I See It!
The Power of Visualization

Marc Garneau • piman@telus.net • Surrey, BC, Canada
@314Piman • diaryofapiman.wordpress.com
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> "*Visualization is fostered through the use of concrete materials, technology and a variety of visual representations. It is through visualization that abstract concepts can be understood concretely by the student.*" The Common Curriculum Framework for Grades 10-12 Mathematics. WNCP (2008)

Compare the following fractions **visually**, not symbolically (ie. no common denominators, no decimals, no cross-multiplying, etc.)

(a) $\frac{4}{5}$ $\frac{4}{9}$

(b) $\frac{3}{4}$ $\frac{9}{10}$

(c) $\frac{3}{8}$ $\frac{4}{10}$

Visualizing Patterns - The Border Tiles Problem

The tiling pattern below is made of a growing pattern of yellow tiles, surrounded by a border of green tiles.

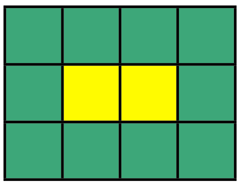


Fig. 1

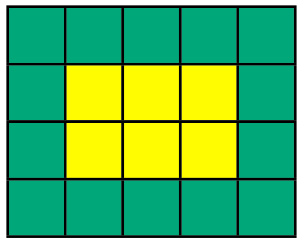


Fig. 2

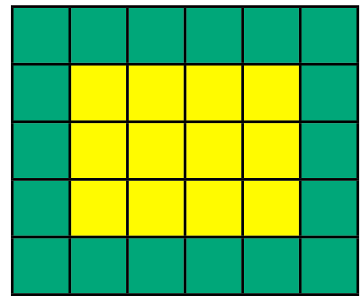


Fig. 3

1. Assuming the pattern continues, describe how you would build the next two figures.

2. Complete the table, showing your calculations.

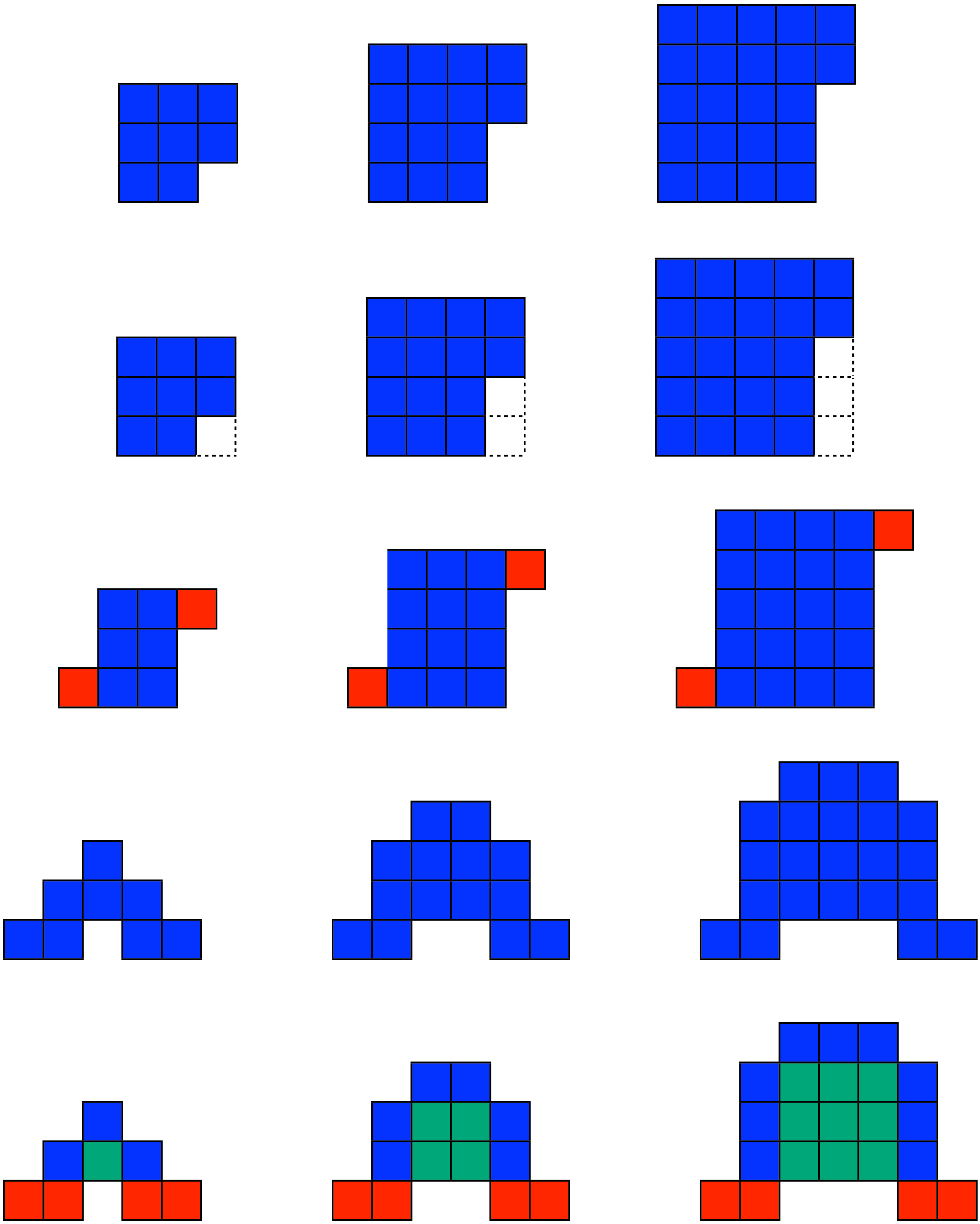
Figure #	Border Tiles (green)	Centre Tiles (yellow)	Total Tiles
1			
2			
3			
4			
5			
10			

