Access Proof Through Geometric Models

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Session 108

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- Mineral Hall D/E (Hyatt Regency)
- Presentation Format: Gallery Workshop
- Grade Band Audience: 9 to 12

Description

- Use origami, constructions, and 3-D models to illustrate proofs of geometric theorems. Provide access for English language learners and special education students. Receive proven lessons that bring access to deductive reasoning to your classroom. Discuss the implications of the Common Core Standards for Mathematics in transforming the geometry classroom.

Warm-Up

- Construct a cylinder with an inscribed sphere of radius 2 cm.
- On 1 cm graph paper, use the compass to draw 2 circles of radius 2 cm. Cut them out.
- Cut a long rectangle of width 4 cm. Wrap this lateral area around the ball and secure with tape.
- Tape the circle bases on top and bottom.

Cylinder with an Inscribed Sphere

- From your construction relate the radius of the sphere to the radius of the cylinder.
- Complete the Graphic Organizer to determine the ratios of the volumes and total surface areas of the sphere to the cylinder.
- Make a conjecture as to whether these ratios apply with any radius.
- Explore the truth of your conjecture.

Construct Viable Arguments

- Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures.

Proofs

- A proof is an argument that uses logic, definitions, properties, and previously proven statements to show that a conclusion is true.
- Deductive reasoning is the process of using logic to draw conclusions from given facts, definitions, and properties.
- Two Column Proofs
- Flow Chart Proofs
- Paragraph Proofs

Proofs Requirements for Students

- Students require a store of properties, theorems, definitions in order to recall.
- Students must logically connect all of the assertions from the hypothesis to the conclusion.
- Students must effectively compose syntax, grammar, symbols and conventions to write a successful mathematical argument.

Making Snowflakes

- Fold a blank paper in half to make a crease along the longer side and open the paper.
- Fold the paper in half along the shorter side and keep the paper folded.
- Fold two more creases by halving the quarter pieces parallel to the first crease and opening.
- Fold and crease the distance between the intersection of the first two folds and the last crease.
- Fold over the opposite distance.
- Cut off excess paper to form and equilateral triangle.
- Cut designs in the triangle and open the snowflake.

Open Ended Prompts

- Before we cut the snowflake we created a triangle. Determine the measure of each angle of our triangle. Explain how you arrived at that measurement.
- Make an assertion about the measurement of the angles formed in our unit circle and provide evidence that supports your assertion.

From Snowflake to Unit Circle

- In a 30 – 60 – 90 degree triangle the length of the hypotenuse is 2 times the length of the shorter leg, and the length of the longer leg is the length of the shorter leg times $\sqrt{3}$
- In a 45 – 45 – 90 degree triangle both legs are congruent, and the length of the hypotenuse is the length of a leg times $\sqrt{2}$
- Use patty paper to support these assertions.

Construct a Unit Circle

- On ¼ inch graph paper construct an x-axis, y-axis, and a circle of radius 10 centered at the origin.
- Draw points at the intersections of the circle with each of the lines x=5, x=-5, y=5, and y=-5.
- Connect each point to the origin.
- Draw in the lines y=x and y=-x.

Mathematical Practice - Modeling

- Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).

Pyramid Model

- Seal a standard 9.20 X 16.51 cm envelope.
- Fold the envelope back and forth along its two diagonals.
- Fold the envelope back and forth along the median that is parallel the shorter side.
- Cut off the half diagonals along the seal of the envelope.
- Fold one side of the envelope into the other.

Special Education Students

- The Standards should also be read as allowing for the widest possible range of students to participate fully from the outset and as permitting appropriate accommodations to ensure maximum participation of students with special education needs.
- Scaffold learning to help students reach higher levels of math achievement.

Prove sides are parallel

- In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope.
- The slope of a line is the ratio of the rise to run.
- If (x_1,y_1) and (x_2,y_2) are any two points on a line, then the slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$

Prove a Quadrilateral is a Parallelogram

- Theorem – If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- 1. Determine if you have a quadrilateral.
- 2. Find the lengths of all sides.
- 3. Show that both pairs of opposite sides are congruent.
- 4. Make your conclusion.

Three Levels of Scaffolding

- Given only ordered pairs, use the formula for slope to determine if the points are the vertices of a parallelogram.
- Given a quadrilateral plotted in the coordinate plane, calculate slopes to determine whether opposite sides are parallel.
- Count squares to determine rise and run of each segment. Divide rise by run to calculate slope. State which segments have equal slopes.

English Language Learners

- It is beyond the scope of the Standards to define the full range of supports appropriate for English Language Learners
- Vocabulary development
- Represent Abstract Concepts with concrete models
- Increase opportunities to develop listening, speaking, reading, and writing skills in English

Prove Geometric Theorems

- 9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

Prove Geometric Theorems

- 10. Prove Theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

Prepare Students to Write a Proof

- Use concrete models to explore the connection between hypothesis and conclusion – Patty Paper
- Scaffold properties that will be needed – Graphic Organizer
- Connect logical pieces into a cohesive argument – Proof Puzzles
- Students write and defend a proof.

Common Core Transforms the Geometry Classroom

- Incorporate Mathematical Practices
- Assessments require students to write their mathematical thinking
- Incorporate English Language Arts Standards
- Mathematical Modeling is prominent throughout the standards