How Soon Is Too Soon? Algebra in Elementary Grades

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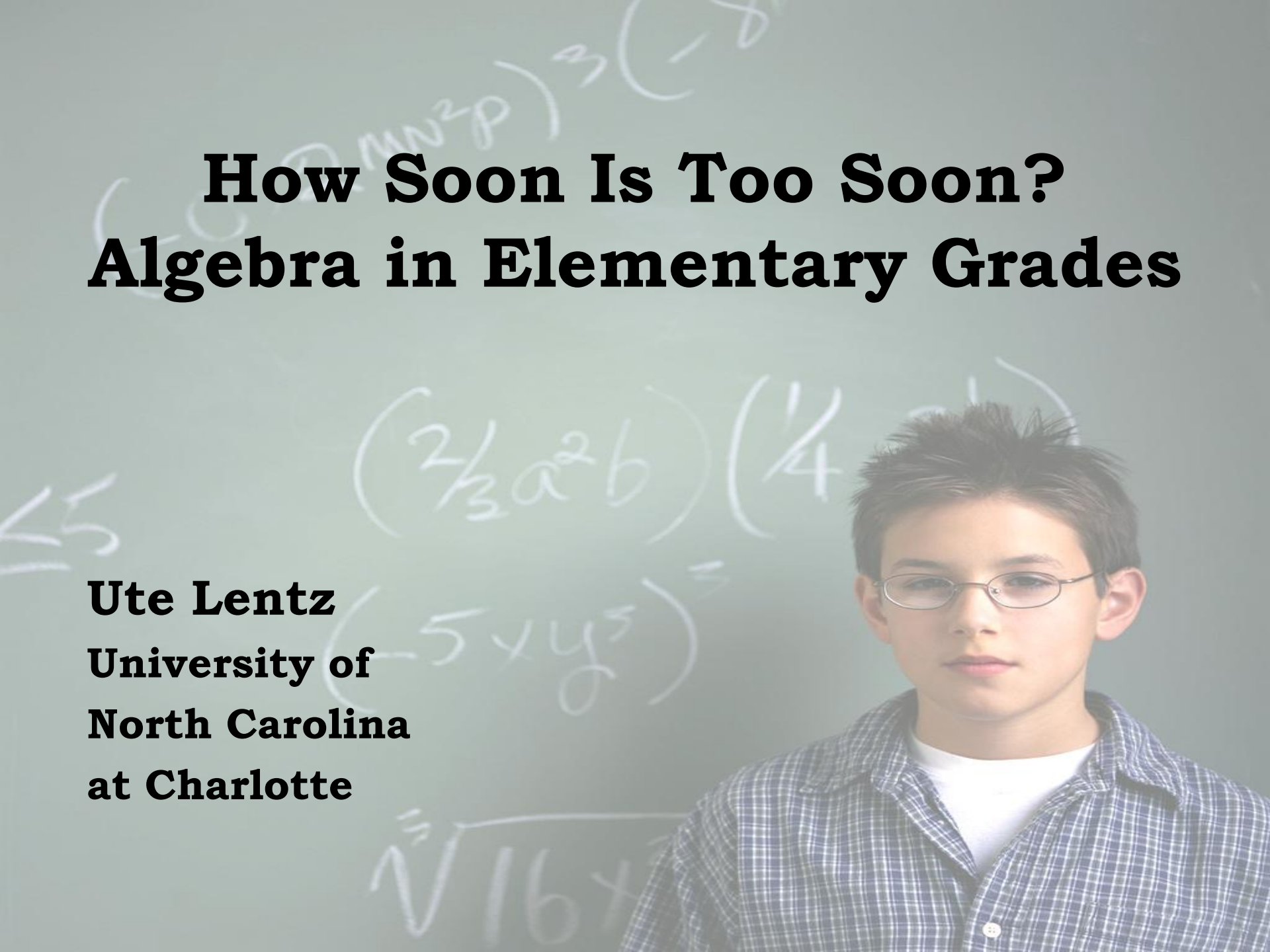
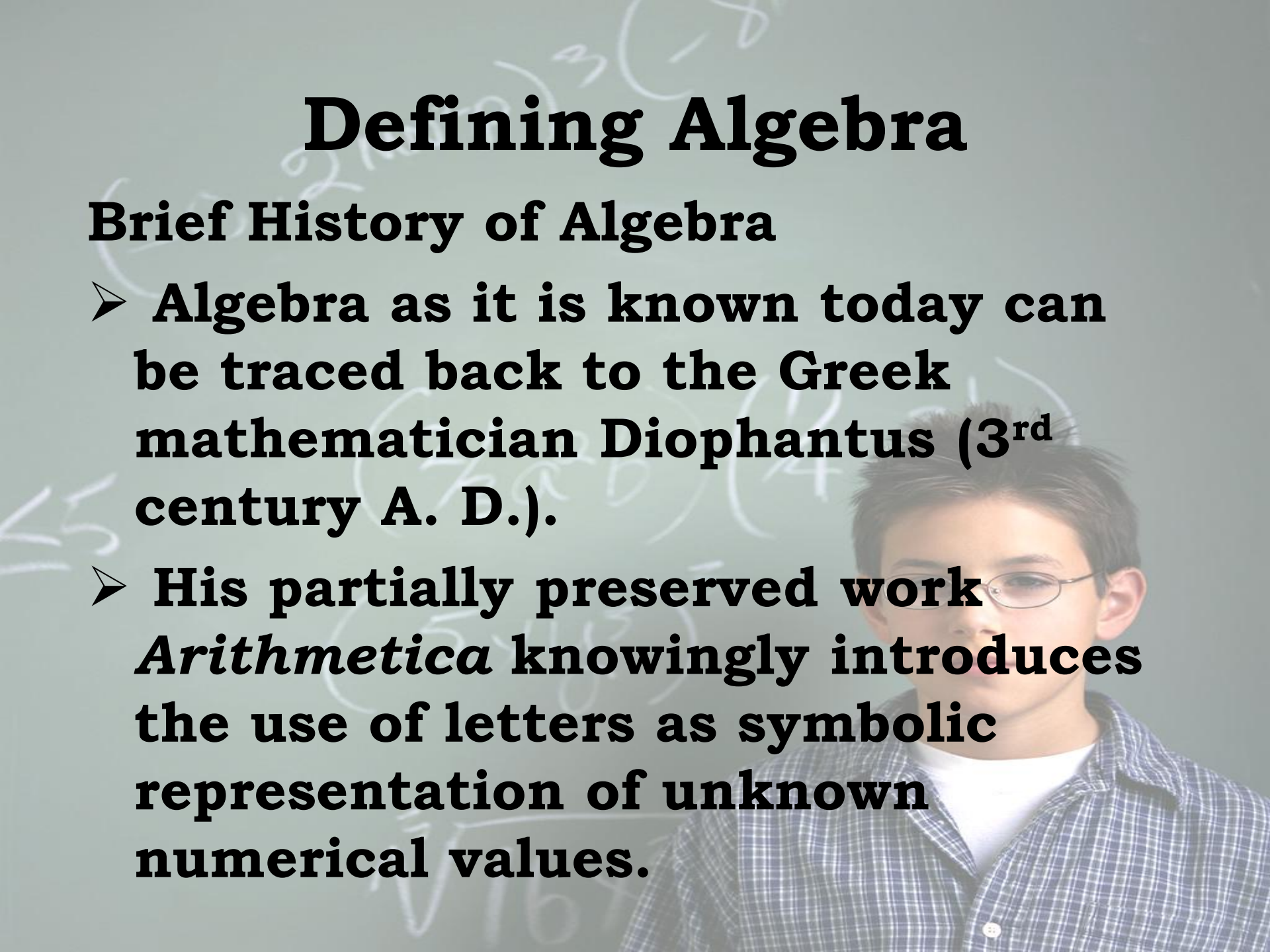


