

Proportional Reasoning?

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Creating Viable
Arguments Through
Inquiry and
Investigation

Ice Breaker

- Who works in a state that has fully implemented the Common Core?
- Who works in a state that has partially implemented the Common Core?
- Who works in a state that has not yet implemented the Common Core?

Defining Proportional Reasoning

Proportional reasoning involves understanding that (a) equivalent ratios can be created by iterating and or partitioning a composed unit, and (b) if one quantity in a ratio is multiplied or divided by a factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship.

(NCTM, 2013)

Are They Proportional?

$$\frac{3}{4}$$

$$\frac{4}{5}$$

$$\frac{5}{6}$$

How do you know?

Are These Proportional?

$$\frac{2}{5}$$

$$\frac{30}{75}$$

$$\frac{226}{565}$$

How did you decide?

Are These Equivalent to $\frac{1}{7}$?

$$\frac{11}{17}$$

$$\frac{14}{47}$$

$$\frac{3}{21}$$

Explain your reasoning.

Ratios and Proportional Relationships 6.RP.3b

Understand ratio concepts and use ratio reasoning to solve problems.

3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
 - b. Solve unit rate problems including those involving unit pricing and constant speed.

Rates and Unit Rates

Video

6.RP.3b I can solve real world problems involving unit pricing.

Tyneshia babysits for her nieces and nephews. She earns \$5 for babysitting for 1 hour. How much will she earn in 4 hours?

1

1

1

1

Hours	Process	Dollars
1		6
2		
3		
4		

6.RP.3b Student Work #1



6.RP.3b I can solve real world problems involving unit pricing.

Find the unit rate in each scenario.

1. Colton purchased 4 pounds of candy for \$3.76.

$$\frac{3.76}{4}$$

- a. How much does 1 pound of candy cost?

$$\begin{array}{r} 4 \overline{) 3.76} \\ \underline{4} \\ 0 \\ \underline{0} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

Each pound cost .94¢.

- b. How much candy could he buy for a dollar?

he could had buy 1 pound for a dollar.

2. Megan and Katie went shopping for socks. Megan said, "Four pairs of socks for \$9 is the better deal." Katie said, "No, five pairs of socks for \$13 is the better deal." Who is getting the better deal? Explain how you got your answer.

$\begin{array}{r} 9.00 \text{ Megan} \\ \hline 4 \\ \hline 2.25 \end{array}$	$\begin{array}{r} 13.00 \text{ Katie} \\ \hline 5 \\ \hline 2.60 \end{array}$	$\begin{array}{r} 2.60 \\ \hline 2.75 \\ \hline 2.60 \\ \hline 0.15 \end{array}$
$\begin{array}{r} 4 \overline{) 9.00} \\ \underline{8} \\ 10 \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$	$\begin{array}{r} 5 \overline{) 13.00} \\ \underline{10} \\ 30 \\ \underline{25} \\ 50 \\ \underline{50} \\ 0 \end{array}$	$\begin{array}{r} 2.60 \\ \hline 2.75 \\ \hline 2.60 \\ \hline 0.15 \end{array}$

*Katie had the best deal I know this because her are \$2.60 and Megan cost \$2.75 And I got my answer by dividing as you see in the top.

6.RP.3b Student Work #2



6.RP.3b I can solve real world problems involving unit pricing.

Find the unit rate in each scenario.

1. Colton purchased 4 pounds of candy for \$3.76.

$$4 \div 3.76 = 0.24$$

- a. How much does 1 pound of candy cost?

1 pound of candy cost 0.24

- b. How much candy could he buy for a dollar?

He could buy 2 pounds of candy for a dollar

2. Megan and Katie went shopping for socks. Megan said, "Four pairs of socks for \$9 is the better deal." Katie said, "No, five pairs of socks for \$13 is the better deal." Who is getting the better deal? Explain how you got your answer.

$$4 \div 9 = 2.25$$

$$5 \div 13 = 2.6$$

Megan is getting a better deal because Megan will get \$2.25 back and Katie will only get \$2.60 back.

A Common Misconception...

A common error in setting up proportions is placing numbers in incorrect locations. This is especially easy to do when the order in which quantities are stated in the problem is switched within the problem statement.

(Progressions for the Common Core State Standards in Mathematics, 6-7, Ratios and Proportional Relationships, 2012)

Geometry 7.G.1

Draw, construct, and describe geometrical figures and describe the relationships between them.

