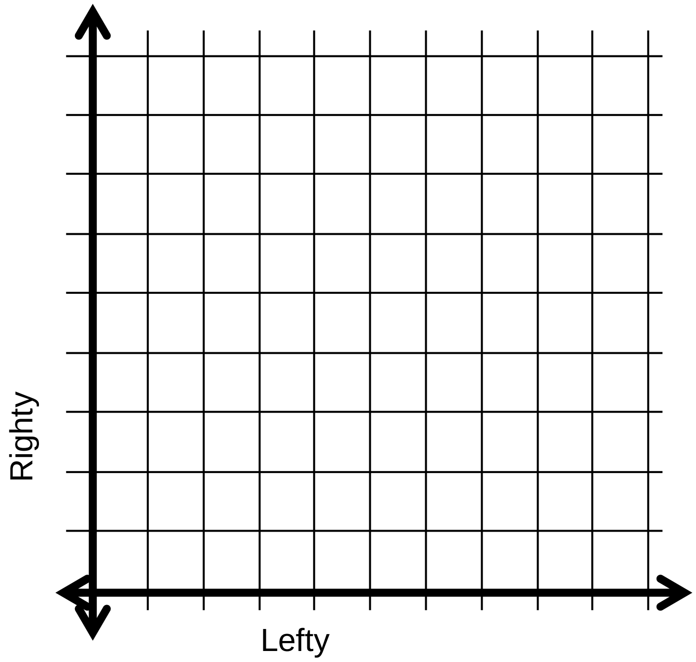
# of circles with X's on the **lefty** side (*x*)												
# of circles with X's on the **righty** side (*y*)												

# of circles with X's on the **lefty** side (*x*)												
# of circles with X's on the **righty** side (*y*)												

EXPLORING LEFTY RIGHTY DATA

1. Make an appropriate scale on the graph so that lefty-righty data will fit on it. Use the same scale for both the horizontal and vertical axes.

2. Graph the line $y = x$. Then graph all coordinate pairs from the table on the previous page.



Using your graph, complete the questions below.

1. How many data points are **on** the line $y = x$? _______ What do these data points mean in the context of the experiment?

2. How many data points are **above** the line $y = x$? ______ What do these data points mean in the context of the experiment?

3. How many data points are **below** the line $y = x$? _______ What do these data points mean in the context of the experiment?

4. Where do most of the data points lie, above or below the line $y = x$? _________ What does this tell us about the class' ability to cross off the circles?

5. An <u>outlier</u> of a data set is a data value that is unusually small or unusually large relative to the overall pattern of values in the data set.

   Do you see any potential outliers in your lefty-righty data set? _______ What does this tell us in the context of the experiment?

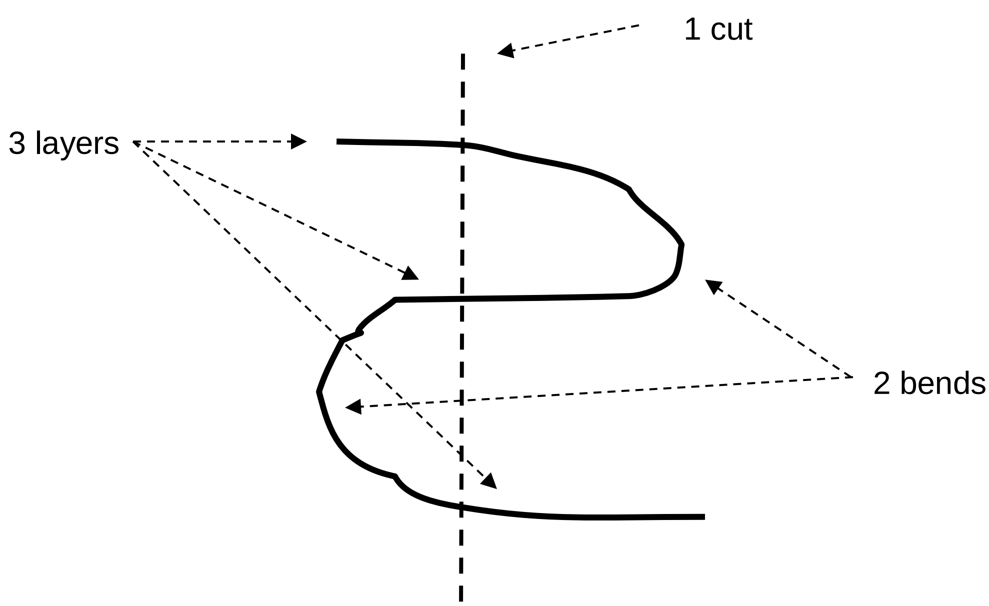
6. <u>Clustering of data</u> refers to a group of numbers where members of each group surround a particular number. Does there appear to be any clustering of the data points? _______ Explain that this means in the context of the experiment.