Defining Algebra

Brief History of Algebra

- **Algebra as it is known today can be traced back to the Greek mathematician Diophantus (3rd century A. D.).**
- **His partially preserved work *Arithmetica* knowingly introduces the use of letters as symbolic representation of unknown numerical values.**

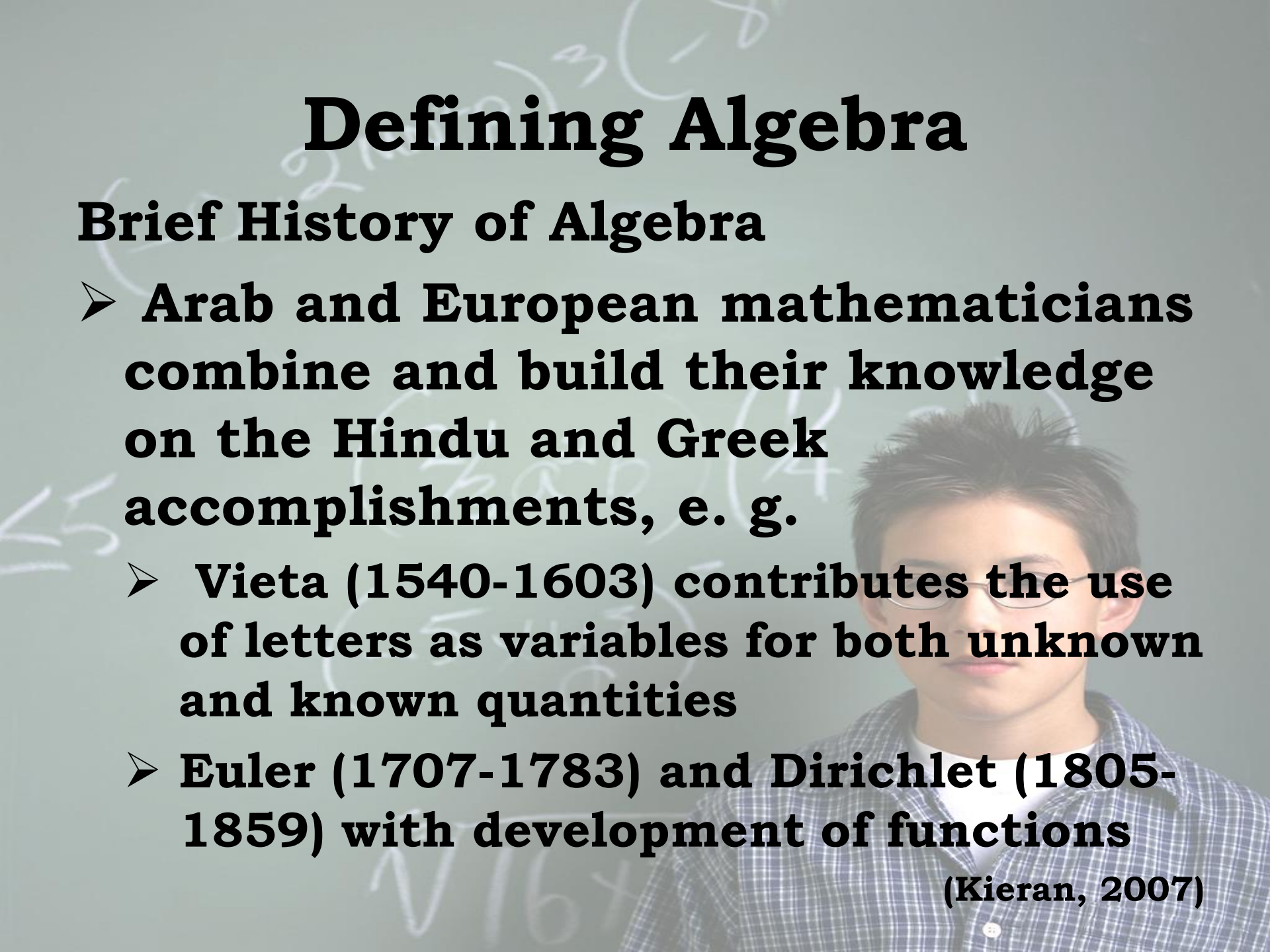


Defining Algebra

Brief History of Algebra

- **Arab and European mathematicians combine and build their knowledge on the Hindu and Greek accomplishments, e. g.**
  - **Vieta (1540-1603) contributes the use of letters as variables for both unknown and known quantities**
  - **Euler (1707-1783) and Dirichlet (1805-1859) with development of functions**