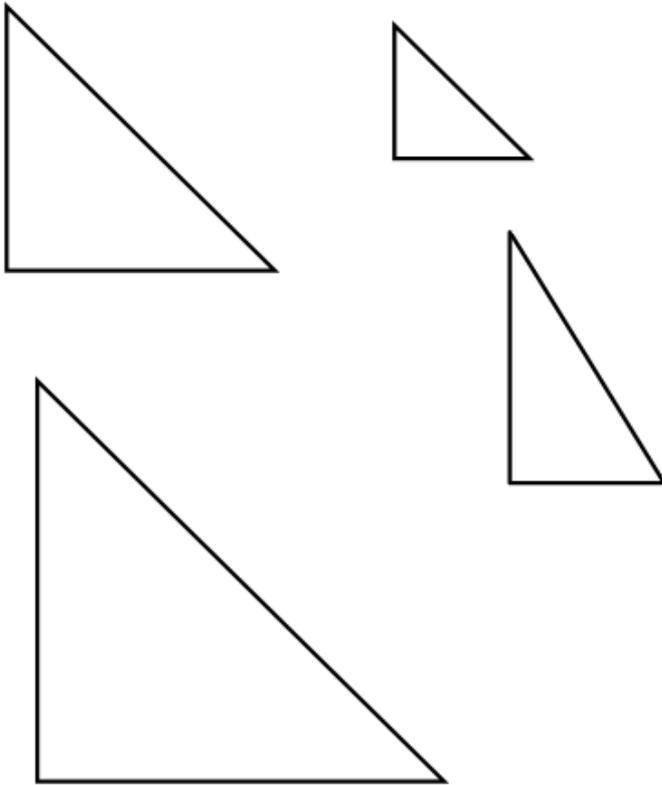
1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Solving for Similar Triangles Video

7.G.1 -



I can use proportional reasoning to identify similar figures.



- Are the triangles proportional?
- Explain how you made your determination?

7.G.1 Student Work #1

Student Work 2

• I can create a scale drawing using a different scale.

Math Probe: Scale Drawings

Using Triangle A as the original, apply a scale of 2.5 to draw Triangle B on the same grid.

I took the # of units on each side and x it by 2.5

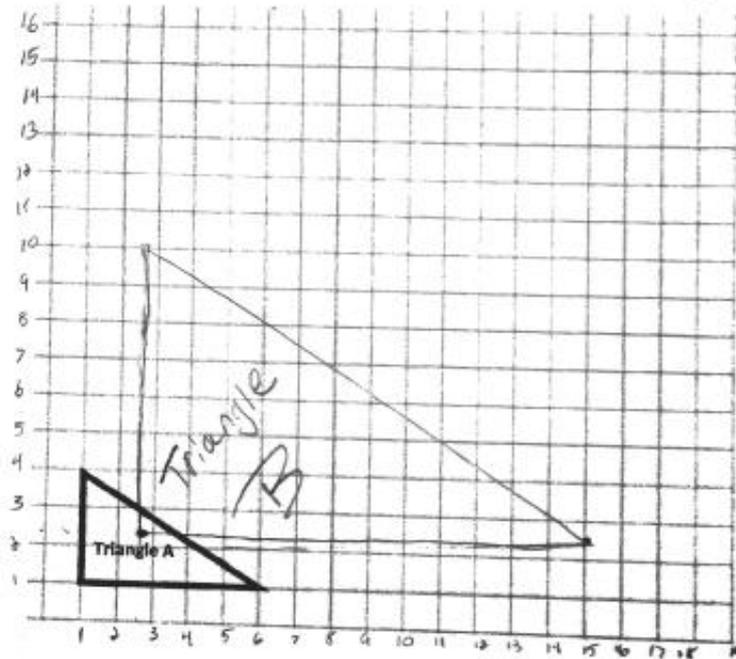
7.G.1 Student Work #2



- I can create a scale drawing using a different scale.

Math Probe: Scale Drawings

Using Triangle A as the original, apply a scale of 2.5 to draw Triangle B on the same grid.



I multiplied the x and y coordinates by 2.5.

$$(1, 1) \rightarrow (2.5, 2.5)$$

$$(1, 4) \rightarrow (2.5, 10)$$

$$(6, 1) \rightarrow (15, 2.5)$$

Cross Multiplication?

Teachers can promote students' proportional reasoning capacities by balancing skills and concepts and by **delaying the instruction of the cross-multiplication algorithm** until students have already gained experience with forming ratios and understanding proportions as an equivalence of ratios.

(NCTM 2013)

Expressions and Equations

8.EE.6

Understand the connections between proportional relationships, lines, and linear equations.

6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .

Rates, Proportions, and Similar Triangles

8.EE.6

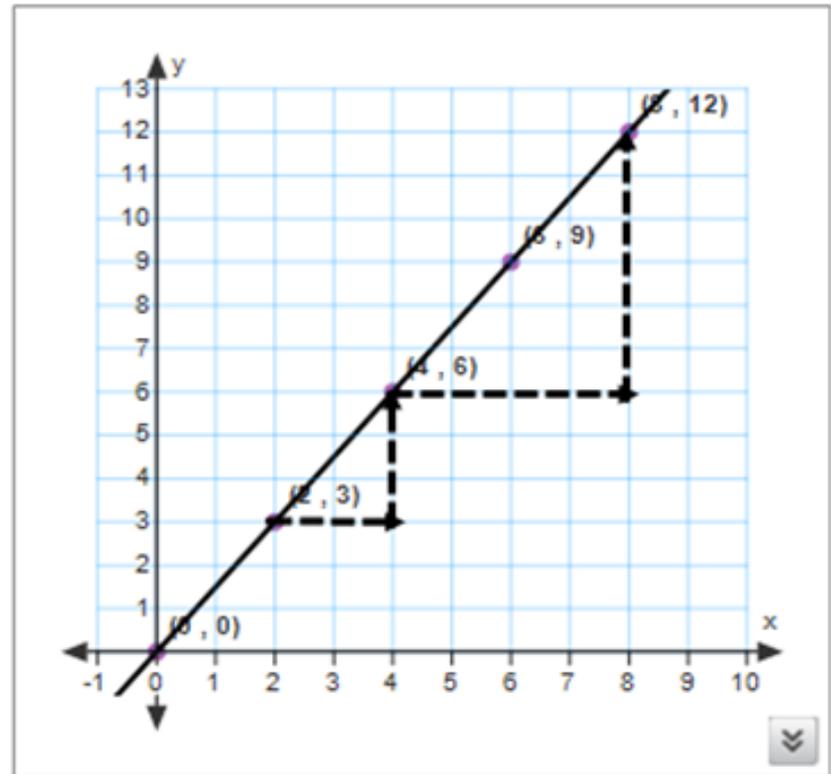


I can use similar triangles to explain slope of a line.

Math Journal Prompt: **Slope**

Use the graph below for questions 1 and 2.

1. How are the triangles related to each other? Explain how you know.
2. Why is the slope m the same between any two points along the line?



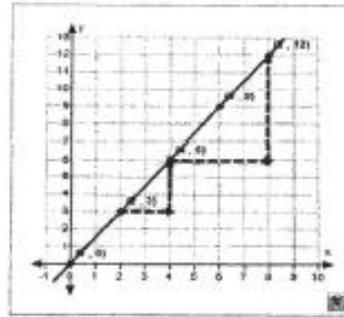
8.EE.6 Student Work #1



I can use similar triangles to explain slope of a line.

Math Journal Prompt: Slope

Use the graph below for questions 1 and 2.



1. How are the triangles related to each other? Explain how you know.

These triangles are related to each other by being right angle triangles, and they both create a rise and a run which is a fraction.
little triangle: $\frac{3}{4}$ Big triangle: $\frac{12}{8}$

their both proportional

2. Why is the slope m the same between any two points along the line?

Because their both on the same slope. And its all gonna end up proportional, its also a linear equation, and has a positive relationship?

8.EE.6 Student Work #2

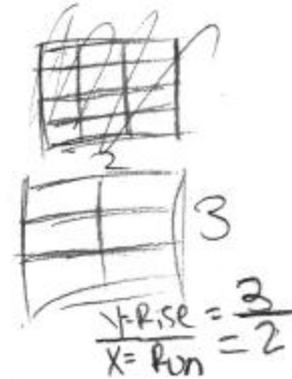
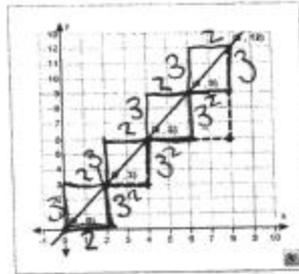


I can use similar triangles to explain slope of a line.

Math Journal Prompt: Slope

Use the graph below for questions 1 and 2.

It's a straight line with the same rate causing the slope to change at the same rate.



How are the triangles related to each other? Explain how you know.

Because it shows the rate of change is constant and the points change at the same rate. the rise and run is the same for all because the growth is the same. $\frac{3}{2} = \text{rise/run}$

2. Why is the slope m the same between any two points along the line?

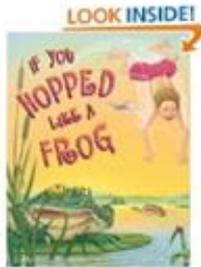
Because the slope is changing at a constant change/rate so with any 2 points you choose you will get the same equation as another set of points and the rise/run is the same change each time $\frac{3}{2}$.

Where we're going...

Teachers who understand the big idea of proportional reasoning can draw on the various related essential understandings to see familiar concepts in new ways. Consider some of the important sense making that can occur when teachers apply this new understanding to linear functions, algebraic equations, and the concept of slope.

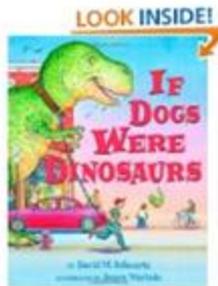
(Developing Essential Understanding of Ratios, Proportions & Proportional Reasoning, 2011)

Literacy Connections



If You Hopped Like A Frog by David Schwartz

ISBN-13: 978-0590098571



If Dogs Were Dinosaurs by David Schwartz and James Warhola

ISBN-13: 978-0439676120



Our New Car: Level 6 (Mathematics Readers) by Lori Barker

ISBN-13: 978-1433334511

Let's Look Back to the Standards

- Where do the Writing, Listening, and Speaking Standards fit into these lessons?
- Mathematical Practices, Ratio and Proportional Reasoning Writing, Listening, and Speaking Standards

Helpful Resources...

- Progressions for the Common Core State Standards in Mathematics for 6-7, [Ratios and Proportional Relationships](#)
- NCTM's, [Teaching Ratio and Proportion in the Middle Grades](#)

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