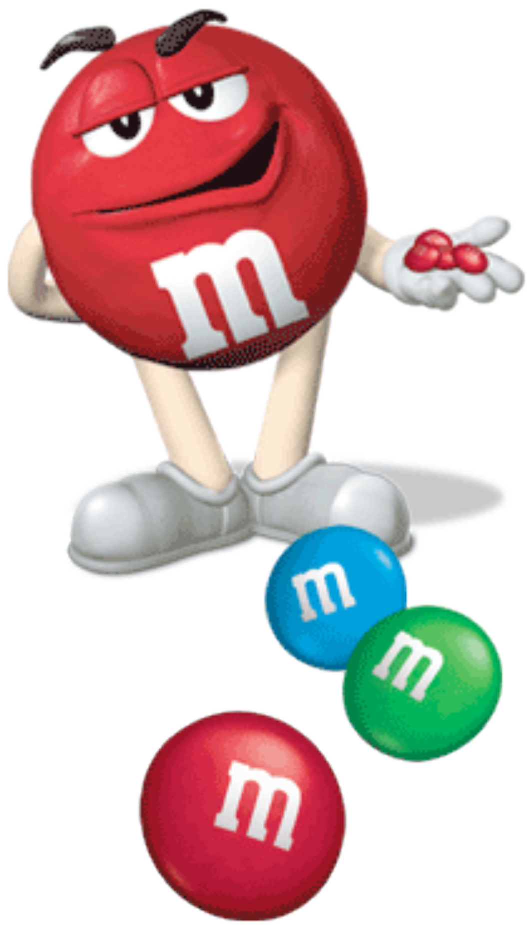
# Everything I Ever Needed to Learn about Statistics I Learned From a Bag of m&m's

## A Handful of Activities for AP and CCSS Statistics



Jason M. Molesky • Lakeville Area Public Schools • Lakeville, MN
jason.molesky@isd194.org • http://apstatsmonkey.com

# Table of Contents

# IntroActivity1 - Data Collection

This activity is a simple, yet effective way to introduce students to the basic concepts they will be encountering in Statistics. I recommend doing this on the first or second day of class as an icebreaker activity, but also as a way to gauge your students' understanding of basic statistical concepts. Take the time to discuss the 4 themes of statistics during the activity - Describing Data, Collecting Data, Probability, and Inference - as a way to introduce students to the concepts they'll be learning throughout the year.

## Materials Needed:

- Trial Size or 1.69 oz bags of m&m's - one per student.
- Data Collection Tables and Dotplots - one per student.
- "Master" Dotplot for each color on the board.

## Activity:

Discuss what students know about the color distribution of m&m's. Is it possible to predict which color will be most popular in a bag of m&m's? Are the colors evenly distributed? If not, which one is most popular? How can we find out? Discuss appropriate methods for data collection to answer these questions. Hand out Data Collection tables and Dotplots. Explain how students are to collect color information - Do a sample bag on the "Master" dotplot on the board. Ask students if they think their bag will be the same as yours...why or why not? Discuss variability.

Hand out bags or have students take theirs out. All students should have the same size bag (trial or 1.69 oz). Students should open bags and record data on their dotplots. All students should record their distributions on the "Master" dotplots on the board. When all students are done, have them compare their dotplots to the "Master" dotplots. Any similarities? Differences? Extremes? What conclusions can we reach from this activity? Can we answer our original questions? Discuss the 4 themes of AP Statistics and how they relate to this activity.

## Color Distribution: (as of July, 2006 - see http://us.mms.com for updated distributions)

| Blue | Brown | Green | Orange | Red | Yellow |
|---|---|---|---|---|---|
| 24% | 13% | 16% | 20% | 13% | 14% |

## Extension:

Keep this data for future reference. Re-display the data using boxplots, stemplots, histograms, for each color or bar graphs and pie charts for the categorical distributions. Discuss benefits/disadvantages of each type of display.

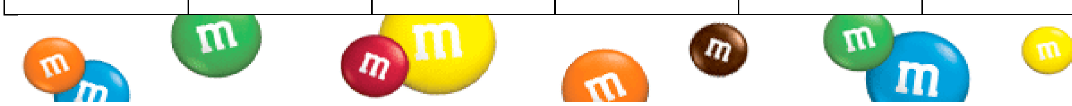
# Data Collection

*Welcome to APStats!* Throughout the course history, mathematics has been called upon to answer some of the world's most pressing questions, including, but not limited to: "What colors can I expect to find in a bag of milk chocolate m&m's?" Your task is to collect data on the m&m color distribution through a carefully designed and controlled experiment in which you will:

- open a bag,
- count the number of m&m's falling into each color category,
- record the numeric results,
- record your color distribution on the class dotplots, and
- properly dispose of the m&m's by seeing whether or not they really do melt in your mouth, not in your hand.