(Kieran, 2007)



Defining Algebra

Historical Progression of Algebra

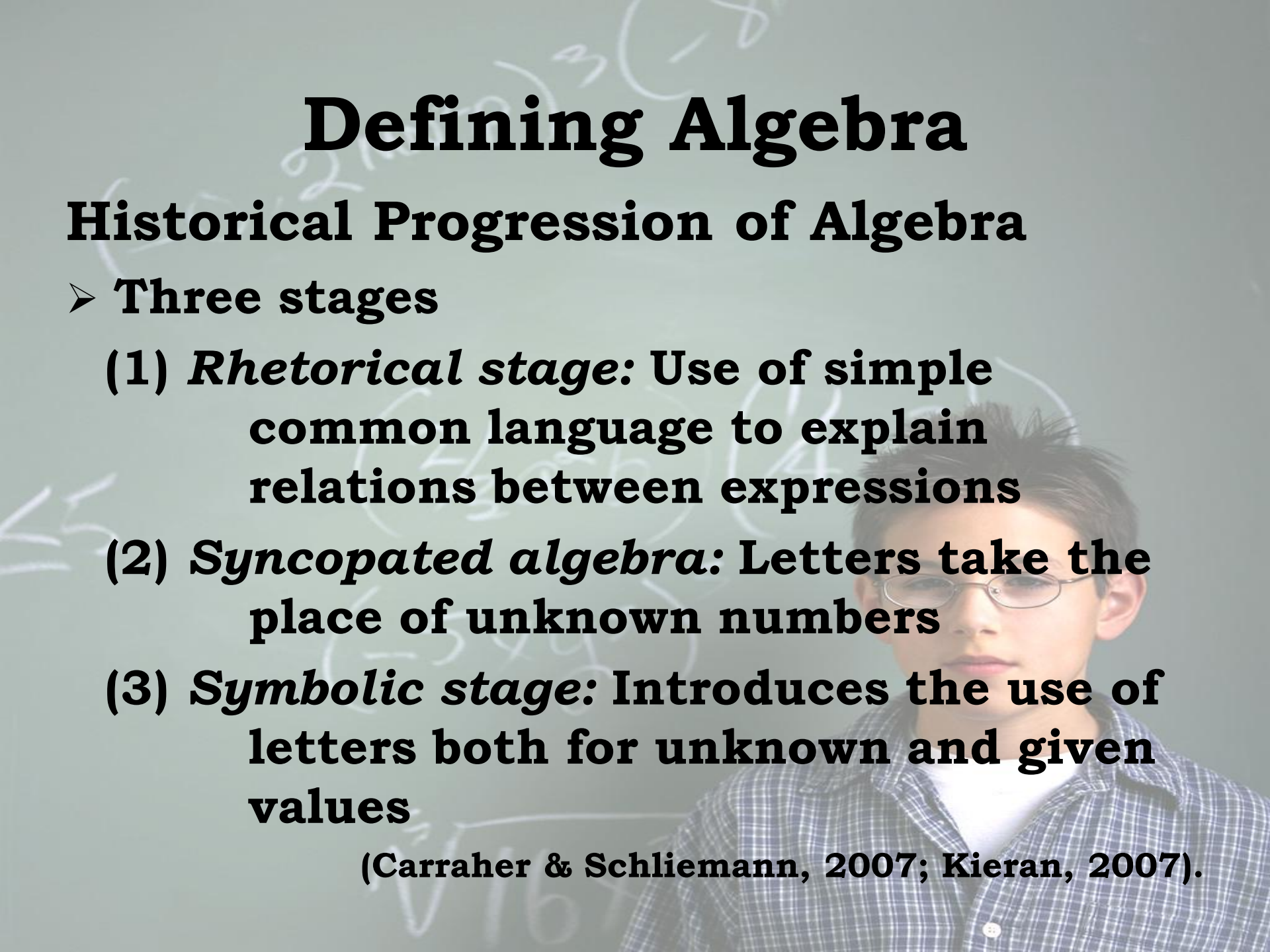
- **Three stages**

(1) *Rhetorical stage:* Use of simple common language to explain relations between expressions

(2) *Syncopated algebra:* Letters take the place of unknown numbers

(3) *Symbolic stage:* Introduces the use of letters both for unknown and given values

(Carraher & Schliemann, 2007; Kieran, 2007).

