Reasonin', wRitin' and aRithmetic: The New 3 R's

Bob M. Drake, Univ. of Cincinnati

and

Lynn Columba, Lehigh Univ.

“Obscurity in writing is commonly an argument of darkness in the mind. The greatest learning is to be seen in the greatest plainness.”

John Wilkins, English mathematician who was one of the founders of the Royal Society, (1614-1672).

Informal Types of Writing

- In-class
  - Free
  - Focused
- Math autobiographies
- Reading logs
- Journals
- Letters

Formal Types of Writing

- Proofs
- Process Papers
- Summaries of journal articles
- Solutions to journal problems
- Research papers
- Lecture notes

Why is writing instruction even needed in mathematics classrooms?

Research has shown that:

Research has shown that:

- Word problems are more compact/dense than ordinary prose.
- Writing in word problems is different than prose.
- Word Problems lack context clues.
- Word <u>meanings are different</u>.
- Word Problems miss continuity of prose.

There are these 5 reasons for using writing, but also more:

- Word problems are more compact/dense than ordinary prose.
- Writing in word problems is different than prose.
- Word Problems lack context clues.
- Word meanings are different.
- Word Problems miss continuity of prose.

Other Findings:

A study was conducted in which three groups of children were compared:

- The first group solved **NO** text book story problems during the year.
- The second group solved **ALL** the text book story problems during the year.
- The third group **WROTE** their own story problems.

Other Findings (continued):

- Children who <u>wrote</u> their own word problems performed better than either of the other groups.

- **Children who <u>SKIPPED</u> word problems out-performed children who <u>SOLVED</u> them during the academic year!!!!**

MPH Problem:

```
    30 mph         "X" mph
     1 mile          1 mile
\__________  __________/
           \  /
         60 m.p.h.
```

A person drives exactly one mile at exactly 30 mph. How fast does he have to drive a second mile to average 60 mph for the two mile trip?

Hint: The answer is NOT 90 m.p.h.!

Student #1's Response

If 30mph is $\frac{1}{2}$ as fast as 60mph and the distance is $\frac{1}{2}$ of the distance then:

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ (30mph is equal to $\frac{1}{4}$ the total average speed),

therefore: $30 = \frac{1}{4}s$; $s = 120$

The average speed for the second mile is 120 mph

I don't really know if this is correct - but it looks good.

How is it that I can multiply mph by distance? Doesn't mph already imply time (speed) over distance? I don't feel as if I could give a good explanation of this to a student - I don't remember getting a good explanation of these concepts in school. I do remember mph questions - I usually got them wrong.

Student #2's Response

When trying to solve this problem we started out trying to find the difference with the number 60 and 30. Now I believe that it can be solved using information we know about time. Driving 60 mph and going 1 mile = 1 minute. That's as far as my thinking extended. I've learned that I'm not good at understanding the logic of setting up a formula.

60mph = 1 min

30mph = 2 min

Division of Fractions Problem

Create a story problem to illustrate why

$$\frac{1}{2} \quad \frac{1}{4} = 2$$

Barb:

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Fred wants to know how many sets of ½ are needed to equal ¼. He knows he has to multiply ½ by something to equal ¼. He sets his problem up like this:

$$\frac{1}{2} \times \_\_ = \frac{1}{4}$$

He knows he must multiply the denominator by something to get 4. He also knows the numerator will be one because 1 x 1 = 1. He decides that 2 x 2 = 4, thus his answer is ½.

½ ¼ = 2

I really have no idea because when dividing fractions, the real computation is done by multiplying. In other words, although you are dividing a fraction by another fraction, you are actually multiplying.

Teresa:

- If you divided one half of a piece of pie into 4 equal areas, how many half pieces would it take?
- If you have 2 sets of ¼ pieces of a pie, how much of a pie would you have?
- How many groups of ¼ do you need to make ½ ?
- One of these is bound to be right!

Diagnosis of Error Patterns

What is Susie’s problem? Diagnose her procedure.

$$\begin{array}{r} {}^{3}\cancel{4}\;\;{}^{1}2\;\;5 \\ -2\;\;4\;\;3 \\ \hline 2\;\;8\;\;2 \end{array} \qquad \begin{array}{r} {}^{1}\cancel{2}\;\;{}^{1}\cancel{3}\;\;{}^{1}4 \\ -1\;\;4\;\;6 \\ \hline 1\;\;9\;\;8 \end{array} \qquad \begin{array}{r} {}^{2}\cancel{3}\;\;{}^{1}\cancel{1}\;\;{}^{1}6 \\ -1\;\;8\;\;8 \\ \hline 2\;\;3\;\;8 \end{array}$$

Susie's Diagnosis

It appears that Susie is working from left-to-right rather than right-to-left as we typically solve mathematics computations. Why does she do this?

What does Susie say?

$$\begin{array}{rrr} {}^{3}\not{4} & {}^{1}2 & 5 \\ -2 & 4 & 3 \\ \hline 2 & 8 & 2 \end{array}$$

Susie’s Diagnosis: What Susie Says

“First, go to the column on the side of the room where the piano is…”

It seems her teacher last year told students to “always begin your work on the side of the room where the piano is located.”

Susie understands that regrouping requires exchanges, but she doesn’t understand why the procedure is supposed to work. The “rule” she uses doesn’t work in this room.

Why Students Fail Homework

On the MPH problem, when Student 2 said,

“Now I believe it can be solved using information we know about time.”

She recognized a shortcoming in her understanding. Writing helped foster that recognition.

Diagnosis of error patterns

$$\frac{1\not{6}}{\not{6}4} = \frac{1}{4} \qquad \frac{1\not{9}}{\not{9}5} = \frac{1}{5} \qquad \frac{2\not{4}}{\not{4}8} = \frac{2}{8}$$

$$\frac{13}{39} = \frac{}{}$$

How will this student solve the remaining problem and, more importantly, what is the error of his thinking? Can you recognize the source of his problem?

The student has been taught to “cancel” when multiplying fractions.

Student:

First you’re supposed to do division of the top and bottom numbers. Division and multiplication are opposites, so instead of dividing both the top and bottom by the same number, I can just do like multiplication and cancel the same number out of both. Since I do it to both, it should cancel out.

7 Reasons to Use Writing:

- Diagnosis of Error Patterns
- Insights regarding where instruction should start
- Why students don't connect strands of math
- Clarify student understanding
- Why Students Fail Homework
- Insights regarding beliefs and attitudes

Thank you!

Bob M. Drake

Bob.Drake@uc.edu

Lynn Columba

hlc0@lehigh.edu