3. Complete the table.

	border tiles	**centre tiles**
function of the figure number (*n*) (more than 1 way?)		
type of relationship		
meaning of terms (context of tile problem)		
sketch a graph		

4. Write the **total** number of tiles as a function of the figure number, and explain the meaning of the terms in the context of the tiling pattern.

All of these tell the same story, but in different ways and at different levels.



Extension Ideas

Design a tile pattern to match the function: $f(n) = n^2 + 2n + 1$

- Draw the first three figures of your pattern.
- Show how your pattern matches the function.

Design your own tile pattern.

- Draw the first three figures of your pattern.
- Write a function that represents your tile pattern.
- Show how your pattern matches the function.

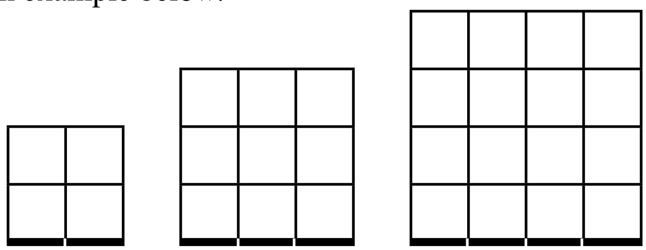
Write a function: $f(n) =$

- Design a tile pattern to match your function.
- Draw the first three figures of your pattern.
- Show how your pattern matches the function.

Exploring Radicals

Use a calculator and notice, $\sqrt{20} = 2\sqrt{5}$. When $\sqrt{20}$ is said to be **simplified**, the result is $2\sqrt{5}$. How do we get this result? This lesson explores a way to simplify radicals.

Given a square of area n, the length of the side of that square is $\sqrt{n}$.
Complete the information for each example below.



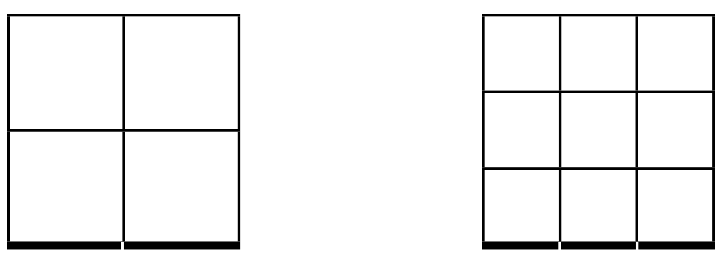
Area = ______ ______ ______

Side Length = ______ ______ ______

But what if the area is not a square number? For example, consider $\sqrt{18}$.
Estimate the value of $\sqrt{18}$? What reasoning did you do to come up with your estimate?