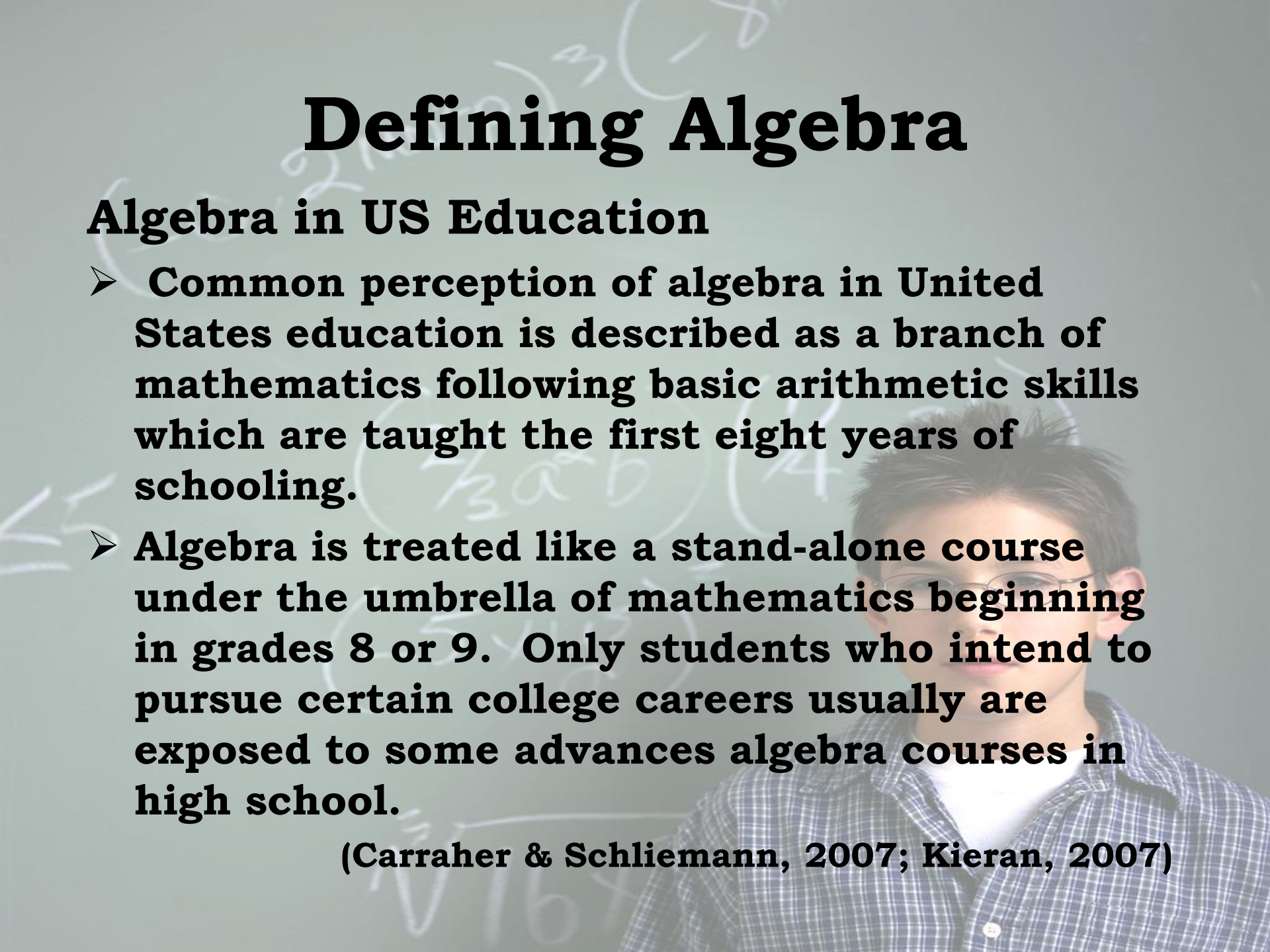


Defining Algebra

Algebra in US Education

- Common perception of algebra in United States education is described as a branch of mathematics following basic arithmetic skills which are taught the first eight years of schooling.
- Algebra is treated like a stand-alone course under the umbrella of mathematics beginning in grades 8 or 9. Only students who intend to pursue certain college careers usually are exposed to some advances algebra courses in high school.

(Carraher & Schliemann, 2007; Kieran, 2007)

