PROBLEMS
that turn
ALGEBRA
PROCEDURES
into
GOOD GROUP WORK

Using Errors:

Gerardo is simplifying expressions with very large exponents. He arrives at each of the results below. For each result, decide if he is correct and **justify** your answer using the meaning of exponents. Giving a rule is not a justification.

a. $x^{150} / x^{50} = x^3$ b. $y^{20}(y^{41}) = y^{61}$
c. $(2m^2n^{15})^3 = 2m^6n^{45}$

d. $\dfrac{x^2+x+3}{x+3}=x^2$

e. $\dfrac{(x+2)(x+3)}{x+3}=x+2$

EQUATIONS: As she was solving equations Alicia made several errors. Find an error, explain the why it is an error, and show her how to complete the solution.

a. $3x - (7 - 5x) = 9x$

$3x - 7 - 5x = 9x$

$-2x - 7 = 9x$

$-7 = 11x$

$7/11 = x$

b. $4 + 3(x - 10) = 2x + 4$

$7x - 70 = 2x + 4$

$5x = 4$

$x = 4/5$

Rewriting Equivalent Expressions and Equations:

For each of the following expressions, write at least three equivalent expressions. Be sure to justify how you know they are equivalent.

a. $(x + 3)^2 - 4$

b. $(2a^2b^3)^3$

c. $x - \frac{1}{x}$

Rewriting Equivalent Expressions and Equations:

Is $6x - 3y = 12$ equivalent to $y = -2x + 4$? How can you be sure. Use each representation, a table, graph, equation to verify.

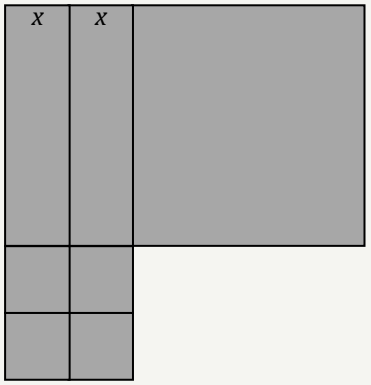
Starting with a Visual Model:

For each of the shapes formed by algebra tiles below:

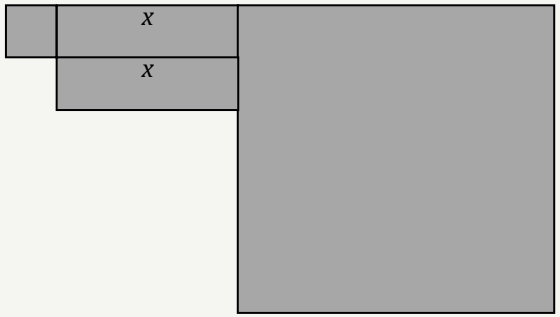
- Use tiles to build the shape.
- Sketch and label the shape on your paper and write an expression that represents the perimeter.
- Simplify your perimeter expression as much as possible.

Areas are given for each figure.