Be sure to record your data accurately as we will be referring to it throughout the course.

| Blue | Brown | Green | Orange | Red | Yellow |
|---|---|---|---|---|---|
| | | | | | |



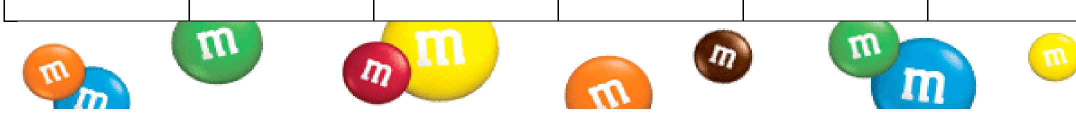

# Data Collection

*Welcome to APStats!* Throughout the course history, mathematics has been called upon to answer some of the world's most pressing questions, including, but not limited to: "What colors can I expect to find in a bag of milk chocolate m&m's?" Your task is to collect data on the m&m color distribution through a carefully designed and controlled experiment in which you will:

- open a bag,
- count the number of m&m's falling into each color category,
- record the numeric results,
- record your color distribution on the class dotplots, and
- properly dispose of the m&m's by seeing whether or not they really do melt in your mouth, not in your hand.

Be sure to record your data accurately as we will be referring to it throughout the course.

| Blue | Brown | Green | Orange | Red | Yellow |
|---|---|---|---|---|---|
| | | | | | |

# m&m's Color Distribution Dotplots

**Blue**

0 5 10 15 20 25 30

**Brown**

0 5 10 15 20 25 30

**Green**

0 5 10 15 20 25 30

**Orange**

0 5 10 15 20 25 30

**Red**

0 5 10 15 20 25 30

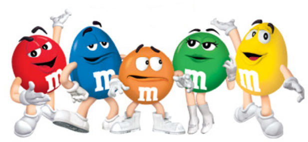
**Yellow**

0 5 10 15 20 25 30

# IntroActivity2 - Data Exploration

## "The Great m&m Experiment"

Adapted from Michelle Hipke



**Question: What percentage of milk chocolate m&m candies are orange?**

The purpose of this activity is to introduce you to the basic concepts we will be encountering in Statistics. Your goal is to answer the question above using only a "sample" of m&m candies. You must justify your conclusion by organizing, plotting, and referencing data collected by the entire class.

### I. Collecting the Data:

Scoop out a sample of M&M candies. Count the total number of M&Ms in your sample. You will need exactly 25 M&Ms, so if you need more, randomly choose a few more to add to your sample. If you have too many, you must randomly choose M&Ms to discard.
*DO THIS WITH YOUR EYES CLOSED! NO PEEKING!*

Calculate the percentage of *orange* M&Ms in your sample: ___________

Record the class data using the following chart:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

### II. Organizing the Data: Organize the data in a meaningful way.

Title: ________________________________________

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

Can you summarize the class data using a basic numeric measure?

## III. Displaying the Data: Display the data using a dot plot.


Title: ______________________________________

4%  8%  12%  16%  20%  24%  28%  32%  36%  40%  44%  48%

## IV. Analyzing the Data:

- Describe some general features of the data.

- What would you consider a "normal" or "typical" percentage of orange M&Ms? Why?

- Does our data reveal the true percentage of orange M&Ms? If so, what is the true percentage? If not, what DOES it reveal about the true percentage?

**Conclusion? How confident are you in your conclusion? Why?**

# Mean and Standard Deviation

After students have mastered constructing and interpreting the shape, outliers, center, and spread of a distribution, they are ready to add numeric summaries to their exploratory data analysis. We can use 1.69 oz bags of m&m's to introduce/reinforce the concepts of mean and standard deviation.

## Materials Needed:

- Ten 1.69 oz bags of m&m's.
- Scale - check with your Science department.
- Data Collection and Calculation Table - one per student.

## Activity:

Review data displays and interpretations of center and spread. If a bag advertises 1.69 oz, does that mean we are guaranteed 1.69 oz of m&m's? What is a typical actual weight? How can we calculate the typical weight found in a bag? What if the weights differ between bags? Discuss variability. How can we describe the variability from bag to bag? Introduce the concepts of mean and standard deviation.

