Multisensory Algebra: Building Solutions, Proof by Construction

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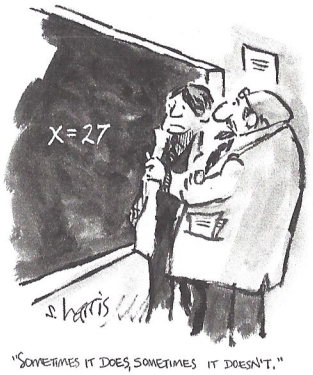
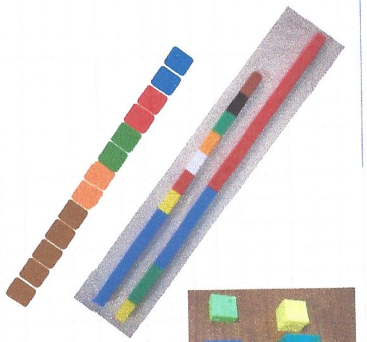
For Many Students Algebra is a Set of Concepts & Procedures that Don't Add Up

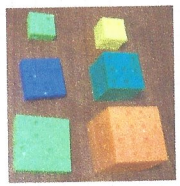


- Algebra can be:
  - Very Abstract
  - Only a set of procedures
  - Not relevant in the student's experience
  - Buttons pushed in a specific sequence- all with no meaning attached

Preview:

- What
  - Multisensory
  - CRA
- Why
  - Rationale
- How
  - Practical use of manipulatives in algebra
- Do-Practical Practice

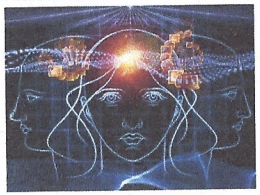




Research Suggests

- The core deficit in mathematics difficulties is numeracy
- Employ the visuospatial sense
- This deficit exists at all levels of math instruction
- Define numeracy…
- Automatic recognition of quantity and quantity relationships
- So, what can we do?

Multisensory Math



- **Why**
  - Learning occurs in many parts of the brain
  - But memory is highly associative
  - All students benefit from hands-on instruction
  - For those students with learning differences, it can provide an essential link

Why: Multisensory

- **Information**
  - can be processed on a modality-specific basis [visual, auditory, kinesthetic etc.]
  - converges and is integrated in the brain
- **Performance enhancement**
  - **is larger for multisensory than unisensory stimuli**

Paul J. Laurienti, M. D., Ph.D. Department of Radiology,

ANSIR, Advanced Neuroscience Imaging Research, Wake Forest University

Meaning:

- The more of the brain that is involved
- The stronger the associations
- The stronger the implications for learning & memory

For the At Risk Student

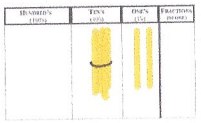
- Difficulties exist
  - Gaps in conceptual understanding
  - Poor numeracy
  - Poor computational skills
  - Inadequate background knowledge
- Algebra teachers experience frustration
  - Teaching concepts to students who have weak skills
  - Explaining complex concepts to students who struggle.

Multisensory Mathematics: UDL

- Addresses the needs of all students
- Uses manipulatives to teach, enhance, integrate and reinforce concepts
- Is research based
- Adapts to any curriculum and to the implementation of common core standards

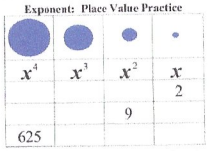
Manipulatives Must Be. . .

- Efficient
  - For the concept being taught
- Effective
  - At illustrating the concept
- Reproducible/ Retrievable
  - In memory, making the concept visualization portable



The Goal of Manipulatives

- Manipulatives are:
  - Time consuming
  - Messy
- The Goal of using them...
  - Is to get rid of them
- But they are essential for many students

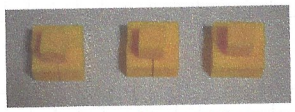


CRA: An Instructional Sequence

- **Concrete:** Illustrates the concept
  - using hands-on instruction, manipulatives
- **Representational:** Pictorial,
  - illustrates the concept in a retrievable or reproducible format
- **Abstract:** Uses only numerals, computational algorithms

Problem Solving Math

- Students may use manipulatives to:
  - Illustrate a variety of algebraic patterns & concepts
  - Model functions
  - Solve problems
  - "Code" patterns & meaning for writing equations from word problems



The Meaning Behind the Math

- Ask any algebra student...
- What is slope?
- What answers do you hear?
- "Rise over Run"
- Slope is "m"
- Slant of a line
- Or. . . $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$
- So what does all that mean?

