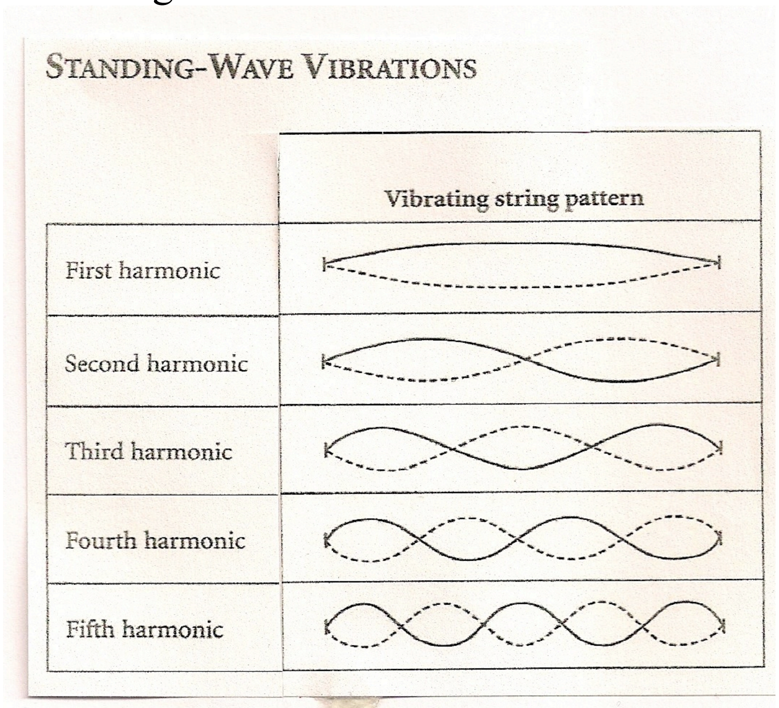
Connecting Math and Music Worksheet 2: Pitch



1. Write each string length ratio below, **using 1 for the length of the 1st harmonic**. Do not simplify.

1st harmonic : 2nd harmonic	2nd harmonic : 3rd harmonic
3rd harmonic : 4th harmonic	4th harmonic : 5th harmonic

2. Express these string length ratios using the smallest possible integers.

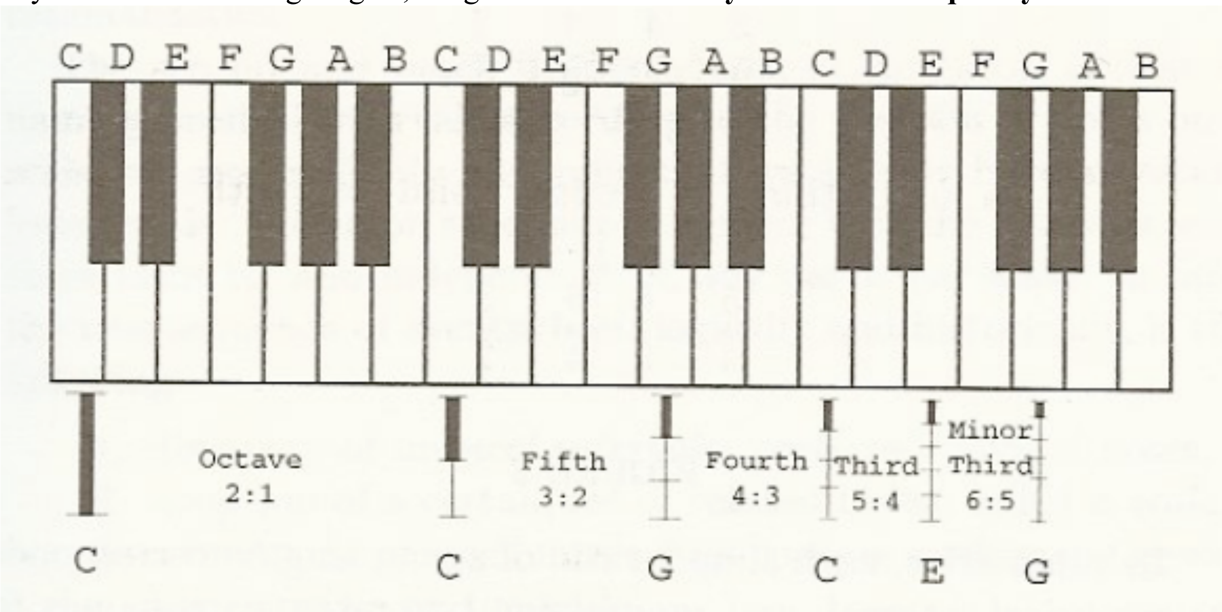
1st harmonic : 2nd harmonic	2nd harmonic : 3rd harmonic
3rd harmonic : 4th harmonic	4th harmonic : 5th harmonic

3. Remembering that frequency is inversely proportional to string length, express the frequency ratios using the smallest possible integers.

