# Modeling Data with Core Math Tools: Enhancing Mathematical Practices Implementation 

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## Session Overview

- Brief overview of the Common Core State Standards for Mathematics (CCSSM)
- Major Shifts
- Important changes for High School
- Mathematical Practices
- Engaging Students in Mathematics
- Introduction to Core Math Tools free software
- Modeling Data from Algebra tasks
- Pay it Forward
- Bouncing Balls
- Coin Experiment


## Major Shifts in CCSSM

- Emphasis on Mathematical Practices
- Acceleration: International Benchmarking
- Focus: Structure
- Coherence: Progressions
- Clarity: Fewer more rigorous


## CCSSM Require Changes

- Major shifts in content at the high school level includes masteries students must show beyond traditional
Algebra 2 content:
- Periodic functions
- Polynomials
- Radicals
- Advanced Probability and Statistics
- Mathematical Modeling
- Assessing Standards for Mathematical Practice


## Eight Mathematical Practices

- All teachers should develop these practices in their students. These are as important as the content standards.
- Similar to NCTM's Mathematical
Processes from the Principles and Standards for School Mathematics and NJ's Process Standards.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

# Key Challenge: Engaging Students In Mathematics 

"I teach bigh school math. I sell a product to a market that
doesn't want it, but is forced by law to buy it."

Dan Meyer, TED Talk, 2010

## Handshake Problem

- There are 30 people in this class. If everyone shakes hands with everyone else, how many handshakes will there be?
- Take a few minutes to think about this individually, then discuss with a neighbor.
- Try to find as many different approaches to solving this problem as you can. Be prepared to share your approaches.


## Valentine Exchange

Annenberg Learner Video:
http://www.learner.org/vod/login.html?pid=910

- Students watch a video of $4^{\text {th }}$ grade students solving a similar task.
- Model the problem, act it out
- Draw pictures
- Use blocks
- Start with a "smaller" problem
- Generalize
- College students are surprised by what $4^{\text {th }}$ graders were able to do (in comparison)


## Connecting to Mathematical Practices



## Candy Boxes

- I had boxes of Snickers ${ }^{\circledR}$ and Butterfingers ${ }^{\circledR}$. The Snickers ${ }^{\circledR}$ came in boxes of 12 and Butterfingers ${ }^{\circledR}$ came in boxes of 10. If I had nine total boxes, how many boxes of each type were there?
- Oh, and I had 102 pieces of candy
- Solve like you were a $5^{\text {th }}$ grader
- Solve like you were an $11^{\text {th }}$ grader
- Can you find connections between the two approaches?



## $5^{\text {th }}$ Grader Solutions

Total

| 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 108 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 10 | 106 |  |
| 12 | 12 | 12 | 12 | 12 | 12 | 12 | 10 | 10 | 104 |  |
| 12 | 12 | 12 | 12 | 12 | 12 | 10 | 10 | 10 | 102 |  |
| 12 | 12 | 12 | 12 | 12 | 12 | 10 | 10 | 10 | 10 | 100 |
| 12 | 12 | 12 | 12 | 12 | 10 | 10 | 10 | 10 | 98 |  |
| 12 | 12 | 12 | 12 | 10 | 10 | 10 | 10 | 10 | 96 |  |
| 12 | 12 | 12 | 10 | 10 | 10 | 10 | 10 | 12 | 10 | 10 |
| 12 | 12 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 94 |  |
| 12 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 92 |  |
| 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 90 |  |

## $11^{\text {th }}$ Grader Solution

## $12 x+10 y=102$ <br> $x+y=9$

## Connections

| X | 9-x | 12*x+10*y | $12 x+10 y=102$ |
| :---: | :---: | :---: | :---: |
| 9 | 0 | 108 | 2 $2 x+10 y=102$ |
| 8 | 1 | 106 | $x+y=9$ |
| 7 | 2 | 104 | $12 x+10(9-x)=102$ |
| 6 | 3 | 102 |  |
| 5 | 4 | 100 |  |
| 4 | 5 | 98 |  |
| 3 | 6 | 96 | $12 x+90-10 x=102$ |
| 2 | 7 | 94 |  |
| 1 | 8 | 92 | $12 x-10 x=12$ |
| 0 | 9 | 90 |  |
|  |  |  | $2 x=12$ |

## Core Math Tools

- To download the free Core Math Tools, go to: nctm.org (click on to left purple cell, "CMT")
- Features include:
- Algebra- CAS and Spreadsheet
- Geometry- Coordinate and Synthetic
- Statistics- Data Analysis and Simulation
- Discrete Math- Vertex-Edge Graphs
- Pre-made data files + Extra Apps


## Pay It Forward

- Motivating the task: trailer for Pay It Forward movie

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## Pay It Forward

a. Make a table that shows the number of people who will receive good deeds at each of the next seven stages of the Pay It Forward process. Then plot the (stage, number of good deeds) data.
b. How does the number of good deeds at each stage grow? How is that pattern of change shown in the plot of the data?
c. How many stages of the Pay It Forward process will be needed before a total of at least 25,000 good deeds will be done?