Pattern Recognition

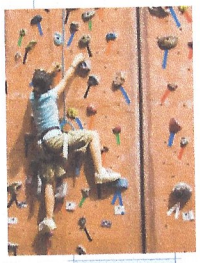
- The **Hole'n the Wall Climbing Gym** charges admission of $12 and $3 per hour for use of the facilities.
- Work with a partner
- Using pipe cleaners and beads, construct a model of Tim's cost after ten hours on site climbing.

Pattern Exploration

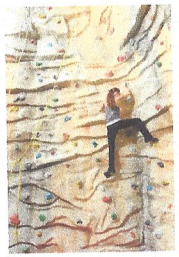
- What does Tim pay when he walks in the door?
- What happens after that?
- How could you describe this pattern?
  - Use your "student words"
- Can you think of other situations like this?
- Tell me the story...

Extend

- Using your construction
  - Create a table of values to post as a price list for the company



X=# of Hours	Y=Total Cost
0	$12



Language is Critical

- Linkage
- Introductory language can be crucial
  - Slope Intercept Form
  - Slope/ Constant Rate of Change
  - Y- intercept / Starting Value
- Sub-skill practice in coding
  - Coding: Identifying a pattern, labeling an element for meaning, applying it to a useful purpose

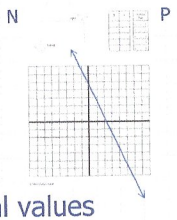
Applications

- Construct a Linking Cube model which represents a depth of 27 feet of water in a city water tank.
- If the water drains at a rate of 2 feet per hour, how much water will be left in the tank after eight hours.
- Use your model to demonstrate an answer
  - You may not use words

Extend

- Construct a table of values which illustrates the rate of change over time.
- Write each pair of values as an ordered pair.
- Graph the values on the coordinate plane.
- Begin by graphing the point of the starting value. (y=mx+**b**)
  - Linked to the equation we begin with "b"

Extension



- Consider the two problems.
  - In which problem are the final values increasing?
  - In which are they decreasing.
- How could we represent the rate of change with positive and negative integers?

Think Words

- Alice makes $12 an hour as a receptionist
- When she arrives, she must first pay $8 to park.
- How much will she have if she only works two hours?
- Explain your reasoning. Can you demonstrate this with Unifix cubes?

Reasoning and Sense Making

- Sometimes the manipulatives are more efficient
- Sometimes the graph tells a better story
- Sometimes manipulatives are just inefficient for the story we need to tell
- The point, is can I "see" the solution by using my visual spatial reasoning?

Translation into Math Speak

- If our equation for this type of situation is $y=mx+b$
- And, "m" represents the constant rate of change,
  - The "each" ...hour, minute, mile,
- And "b" represents the starting value
- Write a model...to represent...

Sub-Skills: **What Works Clearinghouse**

- For students who struggle
- Devise a practice sheet exclusively for identifying "m" & "b" and writing the equation
- Practice sub-skills before computations
  - Identify the Constant Rate of Change (m)
  - Identify the Starting Value
- Write the equation to model the situation

http://ies.ed.gov/ncee/wwc/PracticeGuide.aspx?sid=2

Spatial Relations (?)

- Some students need to "see" the growth of quantity by magnitude.
- Gaps in place value and number line concepts keep students from estimating properly or seeing quantity relationships.

Think Construction

- One bacteria cell divides using a process of mitosis
- If each bacteria cell divides in the same way each minute, how many bacteria will exist after eight minutes?
- Use any manipulative on the table to model your solution.

Patterns of Growth

- Divide your table into three groups: front, left and right of the podium
- Participants in front- construct a model of the two and three times tables using unifix cubes.

Patterns of Growth

- Participants seated to the right of the stage
  - Will use the Unifx cubes to construct a pattern of 2^x power.