1st harmonic : 2nd harmonic	2nd harmonic : 3rd harmonic
3rd harmonic : 4th harmonic	4th harmonic : 5th harmonic

continued on reverse side

For the remaining exercises, we are concerned only with frequencies. The ratios below the keyboard refer to string lengths, so ignore them. **Use only the bolded frequency ratios**.



Freq. 64 128

Frequency Ratios **1 : 2** **2 : 3** **3 : 4** **4 : 5** **5 : 6**

4. Suppose the leftmost C above has frequency 64 hertz (**hertz = cycles per second**). Its frequency, 64, and the frequency of the next C, 128, are given. Label the frequencies of the remaining labeled notes.

   [64 hertz is close to the frequency of the second C from the left on a piano keyboard. It's frequency is actually about 65.4, so this is close. We rounded to 64 for ease of calculation.]

5. Label the G above the leftmost C and record its frequency. Is its frequency consistent with the G an octave above it? _______ Explain.

6. C-E-G is a C-major chord. For this chord, the ratio of frequencies, using whole numbers, is ___ : ___ : ___. Explain your thinking.

Lew Douglas, Co-Director Bay Area Mathematics Project

Connecting Math and Music Worksheet 3: Sound Volume

Measuring sound volume involves measuring the power of sound waves. The power of the waves is the rate of energy received, not its amount.
Here's an analogy: When you water your garden, the *power* of the water corresponds to how fast it comes out, determined by how much you open the nozzle. The *energy* corresponds to the total amount of water you use. You can use only a little water (energy) with a nozzle opened wide (powerful) if you water for just a few seconds.

1. The situation is similar with light bulbs. The100-watt label on a light bulb measures the power. *Watt-hours* measure energy.
   1000 watt-hours = 1 kilowatt hour = 1 kwh, the standard unit of energy on your electric bill. If a 75-watt light bulb is on for 20 hours, how much energy is used?

Sound volume corresponds more closely to the *power* of a sound wave than to its energy. As an analogy, if someone turns a stream of water on you, you feel it more if the hose nozzle is opened wide. But the "kick" from a strong stream of water also depends on the area of the receptor. If you block a strong stream of water with a 3 ft. by 3 ft. piece of plywood, you'll get a much stronger "kick" if you hold the plywood perpendicular to the stream than if you hold it "edge on." To give the ear more receiving area, people "cup" their ears with their hands to hear better.

Power, sometimes called power intensity, is measured in watts. But the above discussion shows that we are interested in *power density*, which for sound is usually measured in *watts per square meter*.

There's another complication, however: our ears hear *logarithmically*.

2. Suppose we hear a sound with a power density p as volume v. If we multiply the power density by 10 so that it becomes $10p$, the sound becomes louder by a certain amount, which is called 1 bel. So our new loudness in bels is $v + 1$. Complete the table below.

Power Density (watts/m^2)	Volume (bels)
p	v
$10\,p$	$v + 1$
$100\,p$	
	$v + 3$

Let P = Power Density in watts/m^2 and V = Volume in bels. Base 10 is understood when the base is omitted, as usual. The general logarithmic function base 10 may be written $y = a\log x + b$. (A form for the general logarithmic function is not given in many texts; it can be derived from $y = c(10)^{dx}$, a possible form for the general exponential function, by solving for x in terms of y.) With our variables, the general form becomes $V = a\log P + b$.

3. Substitute the values from the first two rows of the table in Ex. 2 into $V = a\log P + b$. Use the resulting equations to show that $a = 1$.

Since $a = 1$, the general equation becomes $V = \log P + b$. But it turns out that the *bel* is too coarse a unit to be useful. 1 bel = 10 decibels = 10 *dB*. Since there are 10 times as many decibels as bels, we have to multiply the volume by 10 if we measure in decibels. Therefore we get the important equation to the right in the box below.

$V = \log P + b$ if V is in <u>bels</u>.	$V = 10\log P + b$ if V is in <u>decibels</u>

Since the constant 10b is not important, let $c = 10b$ for simplicity. Thus our general equation becomes $V = 10\log P + c$.