Suppose the area of each square below is 18. For each one:

- What is the area of each small square?
- Represent the length of <u>each side</u> of the small square using $\sqrt{\ }$.



Area (small square): ______ ______

Side length (small square): $\sqrt{\quad}$ ______ $\sqrt{\quad}$ ______

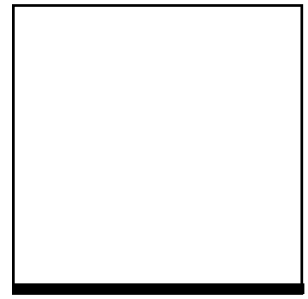
Of the two ways to express the side length of the small square, which one is the simplest? ______

How many of these side lengths does it take to make up the side length of the larger square? ______

Considering your answers to the past 2 questions, what is another way to write $\sqrt{18}$? ______

Would it make sense to divide this square into more squares (16, 25, 36, etc.)? Explain.

Suppose the area of the square on the right is 75.
What do you think is the best choice for the number of small squares? Why?



Sketch the small squares. What is the area of each small square? __________

What is the side-length of each of the small squares, and how many side lengths make up the base of the larger square?

What is another way to write $\sqrt{75}$? __________

Describe a general strategy for simplifying radicals.

Applying the same strategy, what is another way to represent each of the following radicals?

$\sqrt{8}$	$\sqrt{12}$	$\sqrt{54}$
$\sqrt{32}$	$\sqrt{50}$	$\sqrt{72}$

Making Connections:

Solving Quadratic Equations

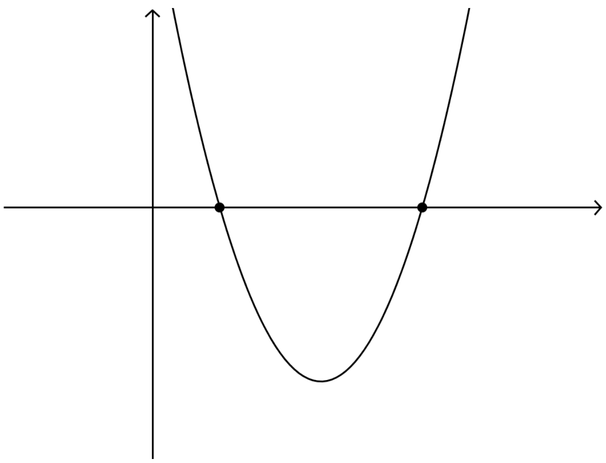
Solve the following equation for *x*...

$$2(x-3)^2 - 8 = 0$$

Graphically

Algebraically

What connections can you find between these two methods?



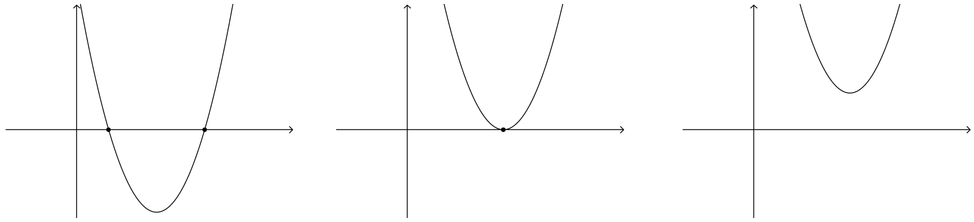
To solve for the zeros of a quadratic function expressed in vertex form, solve the following equation for *x*:

$$a(x-p)^2+q=0$$

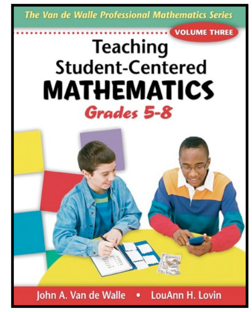
Resulting formula: $x =$

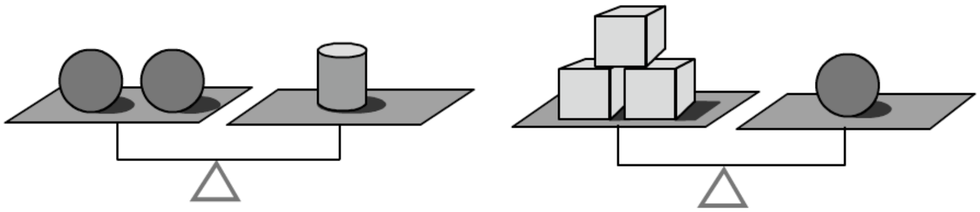
Explain and illustrate the meaning of this formula in terms of the graph of a quadratic function.

Explain and illustrate the nature of the roots of a quadratic function in terms of this formula.

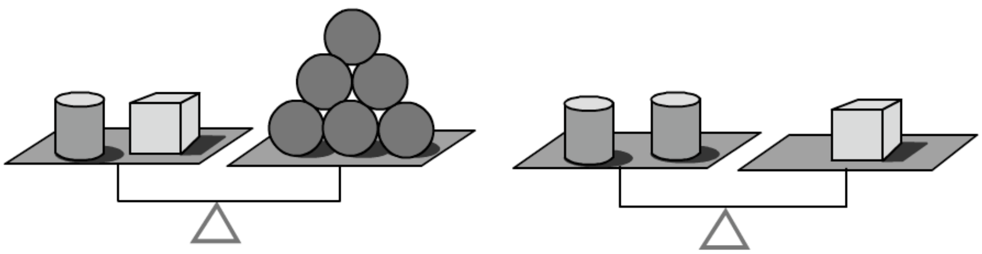


Balance Problems

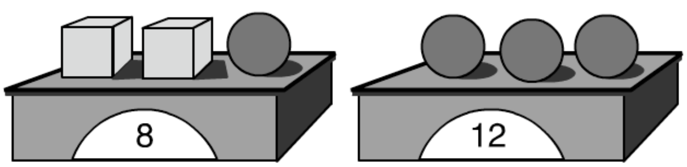




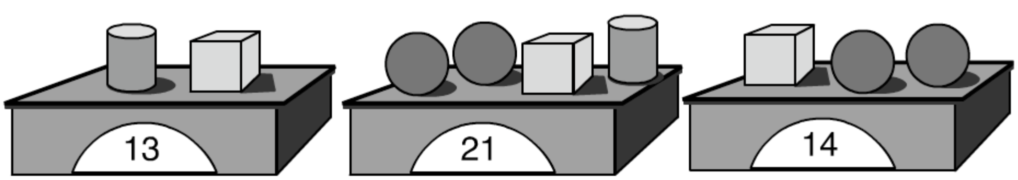
Which shape weighs the most? Explain.
Which shape weighs the least? Explain.



What will balance 2 spheres? Explain.



How much does each shape weigh? Explain.



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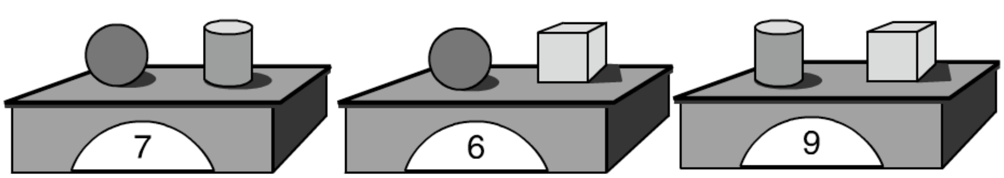
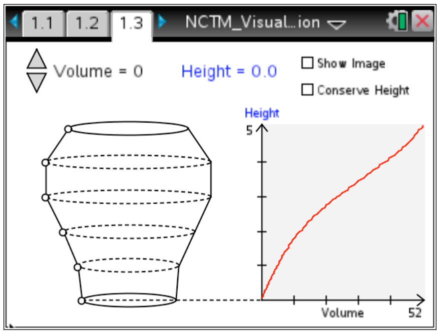


How much does each shape weigh? Explain.