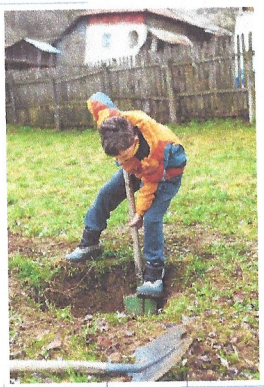
- Participants seated to the left of the stage will construct a pattern of 3^x

A Digression

- Non Math Examples
  - Using non-math examples can sometimes be useful in explicating a concept!
  - According to math researcher, Steven G. Feifer, D.Ed. of Georgetown University, students need to be able to "See" math
  - He calls it: visual spatial functioning

The Neuopsychology of Mathematics-www.SchoolNeuropsychPress.com

What do we mean by Negative?



- How much dirt is in a hole 2 feet by 4 feet by 4 feet?

What is the GCF?

- What is the greatest common/ or shared trait/factor?

Linking the Language

- Emphasize the terms **factor** and **multiple** at a basic level.
- Perform multiplication, division and prime factorization using craft sticks
- Define: prime and composite, factor & multiple

Prime Factorization: Demo

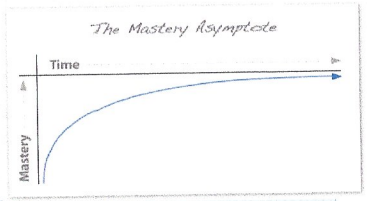
- Using craft sticks
- Find all the factors of one of the following numbers:
  - 24
  - 36
  - 28
- Use craft sticks to illustrate prime & composite numbers

How Close is Close?

- With your partner,
- Use scissors and a strip of paper
- Fold and cut the strip of paper to half its length.
- Trade places
- Fold and cut in half again
  - And again
    - And again
      - And again

What if...

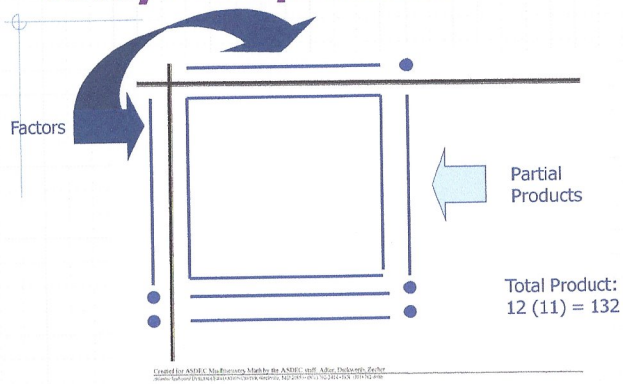
- What if this construction modeled the length of your steps as you approached a destination?
- Theoretically, would you ever reach it?



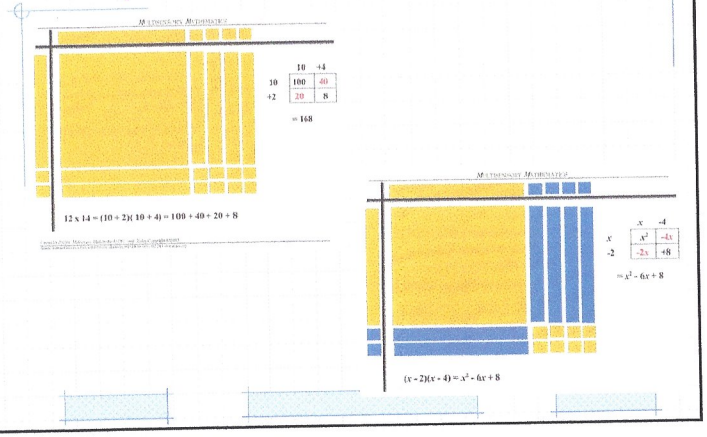
Construction: Arrays

- Using Base Ten Blocks
- Construct an array which represents one of the following:
  - Two groups of three
  - Three groups of ten
  - Two groups of twelve
  - Four groups of thirteen
  - Twelve groups of fourteen

Activity: Representational Array Multiplication



Multiplication Models



Division

- Division is "finding the missing factor
- Given 24 blocks and a divisor of 2, what would the array look like?
- Given 36 blocks and a divisor of 12?
- Arrange 169 blocks into a rectangle or a square without any "leftovers"
- What are the factors?

Division

- Assemble the Quantity 169
- Using 12 as the divisor (the left side factor)
- Arrange the dividend in a rectangle or square to fit the boundaries of the first factor.