Carefully weigh the contents of ten bags of m&m's. Record each bag's weight in the table on an overhead. Display the weights on a dotplot on the board and calculate the average weight. Note this on the dotplot and discuss its relative location

Discuss the variability of the recorded weights and note the distances/deviations from the mean weight on the dotplot. How can we calculate a 'standard' measure of this distance? Using the Standard Deviation Calculation Table, illustrate the computation of the standard deviation. Discuss 'deviation', 'squared deviation', their sums, etc. as you calculate the measure.

## Extension:

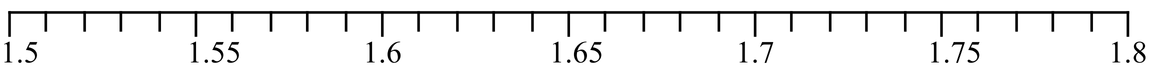
How else could we describe center and spread? Discuss and illustrate median, quartiles, range, and interquartile range. Determine min, Q1, Q2, Q3, and max and display data with a boxplot. Note how each measure of the 5-number summary is shown on this display.

# Am I Getting What I Paid For?

## Mean and Standard Deviation of m&m Bags Labeled 1.69 oz.

Do m&m bags labeled 1.69 oz. actually contain that weight? If not, what is a typical weight and how much variability is there from bag to bag? To answer those questions, we will collect data on ten bags of m&m's. Your teacher or a volunteer will weigh each bag and report the weights. Your task is to use the tables below to calculate and interpret the mean and standard deviation for the weights of bags labeled 1.69 oz.

As the data is collected, construct a dotplot and fill in the table below.




**Weights of "1.69 oz" Bags of m&m's**

| Bag # | $x$ | $(x-\bar{x})$ | $(x-\bar{x})^2$ |
|---|---|---|---|
| **1** | | | |
| **2** | | | |
| **3** | | | |
| **4** | | | |
| **5** | | | |
| **6** | | | |
| **7** | | | |
| **8** | | | |
| **9** | | | |
| **10** | | | |
| **n=10** | $\sum x =$ ______ | $\sum(x-\bar{x})$ | $\sum(x-\bar{x})^2$ |
| | $\bar{x} =$ ______ | = ______ | = ______ |

**Standard Deviation s=** $\sqrt{\dfrac{\sum(x-\bar{x})^2}{n-1}}$ **s=**______________.

# Standard Normal Calculations

This activity can be used to illustrate practical uses of standard normal calculations and their interpretations. It is meant to reinforce the concept of normal distributions and individual observations as well as introduce students to the basic idea of inference. While students have not studied sampling distributions or inferential procedures, they should be able to use what they know about variability and normal distributions to make some basic inferential conclusions.

## Materials Needed:

- 2 1.69 bags of *milk chocolate* m&m's
- At least 1 1.69 bag of *peanut butter* m&m's

## Activity:

Students should be familiar with the concept of normal distributions and should have a basic understanding of sampling variability. Discuss the fact that the proportion of yellow m&m's in an individual bag will vary from bag to bag.

Note that the proportion of yellow m&m's in individual bags varies according to approximately N(0.14, 0.05). Sketch the distribution, noting where each standard deviation falls. Discuss the Empirical (68-95-99.7) Rule as it relates to the proportion of yellow m&m's in a bag. Ask for student impressions of 'extreme' proportions. That is, at what point would they suspect the claim that 14% of m&m's are yellow? Why would they suspect it? Estimate the percent of bags that have less than 10% yellow m&m's, greater than 20% yellow, etc.

Open *milk chocolate* m&m's and note proportion of yellow candies. {Note: if this proportion is exactly 14%, you may wish to use another bag} Locate and plot this proportion on the normal curve. Calculate and interpret the standardized z-score and corresponding proportions above and below this value. If you have 2 bags, repeat the calculations and find the proportion of bags that would fall between the two observations.

After exhausting all possible calculations with the *milk chocolate* m&m's, open and find the proportion of yellow *peanut butter* m&m's. {Note: 20% of peanut butter m&m's are yellow, as compared to 14% of milk chocolate m&m's--do not tell students this}. Locate this proportion on the N(0.14, 0.05) distribution and discuss whether or not they think yellow peanut butter m&m's are produced in the same proportion as yellow milk chocolate m&m's.