a. $x^2 + 2x + 4$

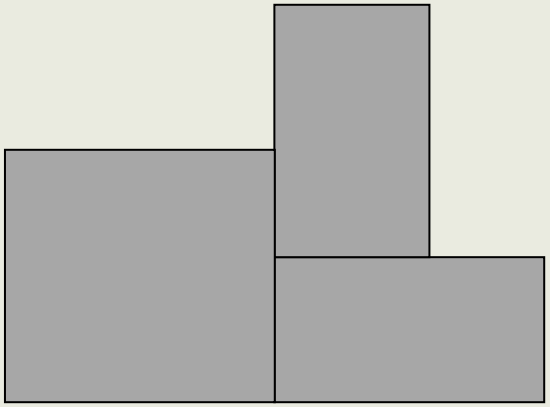


b. $y^2 + 2x + 1$



c. $y^2 + 2xy$

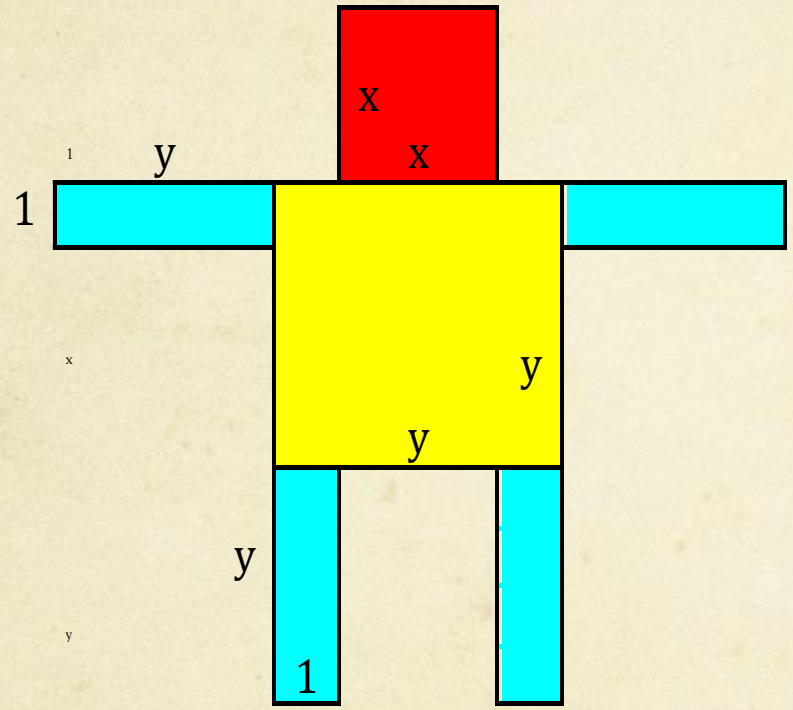
If the perimeter of the shape at right is 32 units, find possible values for x and y. Is there more than one possible solution?



d. For each figure, find a way to demonstrate algebraically that all of your expressions are **equivalent**.

e. Explain how you used the Distributive, Associative, and Commutative Properties in part d.

Work with your partner to write an expression for the perimeter. Simplify your expression.



Decide on a Strategy to Solve:

With your team, solve $4(x - 3) = 20$ for x in at least two different ways. Explain how you found x in each case and be prepared to share your explanations with the class.

For each equation below, with your group decide whether it would be best **to rewrite, look inside,** or **undo.** Then solve the equation, showing your work and writing down the name of the approach you used.

a. $100x^2 + 100x = 2000$

b. $\sqrt{3x+3} = 6$

c. $\frac{x}{3} - \frac{x}{9} = 6$

d. $\frac{2x-8}{10} = 6$

e. $8 - (2x + 1) = 3$

Confronting Points of Confusion:

Angelica and D'Lee were working on finding roots of two quadratic equations: $y = (x - 3)(x - 5)$ and $y = 2(x - 3)(x - 5)$. Angelica made an interesting claim: "Look," she said, "When I solve each of them for x, I get the same solutions. So these equations must be equivalent!"

D'Lee is not so sure. "How can they be equivalent if one of the equations has a factor of 2 that the other equation doesn't?" she asked.

- Who is correct? Is : $y = (x - 3)(x - 5)$ equivalent to $y = 2(x - 3)(x - 5)$? How can you justify your ideas using tables and graphs?
- Is $0 = (x - 3)(x - 5)$ equivalent to $0 = 2(x - 3)(x - 5)$? Again, how can you justify your ideas?

Solve a Really Hard Problem

Now that you have the skills necessary to solve many interesting equations and inequalities, work with your team to solve the equation below.

$$(\sqrt{|x+5|}-6)^2+7=23$$

Work Backwards

Work with a partner, and the simple equation $x = -4$. Perform at least five operations on both sides of the equation to get an equivalent equation that is a more complicated. When you and your partner have agreed on a complicated equation, trade your equation with your team-partners and see if they can solve it.

Now start with $5x - 7 = 0$.

BACKWARDS PERIMETER: Use the xy tile and two y tiles to build an arrangement for each of the following perimeters.

a. $2x + 4y$

b. $4y + 4$

c. $2x + 6y$

Play a Game: EXPONENT CONCENTRATION

- Split your team into two pairs and decide which is Team A and which is Team B. Your teacher will distribute a set of cards for a game described below.
- Arrange the cards face down in a rectangular grid.
- Team A selects and turns over two cards.
- If Team A thinks the values on the cards are equivalent, they must justify this claim to Team B. If everyone in Team B agrees, Team A takes the pair. If the values are not equivalent, Team A returns both cards to their original position (face down). This is the end of the turn for Team A. Team B repeats the process.
- Teams alternate until no cards remain face down. The team with the most matches wins.

Discussion Generators

- **Identify/Explain Errors.**
- **Rewrite Equivalent Expressions; Justify.**
- **Create then Reconcile Multiple Representations of Visual Situations.**
- **Confront a Point of Confusion**
- **Decide on a Strategy to Solve**

- **Solve Really Hard Problem**
- **Reverse the problem. Work Backwards. Give an Example of.....**
- **Play a Game**

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