- Division is the process of finding the other factor
- And any "left overs" or remainders

Multiplying Polynomials

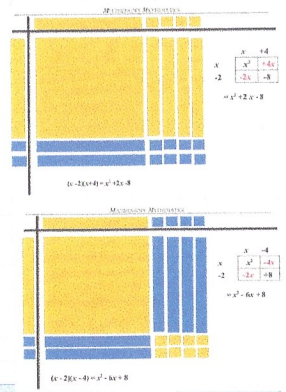
- Use the base ten blocks to multiply a two digit number by a two digit number
- 12 x 13
- Now let's call the ten rod "x"
- $(x+2)(x+3)$
- Using the colored pencils
- Draw this array on your "mini arrays" sheet

Using the Array to Factor

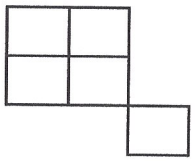
- Using the base ten blocks
- The flat = x^2
- The rod = x
- The units are the constant
- Assemble in the center of the array
- $x^2 + 5x + 6$
- Place the x^2 and the constant in diagonal positions.
- How would you arrange the rods to form a rectangle or square?

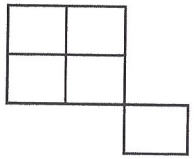
Using the Array to Factor

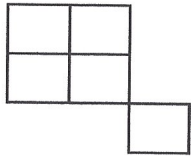
- Using two colors of base ten blocks allows for negative numbers
- How would you represent $x^2 + 5x - 6$?
- Using two colors of pencils could you draw this on your mini arrays?

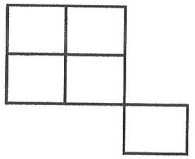


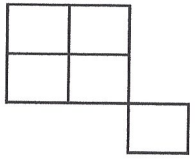
Practice with Place Value Based Multiplication: The Box

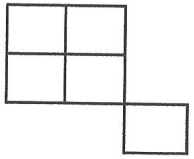












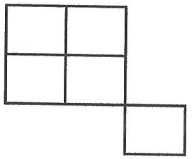
Fractions

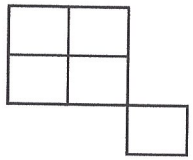
Multiply by the complex conjugate.