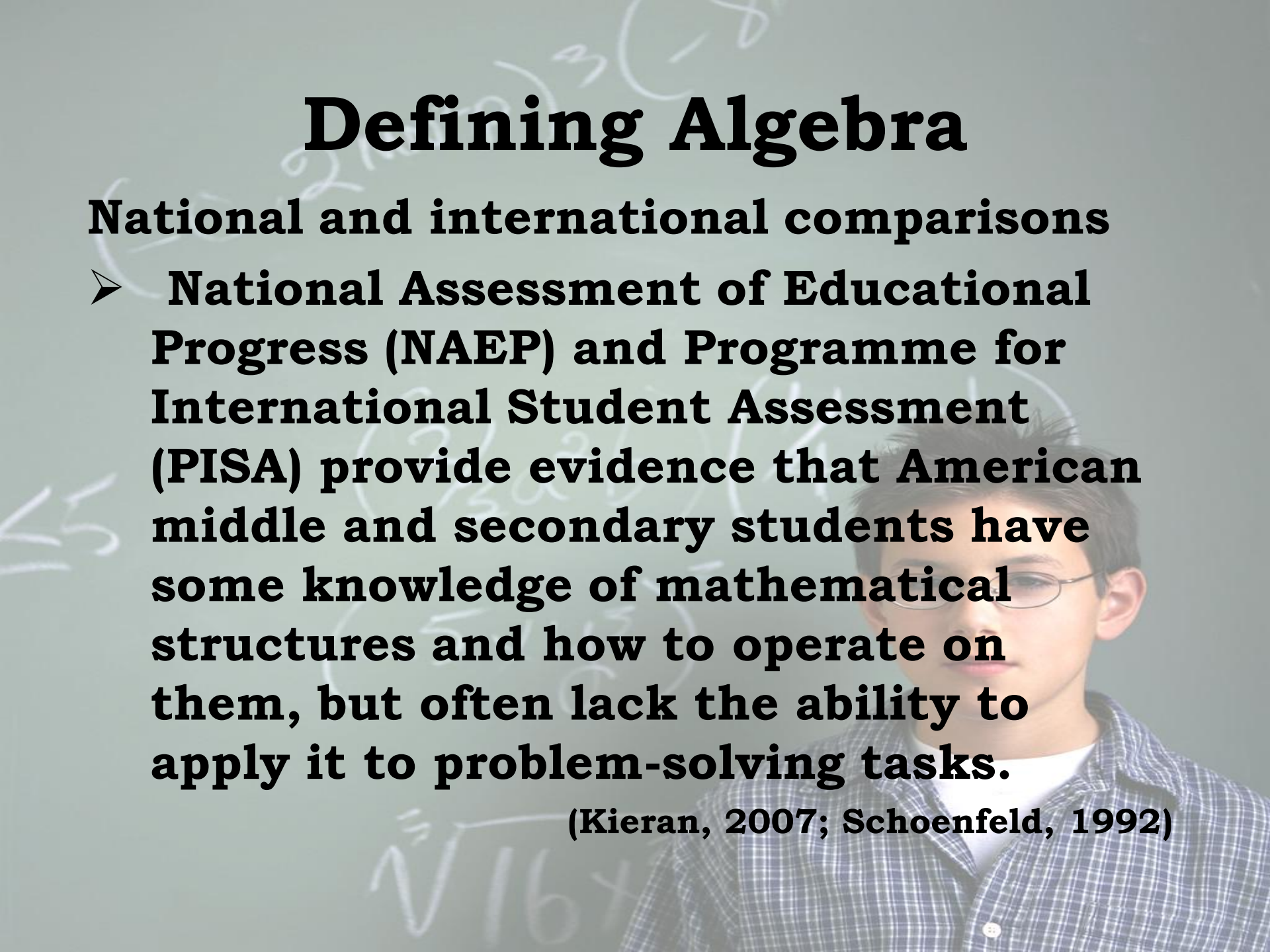


Defining Algebra

National and international comparisons

- **National Assessment of Educational Progress (NAEP) and Programme for International Student Assessment (PISA) provide evidence that American middle and secondary students have some knowledge of mathematical structures and how to operate on them, but often lack the ability to apply it to problem-solving tasks.**

(Kieran, 2007; Schoenfeld, 1992)



Defining Algebra

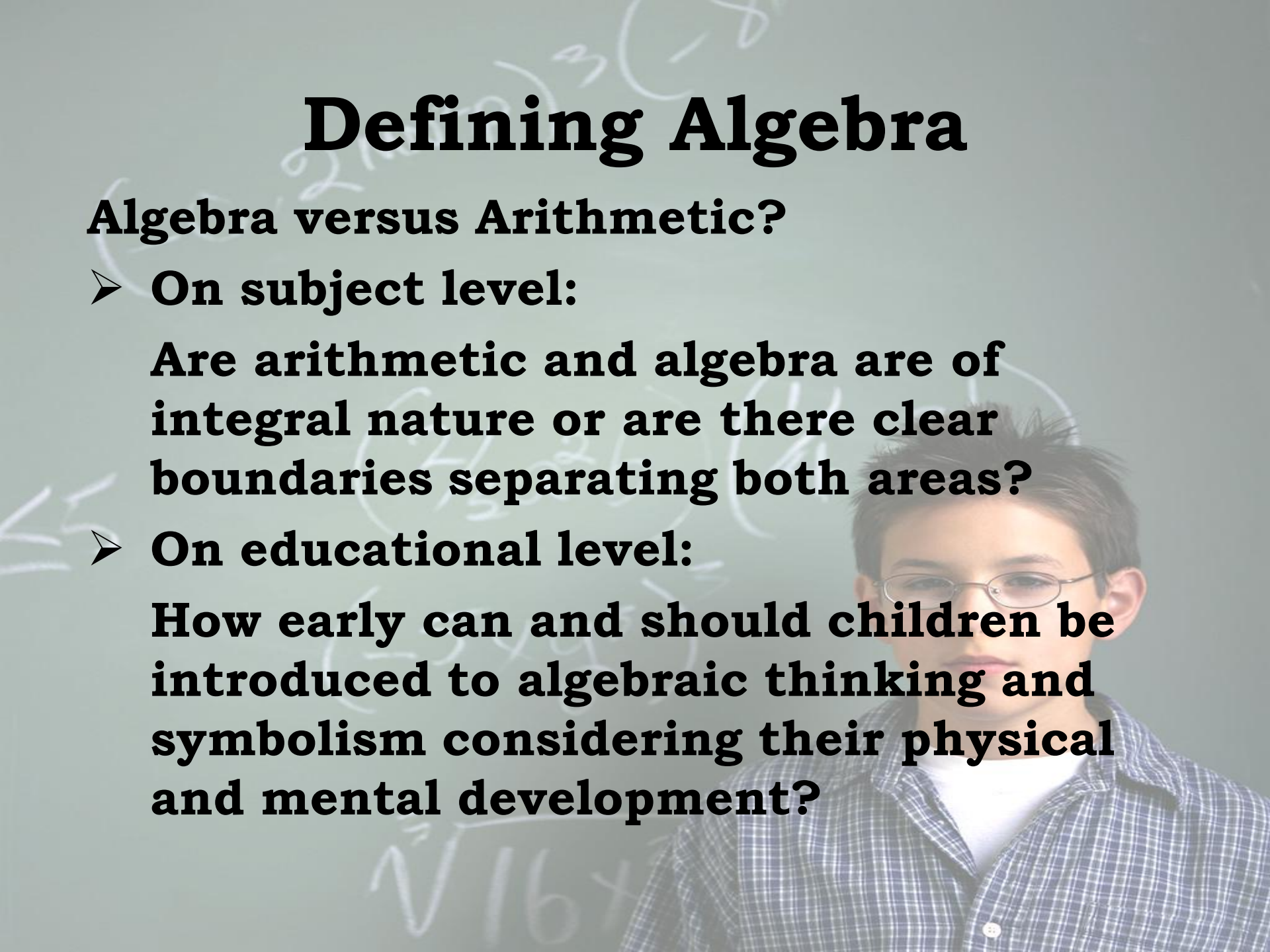
Algebra versus Arithmetic?

- **On subject level:**

  Are arithmetic and algebra are of integral nature or are there clear boundaries separating both areas?

- **On educational level:**

  How early can and should children be introduced to algebraic thinking and symbolism considering their physical and mental development?