CUTTING THE ROPE

(From MathLinks: Linear Functions 4.3)

INTRODUCTION

- Start with one long piece of rope with 3 “layers” and 2 “bends.”
- If 1 vertical cut is made through all 3 layers as shown:



1. How many pieces of rope are there?

2. Draw a 2^{nd} vertical line that cuts through each layer again.
   How many pieces of rope are there now?

CUTTING THE ROPE:
PICTURES, NUMBERS, AND SYMBOLS

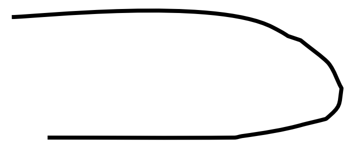
Explore cutting the rope for different numbers of layers and cuts.

1. # of layers = 1

# of cuts(c)	# of pieces (p)

Rule for any number of cuts:
p =

2. # of layers = 2



# of cuts(c)	# of pieces (p)

Rule for any number of cuts:
p =

3. # of layers = 3

# of cuts (c)	# of pieces (p)

Rule for any number of cuts:
p =

4. # of layers = 4

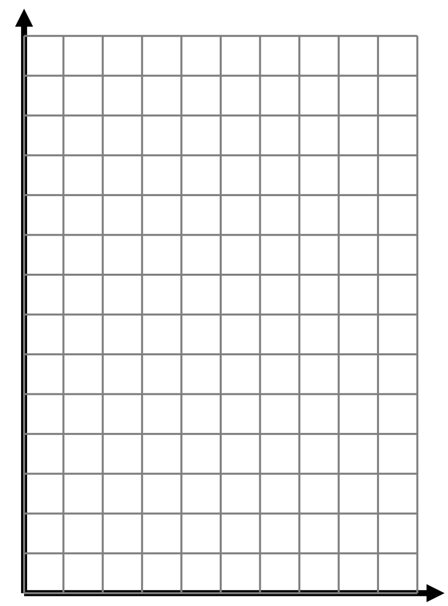
# of cuts (c)	# of pieces (p)

Rule for any number of cuts:
p =

CUTTING THE ROPE:
GRAPHS, SYMBOLS, AND WORDS

1. For each table, plot the points on the grid. Graph each set of points with a different color.

2. What is the same about each graph?



3. What is different?

4. What does the y-intercept represent for each graph in terms of the cutting rope experiment?

5. What does the slope tell us?

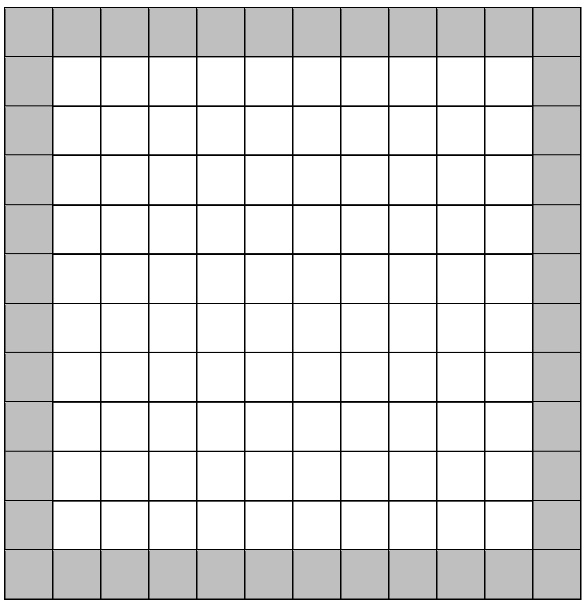
6. Looking at all four tables, write a rule that can be used to find the total number of pieces (p) for any number of layers (ℓ) and any number of cuts (c):

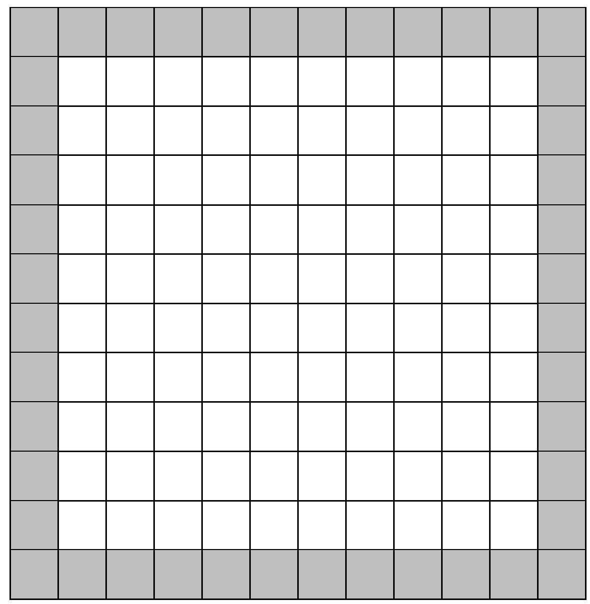
$p =$ ________________

THE BORDER PROBLEM

(Adapted from *MathLinks* Grade 7 Common Core Supplement: Lesson 3.2)