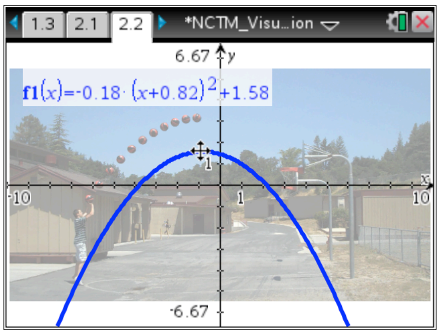
Some TI-Nspire Visualization

All of these problems are collected in the document NCTM_Visualization.tns
(*bit.ly/NCTMVisualization*)



Fill the Urn

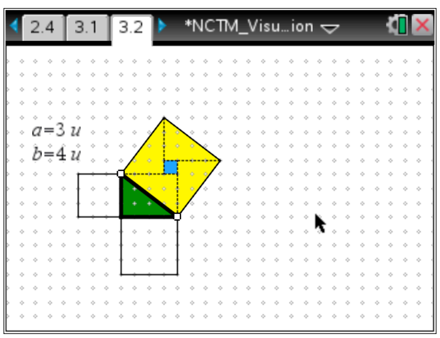
A graph of the height as a function of the volume is shown.
The shape of the urn can be changed.
How will changing the urn's shape change the graph?



Will it Hit the Hoop?

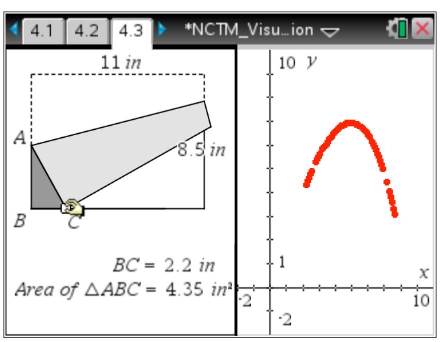
An Nspire-ized version of one of Dan Meyer's problems (blog.mrmeyer.com/?p=8483).
To model the path of the ball, graph a quadratic function and manipulate it by grabbing:

- To translate it, grab near the vertex.
- To stretch it, grab towards outsides.



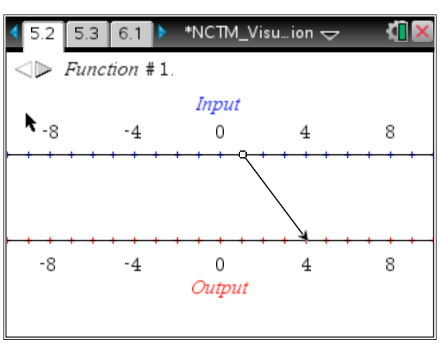
Exploring the Pythagorean Theorem

Visualize spatial relationships that make up the square on the hypotenuse, and use these expressions to determine the area of that square.



The Paper Folding Problem

An 8.5" x 11" piece of paper is folded by moving a corner to the opposite side, thus creating a triangle in the corner.
What kind of function will model the relation of the area of the this triangle to the position of point C?
What position of point C will maximize the area?



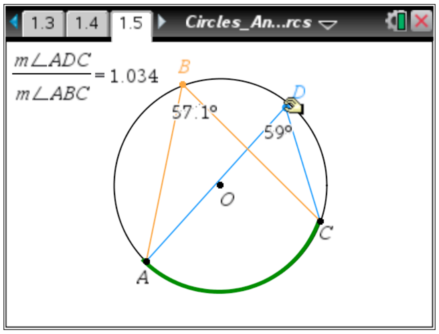
Dynagraphs

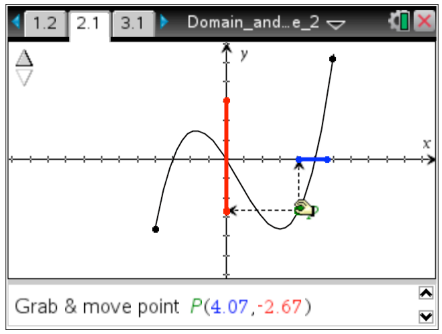
This is a different visualizations of functions. Instead of a coordinate grid, the relation of *y* to *x* is shown as a mapping from one number line to another.
You can check your answer by looking on page 5.3.
To get a new function, click on the slider. There are a total of 9 to try.