# Are These m&m's "Normal" or Just "Plain"? Standard Normal Calculations

We have observed variability in color distributions from bag to bag of "plain" milk chocolate m&m's. According to the m&m website, 14% of milk chocolate m&m's are yellow. Does that mean we are guaranteed 14% of the candies in each bag will be yellow? Should you be concerned if only 10% are yellow? What if all of them are? At what point would you suspect the advertised proportion? We will discuss each of these questions as we explore standard normal calculations with some sample bags of m&m's.

**Background Information:**

We know the proportion of yellow m&m's varies from bag to bag. Suppose these proportions follow an approximately normal distribution N(0.14, 0.05). Sketch this distribution below and note 1, 2, and 3 standard deviations above and below the mean. Interpret the Empirical (68-95-99.7) Rule in the context of this situation.

**Sample Information:**

Our bag of m&m's contained _____ candies. There were _______ yellow m&m's.

The sample proportion of yellow candies for our bag is _____/______ = _______.

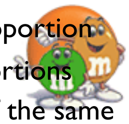
**Standard Normal Calculation:**

Recall, a "z-score" is a value that tells us how many standard deviations above or below the mean a particular observation falls. To find this value, we must subtract the mean from our observation and divide the result by the standard deviation. That is,

$$z = \frac{x - \bar{x}}{s} = \frac{\square - \square}{\square} = \square$$

We can use this z-score to determine what percent of bags of m&m's (of the same size) would have a yellow proportion less than our observed proportion. Sketch two normal distributions for yellow proportions below and note our observed proportion on each curve. Using your z-table, determine the proportion of bags of the same size that would have fewer yellow candies. Shade this area on the first curve. On the second curve, shade and calculate the proportion of bags of the same size that would have more yellow candies.

Suppose we had a second bag of m&m's. We would expect about 14% of the candies in the second bag would be yellow. However, like the first bag, there is a chance that proportion will not equal 0.14 (or the proportion in the first bag, for that matter). Use the proportions from the first bag and from a new bag to determine what percent of bags of m&m's of the same size will have a yellow proportion *between* those two values.



| | **Bag 1** | **Bag 2** |
|---|---|---|
| **Yellow Proportion** | | |
| **z-score** | | |
| **% Bags Below Observed** | | |

Sketch the two observed proportions on the normal distribution N(0.14, 0.05) and note the percent of observations we would expect to see *between* the two observed proportions.

**What About Peanut Butter m&m's?**

Do peanut butter m&m's follow the same color distribution as milk chocolate m&m's? If so, we would expect about 14% of the candies in a peanut butter m&m bag would be yellow. Would you be surprised if 15% were yellow? What about 20%? 30%? At what point would you suspect the color distribution for peanut butter m&m's may be different?

Open a bag of peanut butter m&m's and note the proportion of yellow: ____/____ =______

Plot this value on the N(0.14,0.05) distribution and calculate the % of observations we'd expect to be *more extreme* than this observation. Based on this %, do you feel you have evidence to suggest the color distribution of peanut butter m&m's may be different? Why or why not?

# NonLinear Transformations

This activity can be used to introduce the concept of transforming nonlinear data to achieve linearity. The class will witness the "decay" of m&m's by shaking a container of the candies and removing those that don't display an "m". The activity continues until all m&m's have "decayed", at which point the data is plotted and discussed, leading to an introduction of transformations.

## Materials Needed:

- One 1.69oz bag of m&m's - not *peanut*...the candies must be flat
- A container with a cover to shake the candies

## Activity:

Discuss the decay of radioactive materials. Students will, most likely, be familiar with exponential decay and the characteristics of the graphs of exponential function. Note that we have been dealing primarily with linear growth and decay, but many situations involve nonlinear patterns. This chapter will introduce us to methods that transform nonlinear data to more linear patterns that can be modeled using the method of least squares regression learned previously.

Since we can't use *actual* radioactive isotopes, we'll simulate the decay using a package of m&m candies. When the candies are mixed up, an "m" up indicates an active candy, while an "m" down indicates a decayed one. Start by counting and placing the m&m's in a container such that all of the candies fit in one layer. Note this number as the number of active candies in round 1. Pass the container to a student and instruct them to shake it and then remove the decayed candies, reporting how many active candies remain. Continue until all candies have "decayed".

Plot the (round, active) data and note the strength, direction, and form. You should have a nonlinear decay form, so modeling with a linear function may not be the most appropriate. Walk the students through the idea of transforming data to achieve linearity. Start by calculating log(round) and log(active). Then, plot (round, log(active)) and note the strength, direction and form. Next, plot (log(round), log(active)) and again note the strength, direction, and form. Students should note that these transformations resulted in somewhat more linear scatterplots.

