Exploring Sequences and Series through Multiple Representations

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The basics

The sequence graphing mode on the TI-Nspire allows for the exploration of the graphical representation of sequences—a concept often only explored numerically and symbolically in the past. Just as with other graphical modes on the calculator the sequence can also be explored in a tabular method as well.

Examples:

A recursively defined sequence: $u_1 = 1; u_n = 1 + 2u_{n-1}$

A recursively defined sequence, $u_1 = 1; u_2 = 3; u_n = u_{n-1} + 2u_{n-2}$

An explicitly defined sequence: $u_n = \dfrac{n}{n+1}$

An example of Newton's Method

In the following example, Newton's Method is restated to allow a view of the convergence to the square root of 12. The square root of any number c can be approximated with this method by allowing $u_n = .5(u_{n-1} + \dfrac{c}{u_{n-1}})$ and u_{nMin} to be the initial guess.

Predator-Prey Models

A great application of the sequence mode of the calculator is to study predator-prey models.

Suppose that we have an ecosystem with foxes and rabbits. Let's assume the following initial conditions:

Initial number of rabbits: 500
Initial number of foxes: 50
Rabbit growth rate without foxes per month: 0.06
Rabbit death rate per foxes per month: 0.0015
Fox population growth rate per rabbit per month: 0.0002
Fox population death rate without rabbits per month: 0.03

Given these conditions then the number of rabbits present in month n is a function of the number of rabbits that were present the previous month, the rabbit population growth rate, the number of foxes and the rabbit population death rate, in this case:

$$R_1 = 500; \ R_n = R_{n-1}(1+.06-.0015*F_{n-1})$$

Likewise the fox population is a function of the number of foxes that were present the previous month, the fox population growth rate, the number of rabbits and the fox population death rate:

$$F_1 = 50; F_n = F_{n-1}(1+.0002*R_{n-1}-.03)$$

Series

Series can be explored using the sequence mode by examining the sequence of partial sums. Since on the TI-Nspire a series must be defined recursively, an adjustment must be made to the sequence rule that is used to determine the partial sums.

For example explore the series: $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}: u_n = \frac{1}{(n+1)^2}; \ v_1 = 1; \ v_n = v(n-1)+u(n-1)$

This handout and the associated TI-Nspire tns file are available through the Rice University School Mathematics Project website.