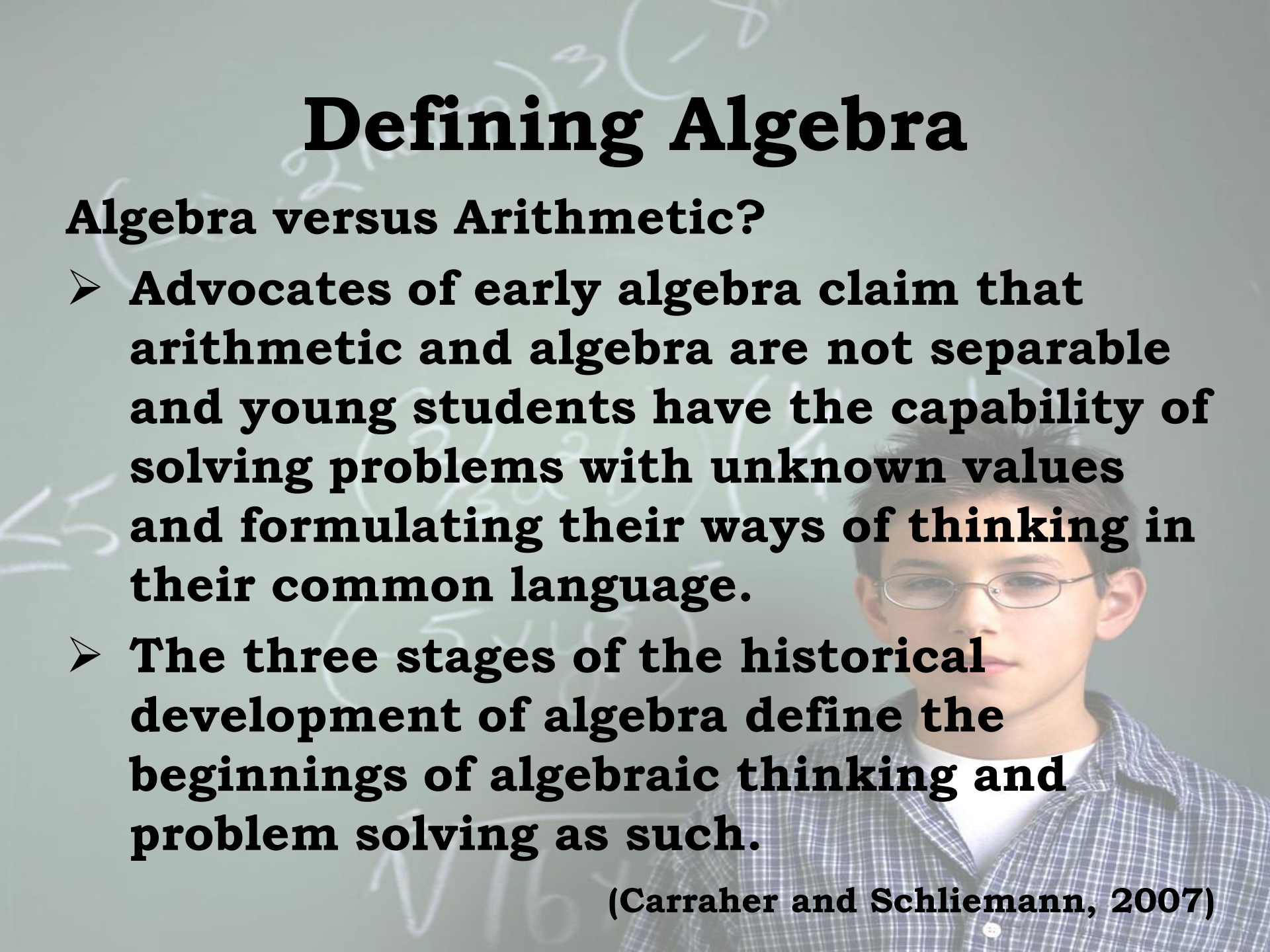


Defining Algebra

Algebra versus Arithmetic?

- **Advocates of early algebra claim that arithmetic and algebra are not separable and young students have the capability of solving problems with unknown values and formulating their ways of thinking in their common language.**
- **The three stages of the historical development of algebra define the beginnings of algebraic thinking and problem solving as such.**

(Carraher and Schliemann, 2007)



