Hot Mathematical Questions

Throughout history, mathematicians have published puzzling questions and challenged each other to answer them. Sometimes they even offer prizes. Right now the Clay Mathematics Institute is offering $1,000,000 to anyone who solves one of seven really tricky problems. Only one has been solved so far, but Grigoriy Perelman refused to show up and take the money. That's good, because in this classroom we can't afford a million dollars. Sorry.

Can you solve and prove one of our class's hot mathematical questions?

Sylvia Glauster – The Ancona School – sglauster@anconaschool.org

Here are a few of the million-dollar questions:

Riemann Hypothesis

Some numbers have the special property that they cannot be expressed as the product of two smaller numbers, e.g., 2, 3, 5, 7, etc. Such numbers are called *prime* numbers, and they play an important role, both in pure mathematics and its applications. The distribution of such prime numbers among all natural numbers does not follow any regular pattern, however the German mathematician G.F.B. Riemann (1826 - 1866) observed that the frequency of prime numbers is very closely related to the behavior of an elaborate function

$$\zeta(s) = 1 + 1/2^s + 1/3^s + 1/4^s + \ldots$$

called the *Riemann Zeta function*. The Riemann hypothesis asserts that all *interesting* solutions of the equation

$$\zeta(s) = 0$$

lie on a certain vertical straight line. This has been checked for the first 1,500,000,000 solutions. A proof that it is true for every interesting solution would shed light on many of the mysteries surrounding the distribution of prime numbers.

P vs. NP problem

If it is easy to check that a solution to a problem is correct, is it also easy to solve the problem? This is the essence of the P vs NP question. Typical of the NP problems is that of the Hamiltonian Path Problem: given N cities to visit (by car), how can one do this without visiting a city twice? If you give me a solution, I can easily check that it is correct. But I cannot so easily (given the methods I know) find a solution.

Navier-Stokes Equation

Waves follow our boat as we meander across the lake, and turbulent air currents follow our flight in a modern jet. Mathematicians and physicists believe that an explanation for and the prediction of both the breeze and the turbulence can be found through an understanding of solutions to the Navier-Stokes equations. Although these equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations.

Descriptions from the Clay Mathematics Institute.
www.claymath.org/millenium

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