$$\frac{(2+i)}{(3-i)}$$

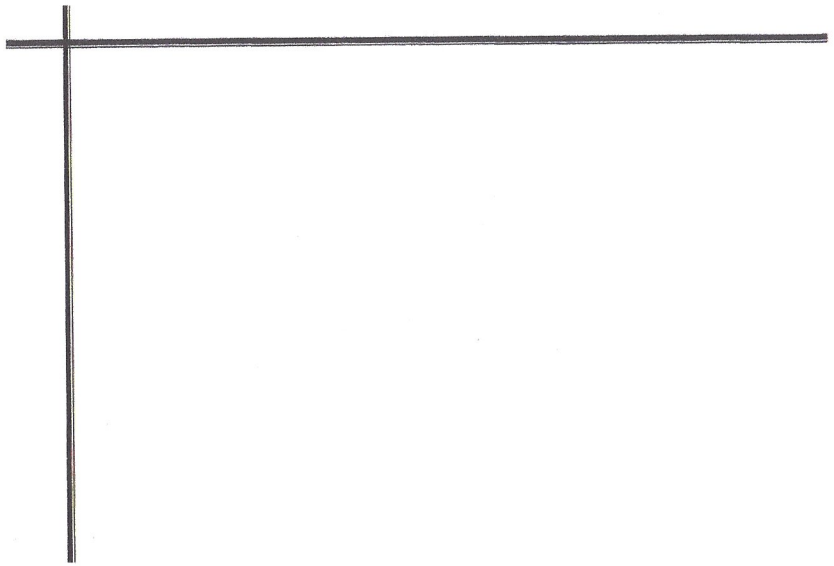
Create a fraction and use the box

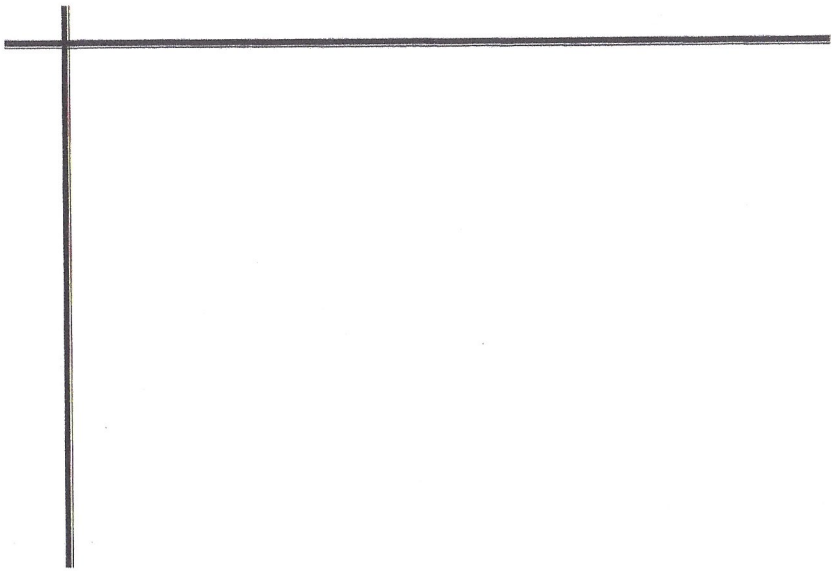
$$\frac{(2+i)}{(3-i)} \bullet \frac{(\quad)}{(\quad)}$$

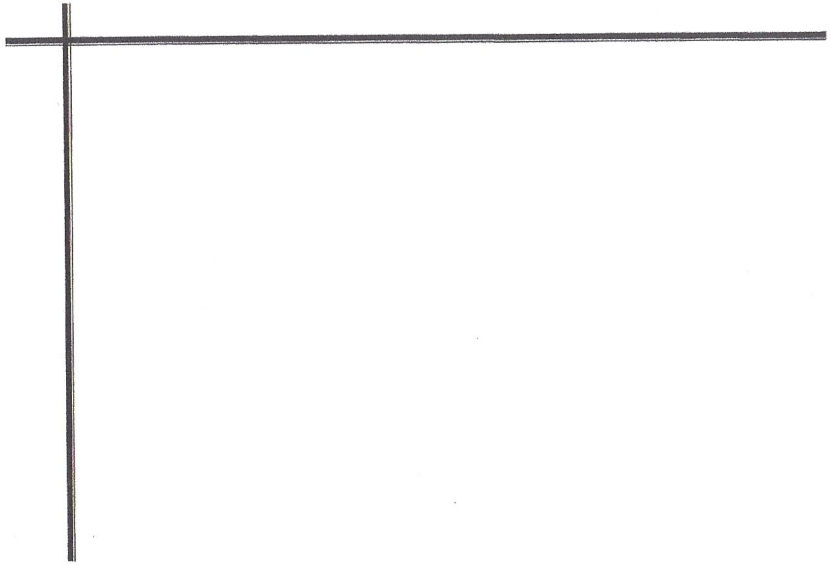


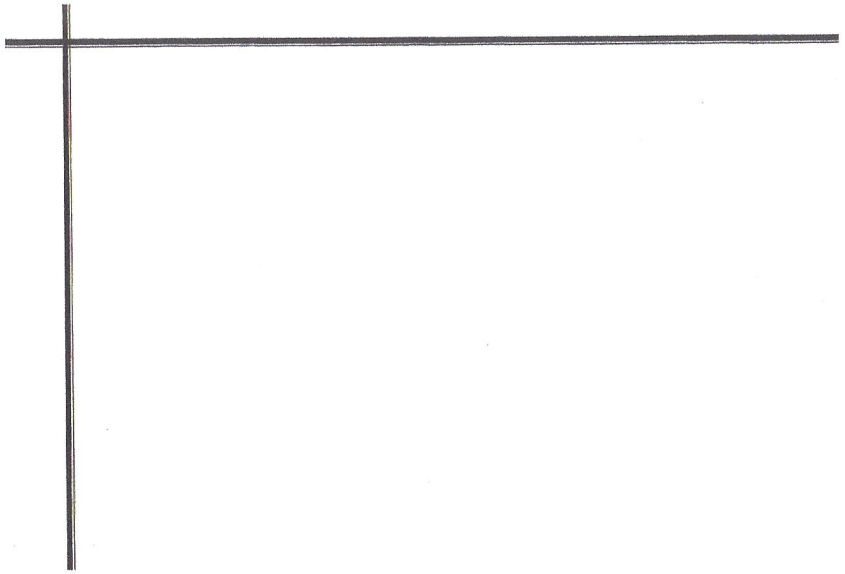


_____ = _____









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