Choose the "most linear" of the scatterplots and find the Least Squares Regression Line. Discuss the form of this prediction model...is it in the terms of the original data? If not, can we transform the equation to a more useful model?

This activity serves as an overview of the concept of transforming data to achieve linearity. It is not meant to highlight all of the details of transformations. A detailed study of transformations and nonlinear modeling should follow during which you can discuss other transformations, how to "back transform", etc.

# "Moleskium" Decay – Nonlinear Transformation Activity

The decay of radioactive isotopes typically creates a nonlinear relationship between time and amount remaining. "Moleskium" is a little-known element found in a short, highly caffeinated math teacher at LSHS. Without proper caffeination, "Moleskium" can decay quite rapidly...trust me, it's not a pretty site when that happens.

We will be simulating the decay of "Moleskium" through the use of m&m candies. As we proceed through this simulation, be sure to dispose of the "decayed" m&m's in a nearby oral cavity or waste-disposal container.

**Disclaimer**: DO NOT eat *actual* radioactive isotopes. These are *pretend* so it's ok.

### "Moleskium" Decay Simulation:

- ☑ Start with a sizeable (n>60) quantity of m&m's in a covered container.
- ☑ Shake the container to simulate typical Mr.M. activity.
- ☑ Open the container and remove all m&m's that have "decayed" {ie, no "m" showing}
- ☑ Count the remaining m&m's and record in the table below.
- ☑ Repeat until all m&m's have "decayed".

☑ Enter the "Round" into L1 and "Isotopes" into L2.

☑ Display the relationship with a scatterplot and interpret in context:

| Round Number | Isotopes Remaining |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |

☑ Use your calculator to find the LSRL for (round, isotopes):

LSRL:________________________________________

☑ Sketch the residual plot for this LSRL and use it to discuss the appropriateness of using a linear model for this relationship.

## “Moleskium” Decay Activity - Continued

We can tell by the scatterplot of (round, isotopes) and by the residual plot of (round, residual) that a nonlinear model may be a more appropriate model for this situation. However, the question that remains is, “Which model is more appropriate? Exponential or Power?” Our eyes are fairly good at judging whether or not points lie on a straight line. However, they are not very good at spotting the difference between power and exponential curves. Therefore, we’ll use the fact that certain transformations of power and exponential data produce linear relationships. If we can determine which transformation does a better job of “straightening out the data” then we’ll have a better idea which model is best.

Use your list editor to define L3 = log(round) and L4 = log(isotopes).

Remember, if (x, logy) is linear, an **exponential** model may be best. If (logx, logy) is linear, a **power** model may be best. All we have to do is plot the two and determine which is “more linear”.

| Sketch (x, logy) and interpret | Sketch (logx, logy) and interpret |
| --- | --- |
| | |

Find the LSRL for the transformed data that exhibits the most linear relationship. Justify “most linear” by considering the correlation and residual plots for each set of transformed data.

Since the LSRL is written in terms of transformed data, transform it back into the original terms of the problem. That is, find a prediction model that will convert “round number” into “isotopes remaining”.

**Final Prediction Model for “Moleskium” Decay:**______________________________

# m&m Experimental Design

This activity can be used to introduce the concept of Experimental Design. Many students enter AP Statistics with a basic understanding of the fundamentals behind designing a controlled experiment. This group activity serves as a means for the teacher to assess student understanding prior to the unit on producing data.

## Materials Needed:

- One 1.69oz bag of m&m's per group (optional)
- Presentation paper for each group (butcher paper, bulletin board paper, etc.)

## Activity:

Discuss the importance of producing "good" data for "good" statistics. In order to have faith in our statistical calculations and inferences, we must have faith in our data. This unit will focus on the methods behind collecting data through sampling and experimental design.

Break the class into groups of four students each. Each group will be given the task of designing an experiment to test the effectiveness of m&m's as a stress reliever. Each group will have 20 minutes to design an experiment to determine whether or not eating m&m's reduces stress. Since this activity is done at the beginning of the unit on experimental design, little instruction will be given. Students are given a chance to develop and share their designs, giving you a chance to determine what students already know as well as a chance to point out errors in their logic and discuss improvements that could be made.

Some students may be able to construct an experiment that incorporates the concepts of control, randomization, and replication. Some may be able to incorporate blocking or other designs without knowing the technical terminology. Other groups will design a basic experiment with flaws in the control or randomization aspects. This can serve as a great discussion starter and provide a basis for defining the key components of a well-defined experiment.