Defining Algebra

Organization of Early Algebra

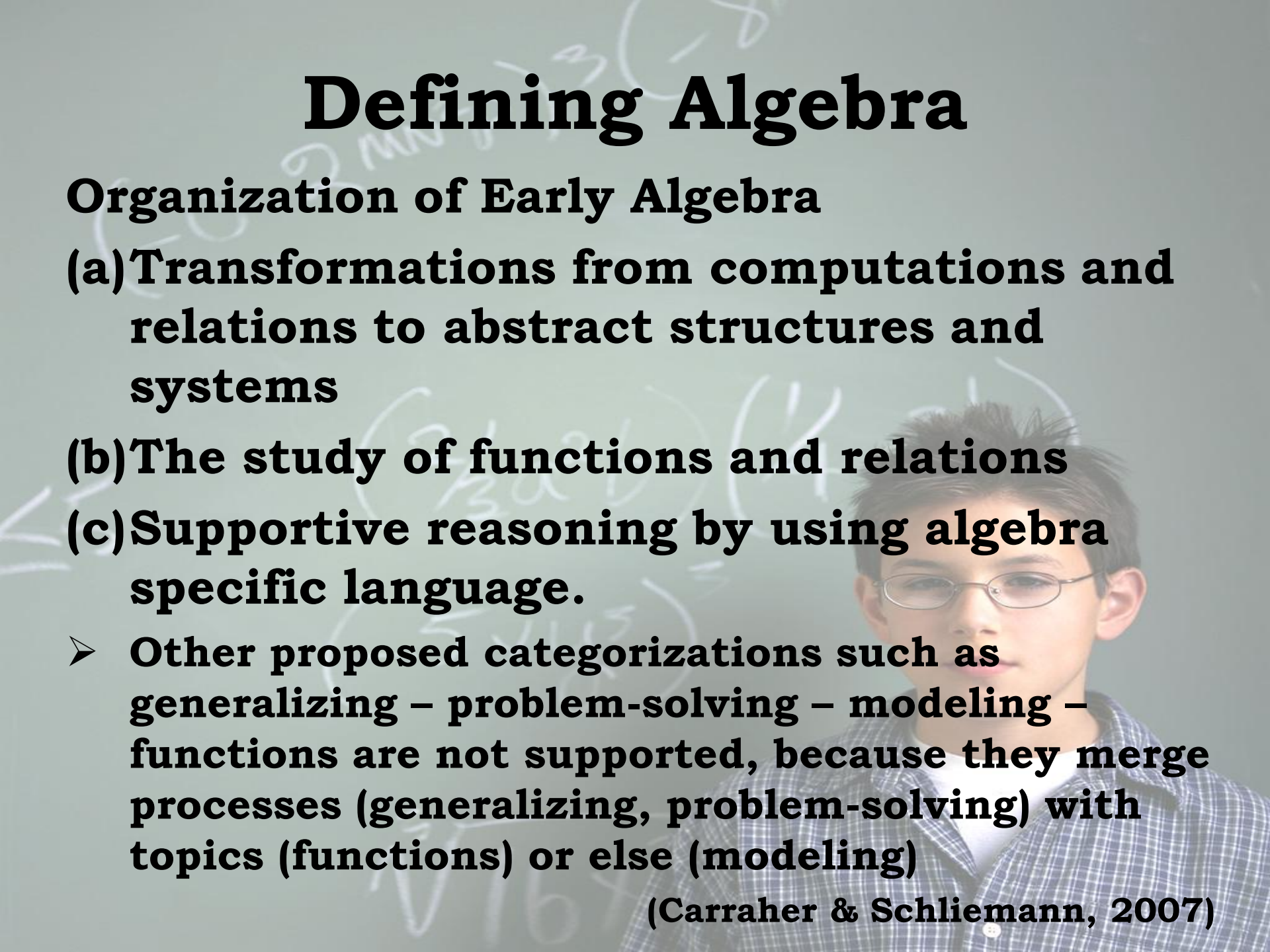
(a) Transformations from computations and relations to abstract structures and systems

(b) The study of functions and relations

(c) Supportive reasoning by using algebra specific language.

- **Other proposed categorizations such as generalizing – problem-solving – modeling – functions are not supported, because they merge processes (generalizing, problem-solving) with topics (functions) or else (modeling)**

(Carraher & Schliemann, 2007)



Defining Algebra

Algebra in the Common Core State Standards

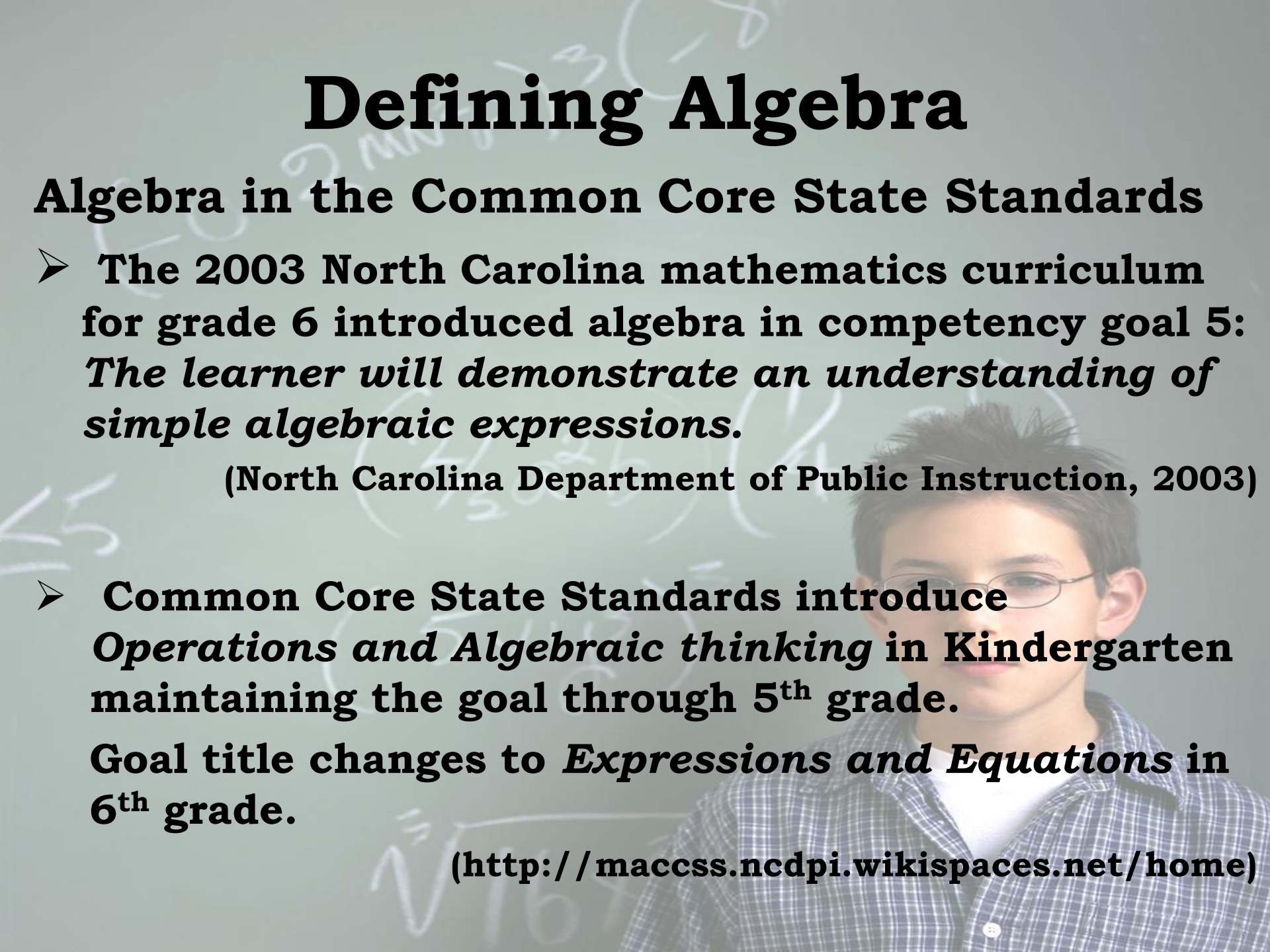
- **The 2003 North Carolina mathematics curriculum for grade 6 introduced algebra in competency goal 5: *The learner will demonstrate an understanding of simple algebraic expressions.***