4. Using the general volume equation $V = 10\log P + c$ for these exercises.
   a. If we double the power, how much louder (in decibels) does the sound get? Hint: Replace P by 2P in the equation and determine the *increase* in V.

   b. If we multiply the power by 10, how much louder (in decibels) does the sound get?

   c. If we multiply the power by 100, how much louder (in decibels) does the sound get?

Lew Douglas, Co-Director Bay Area Mathematics Project

References for Connecting Math and Music

I. Books and Journals

A. *Fermat's Last Theorem A Supplement to the Video*, Edited by Robert Osserman in 1995 for the Mathematical Sciences Research Institute, 1000 Centennial Drive, Berkeley, CA 94720. Section 6 – *Rational and Irrational: Music and Mathematics*. Reprinted from *Essays in Humanistic Mathematics*, MAA 1993 (I will post a scan of the article, which is great, but quite dense.)

B. *Functional Melodies: Finding Mathematical Relationships in Music*, by Scott Beall, ©2000 by Key Curriculum Press. http://scottbeall.com/book.htm

C. *Math and Music Harmonious Connections*, by Trudi Hammel Garland and Charity Vaughan Kahn, ©1995 by Dale Seymour Publications. Easy reading. Can be located by a web search.

D. *The Acoustical Foundations of Music 2nd edition*, by John Backus, ©1977 by W.W. Norton and Co, Inc. Old, but has good information on Volume and Sound Waves. Can be located by a web search.

E. *Music: A Mathematical Offering*, by David J. Benson, ©2007 Cambridge University Press. Comprehensive. Includes some quite advanced math. Can be located by a web search.

II. Freeware

Metronome for Mac
http://members.ozemail.com.au/~ronfleckner/metronome/

III. Apps

Piano HD – Has one or two keyboards you can play. Very useful.
GarageBand - an inexpensive, well-known music studio that does a little of everything
Figure - a $1 app with drum, bass, and lead tracks for beginning music creation
miniSynth 2 - a FREE, but powerful little synthesizer
DM1 - a good all-purpose drum machine
Audiobus - an app that allows you to use the above apps together
WaveWindow – oscilloscope app for Mac. Oscilloscope apps exist for other platforms as well. Do a search.

IV. Musical Notation

Reference Guide
http://www.treblis.com/Notation/Music.htm

V. Wikipedia Links
   A. Examples of well-known songs using every ascending and descending pitch interval: http://en.wikipedia.org/wiki/Ear_training#Interval_recognition
   B. Music of the Spheres (Musica Universalis): http://en.wikipedia.org/wiki/Music_of_the_spheres
   C. Numerical representation of chords: http://en.wikipedia.org/wiki/Chord_chart#Nashville_notation
   D. Explanation of 12-tone "serial" composition: http://en.wikipedia.org/wiki/Twelve-tone_technique
   E. Musical Mode: http://en.wikipedia.org/wiki/Musical_mode#Modern (skip historical discussion)
   F. Musical Scale: http://en.wikipedia.org/wiki/Musical_scale
   G. List of Musical Scales and Modes: http://en.wikipedia.org/wiki/List_of_musical_scales_and_modes
      Non-Western music has more complex and very rigorous systems of scales and modes constructed from the basic 12 tones, such as Turkish makam (http://en.wikipedia.org/wiki/Makam), Middle Eastern hejaz, Arabic maqam, Indian rajas, etc.
   H. Rhythm: http://en.wikipedia.org/wiki/Rhythm
   I. Musical Meter: http://en.wikipedia.org/wiki/Compound_meter_(music)#Simple_meter
   J. Irrational Rhythm: http://en.wikipedia.org/wiki/Irrational_rhythm
   K. Polyrhythm: http://en.wikipedia.org/wiki/Polyrhythm
   L. Syncopation: http://en.wikipedia.org/wiki/Syncopation

VI. Other Web Links
   A. Decibel Comparison Chart: http://www.gcaudio.com/resources/howtos/loudness.html
   B. Frequency and decibels in music: http://www.mikfielding.com/Hz's_dB's.shtml
   C. Body Music with Crosspulse: http://www.crosspulse.com/aboutkt.html
      After examining this page, click on “International Body Music Festival” in left column. Click on “Schedule + Tickets,” the second item on the left. Under Thurs. Nov. 3, you’ll find “Teacher Training” from 4-6 pm. I went to this workshop, which I enjoyed very much. The activities are suitable from 2nd or 3rd grade on up. At the bottom of the workshop description, you’ll find a “Print Handouts” link that will take you to the workshop handout.
   D. The Correlation Between Music and Math: A Neurobiology Perspective, by Cindy Zhan http://serendip.brynmawr.edu/exchange/node/1869