# m&m's as Stress Relief? Intro to Experimental De

Your research firm has been asked to look into your teacher's claim that eating m&m's reduces stress. Being good statisticians, you know that in order to test this claim, you'll need to collect some data. Your group's task is to design a **completely controlled randomized experiment** to collect data to test the effectiveness of eating m&m's for stress reduction.

Your group has the following resources at its disposal:

- 200 volunteers (100 male and 100 female of varying ages and backgrounds)
- 1 month to conduct the experiment
- Enough m&m's to give all volunteers one bag per day
- Enough generic m&m's (ChocoBites) to give all volunteers one bag per day
- A valid clinical stress assessment multiple choice test (stress measuring tool)
- Other reasonable resources your group deems necessary

Using these resources, or others you feel are necessary, design an experiment to test the claim. Your design should be detailed enough that someone with no statistical knowledge could replicate the experiment. Further, your experiment should take the following into consideration:

- Males and females deal with stress differently.
- Some people feel chocolate, in general, helps reduce stress
- Stress differs from person to person: work, family, school, relationships, etc.

Use a sheet of bulletin board paper to present your design. Your presentation should include an experimental question (What are you trying to answer?), a diagram, a written description of your experiment (Who is involved? How will you carry this out?), and a discussion of possible confounding variables.

Use the space below to draft your design...consider the following questions:

**What is your experimental question? What are you trying to answer?**

**What are some possible causes of stress? What are some possible stress-reducers *other than* m&m's?**

**What will you measure to determine if stress is being reduced? How will you be able to tell if it is being reduced?**

**What are some possible problems with your experiment?**

# Random m&m's

*{Adapted from "Statistics in Action" by Watkins, Scheaffer, Cobb}*

**DO NOT TURN THIS SHEET OVER UNTIL YOU ARE TOLD TO DO SO!**

**Goal**: Estimate the average number of m&m's per pile for the 100 piles pictured on the back.

1) When I give you the signal, you will have 10 seconds to look at the back side of this sheet and make a guess as to the average number of m&m's per pile. Do not use a pencil or paper...just guess.

Guess:___________ Enter this guess on the appropriate dotplot on the board.

2) Select 5 piles that are, in your judgment, representative of the entire population. Calculate the average pile size and enter the result on the dotplot on the board.

Representative Average Number:______________
Enter on the appropriate dotplot.

Compare the two distributions...

3) Use a random number table or your calculator to select a SRS of 5 distinct piles. Calculate the average number of m&m's for these piles and enter the information on the dotplot. Repeat this process until you have 5 sample averages.

SRS 1: Average Area:____________

SRS 2: Average Area:____________

SRS 3: Average Area:____________ Plot each average on the appropriate dotplot.