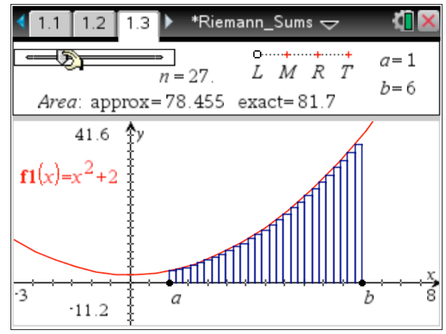
Some Sources to Sample ~ Visualization through Technology

Math Nspired (*education.ti.com/calculators/timathnspired/*)

- Action-Consequence TI-Nspire documents & lessons, organized by course and key concepts

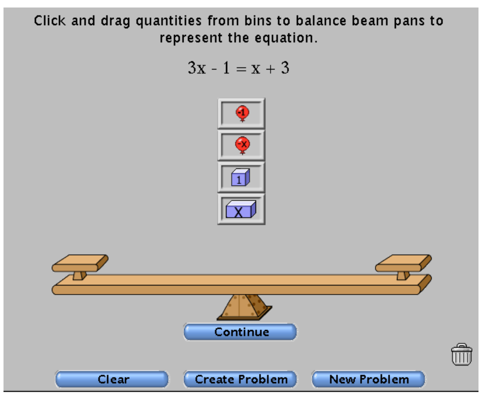


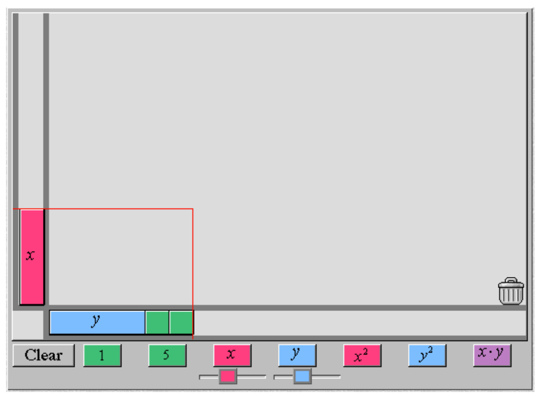




National Library of Virtual Manipulatives (*nlvm.usu.edu*)

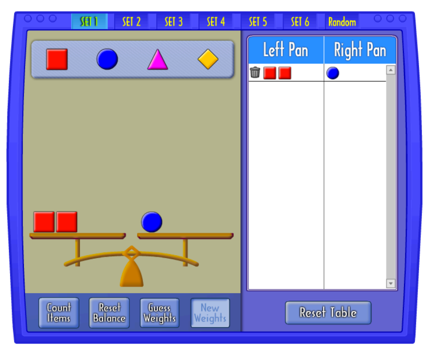
- Virtual manipulatives organized by concept & grade band





NCTM Illuminations (*illuminations.nctm.org*)

- Activities organized by concept & grade band



And a few more things to check out

- *visualpatterns.org* - predict later terms, and determine expressions for patterns represented visually. Created by Fawn Nguyen. Also check out her blog: *fawnnguyen.com.* Speaking of blogs, there are a lot of really great ones, with ideas and how they turned out in the classroom.
- *www.cut-the-knot.org* - Interactive Mathematics Miscellany and Puzzles
- *www.explorelearning.com - Gizmos* (*lots of great activities, but at a cost*)
- *www.desmos.com* - FREE online graphing calculator, with simple setup for sliders, & much more
  - Also check out *www.dailydesmos.com*
- *www.geogebra.org* - FREE interactive geometry, algebra, stats & calculus software
  - Also check out *geogebratube.org* for user-submitted activities
- *www.dynamicgeometry.com* - The Geometer's Sketchpad Resource Center
- iPad apps - only a small fraction of them are decent, even fewer are really good. My favourites are TI-Nspire and Sketchpad Explorer. Note also that a Geogebra app is coming, and both Geogebra & Desmos work on the web.