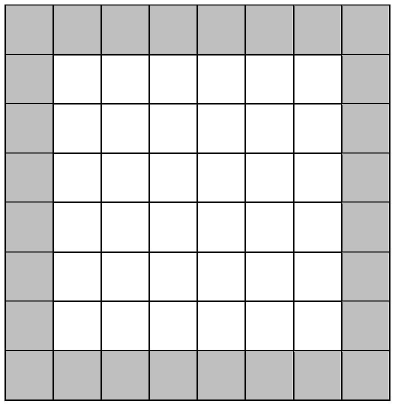
1. Other than simply counting all the way around, list at least two different ways you can think of to count the number of shaded border squares for a 12 x 12 grid. Write a numerical expression for each.





THE BORDER PROBLEM (Continued)

2. Based on the expressions generated for problem 1, list at least three different ways to count the number of shaded border squares for an 8 x 8 grid. Write a numerical expression for each.



3. Based on the expressions generated for problems 1-2, list at least three different ways to count the number of shaded border squares for a 4 x 4 grid. Write a numerical expression for each.

4. Based on the expressions generated for problems 1-3, list at least three different ways to count the number of shaded border squares for an $n \times n$ grid. Write a variable expression for each.

REPRESENTATIONS OF THE BORDER PROBLEM

1. Turn your paper to “landscape” orientation to complete this table. List all of the different expressions that the class generated for each square grid. Organize the expressions so that the related ones from each grid size are in the same row. Then write each expression in simplest form.

n x *n* grid					
4 x 4 grid					
8 x 8 grid					
12 x 12 grid					
	a	b	c	d	e

2. What do you notice about all of the expressions for each grid?

EXPLORING FURTHER

Let's continue to explore the border problem.

1. Make a table of values that shows the side length of a square (n) and the number of squares in the border (B)

Side length of a square (n)		2	3	4	5	6	7	8	9	10	11	12	n
Number of squares in the border (B)												44	

2. Write an equation that can be used to find the number of border tiles (B), given the length of the square (n).

3. Use your rule to find the number of border tiles when the side of the square is 1 unit?

   Interpret this answer in the context of the problem.

Now let's think about the grid in a little different way. Instead of counting border tiles, we are going to look at the perimeter of the square.

5. What is the perimeter of the square created by a 12 x 12 grid?

6. What is the perimeter of a square created by an n x n grid?

7. Write an equation for the perimeter (P) of a square created by n x n grid.

8. Compare the equation for the number of border tiles in a square of side n (problem 2 above) to the equation for finding the perimeter (P) of a square of side n (problem 7 above). Are they the same? Explain.

COMMON CORE STATE STANDARDS-MATHEMATICS

Lefty-Righty Experiment

8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

MP2 Reason abstractly and quantitatively

MP3 Construct viable arguments and critique the reasoning of others

MP4 Model with mathematics

Cutting the Rope

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

MP1 Make sense of problems and persevere in solving them

MP4 Model with mathematics

MP7 Look for and make use of structure

The Border Problem

6.EE 4 Identify when two expressions are equivalent (i.e. when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

6.EE 6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

7.EE 2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that “increase by 5%” is the same as “multiply by 1.05.”*

MP2 Reason abstractly and quantitatively

MP4 Model with mathematics

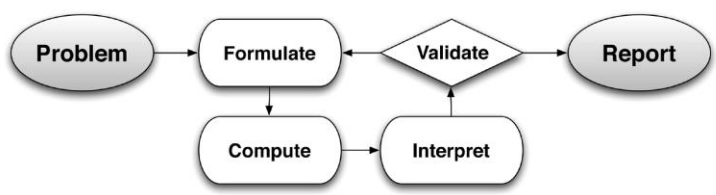
MP8 Look for and express regularity in repeated reasoning

THE BASIC MODELING CYCLE

From The Common Core State Standards for Mathematics

Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations in order to understand them better. This understanding helps us to make better decisions.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



COGNITIVE DEMAND SPECTRUM

Memorization	Procedures without Connections	Procedures with Connections	Doing Mathematics
↓	↓	↓	↓
memorized procedures in routine ways	no understanding or meaning of concepts	understanding or meaning attached to concepts	engagement with concepts; tasks stimulate students to make connections to meanings, representations, and other mathematical ideas