SRS 4: Average Area:____________

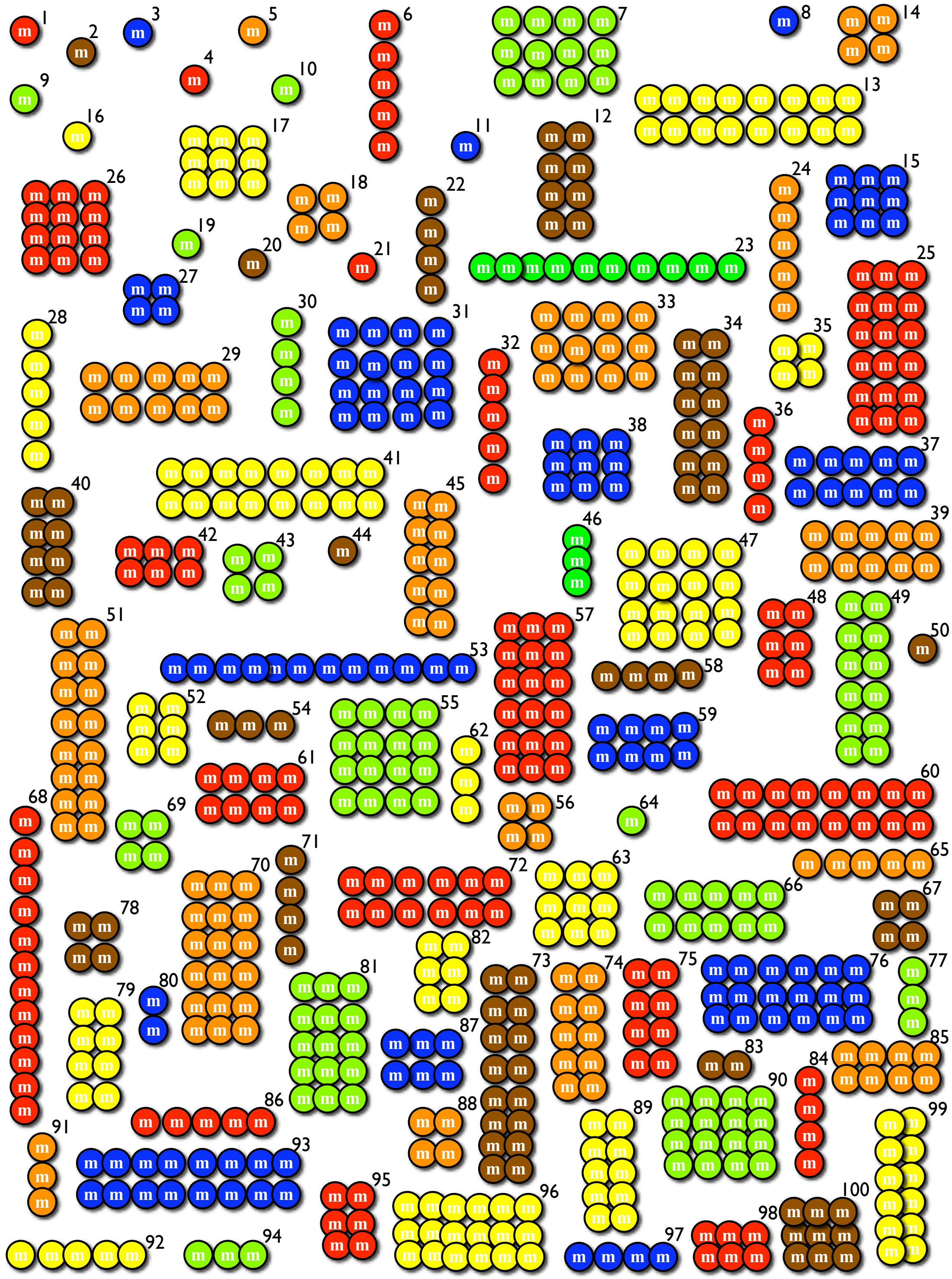
SRS 5: Average Area:____________

**The true average number of m&m's for the 100 piles is:___________**

What is the point of this exercise?

# Random m&m's

{Adapted from *Statistics in Action* : Watkins, Scheaffer, Cobb}



# The Game of "Greed"

*The point of this game is to gain as many points as possible. Points are earned by rolling two dice–the sum of the dice equals the points earned for the roll. To earn points, a player must be actively "in the game". Once a player opts "out of the game," they maintain their current point total, but can not earn any more points until the next round. Players who are "in the game" continue earning points until they opt out or until the "greed point" is rolled, whichever occurs first. The "greed point" determines the end of a round. Any player "in the game" when the greed point is rolled lose all points earned for that round.*

## The Game of Greed

- The teacher should establish a "greed point" that will end the round. For example, if a "2" or "9" is rolled, the round ends and all players in the game lose their points for that round.
- Everybody stands up. The teacher rolls two dice. The result is the starting score for all students.
- If a student is satisfied with the score, they can opt out and keep those points for their total.
- If a student wishes to continue earning points, they can stay in the game for another roll.
- Continue rolling. After each roll, students may opt out and keep their points or stay in the game to earn more points for the round.
- The round ends when all students opt out or when the greed point is rolled.
- The Student with the most points wins a bag of m&m's!

Use the following table to track your scores. Add additional lines if necessary. Add all 4 round totals to determine your game score.

| Roll | Score | Roll | Score | Roll | Score | Roll | Score |
|---|---|---|---|---|---|---|---|
| 1 | | 1 | | 1 | | 1 | |
| 2 | | 2 | | 2 | | 2 | |
| 3 | | 3 | | 3 | | 3 | |
| 4 | | 4 | | 4 | | 4 | |
| 5 | | 5 | | 5 | | 5 | |
| 6 | | 6 | | 6 | | 6 | |
| 7 | | 7 | | 7 | | 7 | |
| 8 | | 8 | | 8 | | 8 | |
| 9 | | 9 | | 9 | | 9 | |
| 10 | | 10 | | 10 | | 10 | |
| 11 | | 11 | | 11 | | 11 | |
| 12 | | 12 | | 12 | | 12 | |
| 13 | | 13 | | 13 | | 13 | |
| 14 | | 14 | | 14 | | 14 | |
| 15 | | 15 | | 15 | | 15 | |
| Total | | Total | | Total | | Total | |
| | | | | | | Game Total | |

Rolling dice is a random phenomenon. While we can't predict *exactly* what will come up on a particular roll, we can be reasonably sure what the distribution of sums will look like for a long series of rolls.