## Now-Next

- Stage $1=3$
- Stage $2=3 * 3$
- Stage $3=3 * 3 * 3$

Stage $2=($ Stage 1$) * 3$
Stage $3=($ Stage 2)*3

- Stage $n=3 * 3 * 3 \ldots * 3$

Next Stage $=$ Now $* 3$
$n$ times
$y=3^{x}$

## CCSSM Content Standards Addressed

## Interpreting Functions

Analyze functions using different representations
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. $\star$
e. Graph exponential.....

## Building Functions <br> F-BF

1. Write a function that describes a relationship..... $\star$

## Linear, Quadratic, and Exponential Models* F-LE

1c. Recognize situations in which a quantity grows or decays...
2. Construct linear and exponential functions... 5. Interpret...

## More Bounce to the Ounce

## - Motivating the task: Sony Bouncing Ball commercial


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## More Bounce to the Ounce

- Task: Suppose a new golf ball drops downward from a height of 27 feet onto a paved parking lot and keeps bouncing up and down, again and again. Rebound height of the ball should be $2 / 3$ of its drop height. Make a table and plot of the data showing expected heights of the first ten bounces of the golf ball in Core Math Tools.
- Algebra->CAS->File->Data->New
- Type "Bounce Number" in Column A and "Rebound Height" in Column B
- To enter fractions in the cell type " $=16 / 3$ ", then hit ENTER
- To plot: Click on blue Graph in the menu pane, Scatterplot, choose Column A for horizontal and Column B for vertical, change window in "Settings" tab


## Bouncing Ball Questions

a. How does the rebound height change from one bounce to the next? How is that pattern shown by the shape of the data plot?
b. What rule relating NOW and NEXT shows how to calculate the rebound height from any bounce from the height of the preceding bounce?
c. What rule beginning with " $\mathrm{y}=$..." shows how to calculate the rebound height after any number of bounces?
d. How will the data table, plot, and rules for calculating rebound height change if the ball drops first from only 15 feet?

## Checking Your Work With Core Math Tools

- Click the $Y=$ tab
- Type in the equation you believe to be correct
- In the Command line, check the box next to "Graph" for the function you just entered
- Click the Graph tab and your function should be listed in red along with your data points

Next $=$ Now ${ }^{*}$ 2/3

$$
y=27 *\left(\frac{2}{3}\right)^{x}
$$

## CCSSM Content Standards Addressed

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## Building Functions <br> F-BF

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## Linear, Quadratic, and Exponential Models* F-LE

1c. Recognize situations in which a quantity grows or decays...
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## Coin Experiment



## THE MONEY MUSEUM

## Coin Experiment

Suppose that you were asked to conduct this experiment:

- Get a collection of 100 coins, shake them well, and drop them on a tabletop.
- Remove all coins that are lying heads up and record the number of coins left.
- Repeat the shake-drop-remove-record process until 5 or fewer coins remain.
(Core-Plus, C1U5L2l3, pg. 323)


## Coin Experiment

a. If you were to record the results of this experiment in a table of (drop number, coins left) values, what pattern would you expect in the data? What function rule would probably be the best model relating drop number $\boldsymbol{n}$ to number of coins left $\boldsymbol{c}$ ?
b. Conduct the experiment, record the data, and then use the curve fitting functions of Core Math Tools to find a function model that seems to fit the data pattern well.
c. Compare the model suggested by logical analysis of the experiment to that found by fitting a function to actual data. Decide which you think is the better model of the experiment and be prepared to explain your choice.

## CCSSM Content Standards Addressed

## Interpreting Functions F-IF <br> Building Functions <br> F-BF <br> Linear, Quadratic, and Exponential Models*

## Interpreting Categorical and Quantitative Data S-ID

6. Represent data on two quantitative variables on a scatter plot, and describe how they relate.
b. Informally assess the fit of a function by plotting \& analyzing residuals

## Making Inferences and Justifying Conclusions S-IC

2. Decide if a specified model is consistent with results from a given data-generating process, e.g. using simulation

## Discussion Questions

- How do you think the slider will impact students' understanding of exponential decay?
- What conceptions do students need to distinguish between exponential growth and exponential decay?


## Learning Goals for Students

- Develop their ability to reason about and communicate their understanding of the primary CCSSM content
- Classroom environment will reflect the Standards for Mathematical Practice, with such observable features as:
- the sharing of strategies
- explanations and justifications
- interpretation and evaluation of each other's ideas
- the appropriate use of manipulatives and other tools
- an awareness of the problem at hand
- the use of mental math
- the regular use of multiple representations


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## Thank you

## Questions and Comments

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