Presentation#236, Kerrigan, Powerful Strategies to Develop Algebraic Thinkers in Middle School

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This interactive presentation will demonstrate how students can gain a beginning understanding of linear functions by studying: in-out boxes (as functions), then number sequences, then finite differences to find the general term, then arithmetic sequences and finally slope and y-intercept. If time permits, we will also explore arithmetic series. Powerful applications will be explored.

Find the 20th term and then the n th term.

2n

n + 1

2n – 1

3n – 1

n + 3

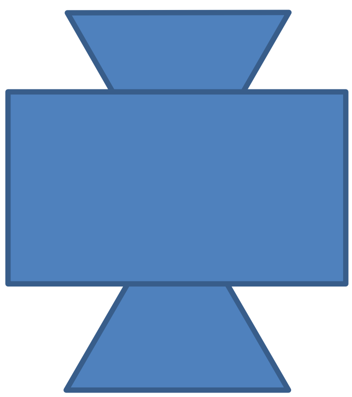
n – 1

2n + 2

5n + 1

7n – 3

INPUT



OUTPUT

n	1	2	3	4			20		n		
F(n)											

These sequences are called arithmetic sequences because there is some common difference (a constant) that you would add on to any term to get the next term.

Let's develop the formula for the general term.

1 2 3 4 5 n

a, a + d, a + 2d, a + 3d, a + 4d, a + **(n –1)d** **so** $t_n = a + (n - 1)d$

where a is the first term, d is the common difference, n is the number of terms and t_n is the nth term.

These word problems were obtained by Googling “arithmetic sequence word problems”. There specific sources are also cited after each problem.

The sum of the interior angles of a triangle = 180 degrees, a quadrilateral = 360 degrees and a pentagon = 540 degrees. Assuming the pattern continues in this manner, what is the sum of the interior angles of a dodecagon? (12 sides) (Regents Exam Prep Center)

After knee surgery, your trainer tells you to return to your jogging program slowly. He suggests jogging for 12 minutes each day for the first week. Each week thereafter, he suggests that you increase that time by 6 minutes per day. How many weeks will it be before you are up to jogging 60 minutes per day? (Regents Exam Prep Center)

If all integers are the same distance apart and if 3 is the first term and 35 is the last term, find the three terms in between.

3 ______ ______ ______ 35

If the 0^{th} term is 32 and the 100^{th} term is 212, what is the nth term and what is the 37^{th} term in this arithmetic sequence?

1/3 ______ 3/8 Find the middle term or the average.

1/3 ______ ______ 3/8 were all terms are the same distance apart.

We have an arithmetic sequence when we let the x values, or the domain, be the Natural Numbers and when we graph this function on the x-y plane, we get an infinite number of dots in a straight line, but many dots are missing.

When we let the domain be the Real Numbers then we get a continuous linear function and when we graph this function on the x-y plane we get a continuous straight line.

When we place + signs between the terms (instead of commas) we call this an arithmetic series (instead of arithmetic sequence). We are usually interested in the sum of an arithmetic series.

What is the sum of the Natural Numbers from 1 to 100 inclusive?

1 + 2 + 3 + ……. + 98 + 99 + 100 = 101 + 101 +

= 100/2 (101) = 50(101) = 5050

In general $S_n = n/2\ (a + t_n)$

1 + 3 + 5 + 7 + 9 + 11 = here a = 1, n = 6, $t_6 = 11$ so $S_6 = 6/2\ (1 + 11) = 36$

You visit the Grand Canyon and drop a penny off the edge of a cliff. The distance the penny will fall is 16 feet the first second, 48 feet the second second, 80 feet the next second and son on in an arithmetic sequence. What is the total distance the object falls in 6 seconds? (Regents Exam Prep Center)

A theatre has 20 seats in the first row, 24 seats in the second row, 28 seats in the third row and so on following this pattern. The theatre has 30 tows of seats. How many seats are in the theatre? (Regents Exam Prep Center)

If you initially put $2500 in the bank and then add $750 to it each month, how much money will you have put in the bank in 20 years?

*** In the problem above, if your interest rate is 1.5% compounded continuously, how much money will you have in 20 years. (This is a much harder enrichment problem for algebra students to consider. What new knowledge would they need to be able to attack this problem?)

In closing, in this presentation, I attempted to demonstrate how students can gain a good beginning understanding of linear functions by studying: in-out boxes (as functions), then number sequences, then finite differences to find the general term, then general arithmetic sequences to see that they are just linear functions with a domain of the Natural Numbers. Slope and y-intercept were explored. Then we looked at arithmetic series.

Finally, we looked at powerful applications like equally placing numbers between any two given distinct numbers; finding interesting sums, showing that the thermometer with Fahrenheit on one side and Celsius on the other is a wonderful example of an arithmetic sequence/linear function. Read more about arithmetic sequence and series theory and applications. Arithmetic sequence and series theory and applications usually gets students quickly involved in material involving important concepts from algebra and engages them in higher order thinking activities.

When students are in algebra II, a similar set of activities involving geometric sequence and series theory should be emphasized. The wonderful applications that occur will add meaning to the skills and concepts being learned.