Your task is to use what you know about rolling dice to create a strategy for the game of greed.

# Goodness of Fit

*{Adapted from "The Practice of Statistics, 2e by Yates, Moore, Starnes}*

This activity can be used to introduce the concept of Chi-square distributions and the "Goodness of Fit" test. Using a bag of m&m's, you will compare the observed color counts to the expected color counts as noted by the m&m/Mars company.

## Materials Needed:

• At least one 1.69oz bag of *milk chocolate* m&m's

## Activity:

Read the statement from the m&m/Mars company Consumer Affairs Department and discuss what you would expect to see in a bag of m&m's if the statement is true. Open a bag and count the total number of candies (do not separate by color...yet). Using the total number of candies, have students calculate the expected number of each color based on the company's claim. Have them enter these numbers on their table.

Count each color and report these values to the students to enter on their tables. Discuss any differences or extreme values. Based on these counts, do we have evidence to suspect the claim may not be true?

Introduce the class to the concept of a "Goodness of Fit" test in which we can test how well an entire distribution of counts fits a hypothesized distribution. For each color, have students calculate $(Observed - Expected)^2/Expected$. Note any extreme values.

If the sample distribution does not vary much from the claimed proportions, these values should be close to zero. As the sample deviates more and more from the hypothesized distribution, these values will increase. The more extreme these values are, the less likely the sample is to be from the hypothesized distribution. Calculate the Chi-square test statistic by adding up all of the component values and determine the likelihood of observing a bag of m&m's with this color distribution assuming the company's claim is true.

**m&m/Mars Company Consumer Affairs Department Announcement**

*"On average, the new mix of colors of m&m's plain chocolate candies will contain 30 percent browns, 20 percent yellows and reds, and 10 percent each of oranges, greens, and blues. While we mix the colors as thoroughly as possible, the above ratios may vary somewhat, especially in the smaller bags. This is because we combine the various colors in large quantities for the last production stage (printing). The bags are then filled on high-speed packaging machines by weight, not by count."*

# Did I Get Enough Blue m&m's?
# Chi-Square Goodness of Fit Calculations

According to the m&m/Mars company, in 1995 *"...the new mix of colors of m&m's plain chocolate candies will contain 30 percent browns, 20 percent yellows and reds, and 10 percent each of oranges, greens, and blues."* However, the mix of colors has been known to change every few years. Your task today is to determine whether or not the current mix of colors matches that of 1995. We want to see if there is sufficient evidence to reject the company's 1995 claim. To do this, we'll be introduced to a new type of test -- the Chi-square Goodness of Fit Test.

- Open a bag of milk chocolate m&m's and carefully count how many of each color are in the sample. (Or, use the data from your teacher's bag) Record the observed data in the "observed" row of the table below.
- Using the statement from the m&m/Mars company, determine how many of each color you expected to see. Note, you'll have to figure this out using the total number of m&m's in your or your teacher's sample bag. Enter these counts in the "expected" row below.

| | **Brown** | **Yellow** | **Red** | **Orange** | **Green** | **Blue** | **Total** |
|---|---|---|---|---|---|---|---|
| **Observed** | | | | | | | |
| **Expected** | | | | | | | |
| **$(O-E)^2/E$** | | | | | | | |

If your bag reflects the distribution advertised in 1995, there should be little difference between the observed and expected counts. To quantify the difference, we'll calculate a total which we'll call "Chi-Square" or $X^2$.

- For each color, perform this calculation: (observed - expected)$^2$ / expected. Enter each value in the last row of the table. Add up all of these "component" values to find $X^2$.
- If this total value is small, we have little evidence to suggest a difference in distributions. However, the larger $X^2$ gets, the more evidence we have to suggest the company's claim may no longer be applicable to bags of milk chocolate m&m's.

To determine the likelihood of observing a difference between observed and expected as extreme as the one we observed, we must look up the p-value on a Chi-square table. Chi-square distributions are skewed right and specified by degrees of freedom. In a Goodness of Fit test, the degrees of freedom equal one less than the number of categories.

Find the p-value for our test by looking up $X^2$ for 5 degrees of freedom. Sketch the curve and observed $X^2$ below. Interpret the result in the context of the problem.