  (North Carolina Department of Public Instruction, 2003)

- **Common Core State Standards introduce *Operations and Algebraic thinking* in Kindergarten maintaining the goal through 5th grade.**

  Goal title changes to *Expressions and Equations* in 6th grade.

  (http://maccss.ncdpi.wikispaces.net/home)



How Children Learn Mathematics

Social Formation of Mind

- **Vygotsky's (1896-1934) psychological and socio-cultural theory informs us that children are very much influenced in their learning processes by their social environment.**
- **Cognitive learning occurs from interpersonal experiences to forming intrapersonal concepts according to age and maturity of the child described as *zone of proximal development*.**

(Moll, 1990)

How Children Learn Mathematics

Five Aspects of Cognitive Learning

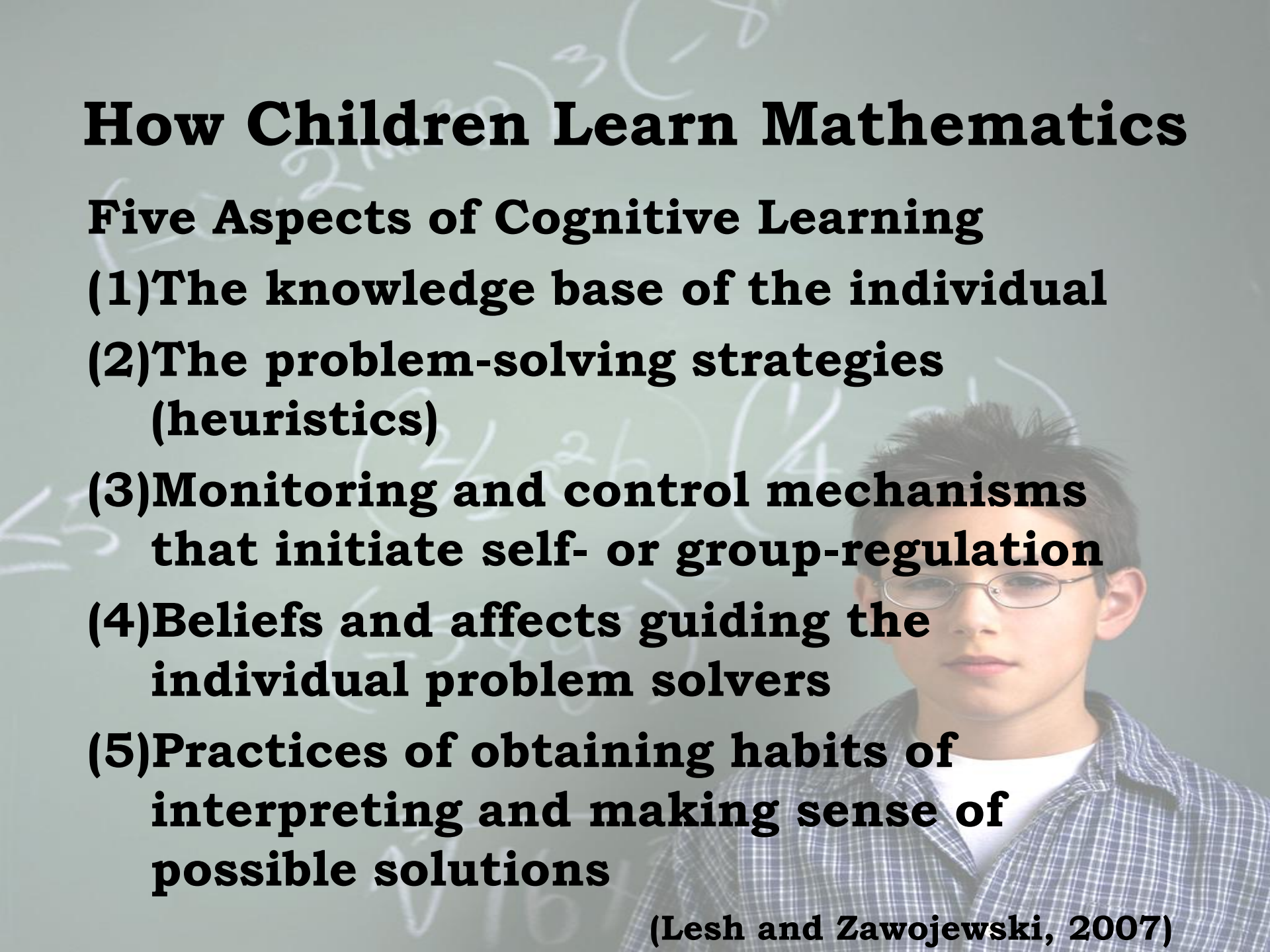
(1)The knowledge base of the individual

(2)The problem-solving strategies (heuristics)

(3)Monitoring and control mechanisms that initiate self- or group-regulation

(4)Beliefs and affects guiding the individual problem solvers

(5)Practices of obtaining habits of interpreting and making sense of possible solutions



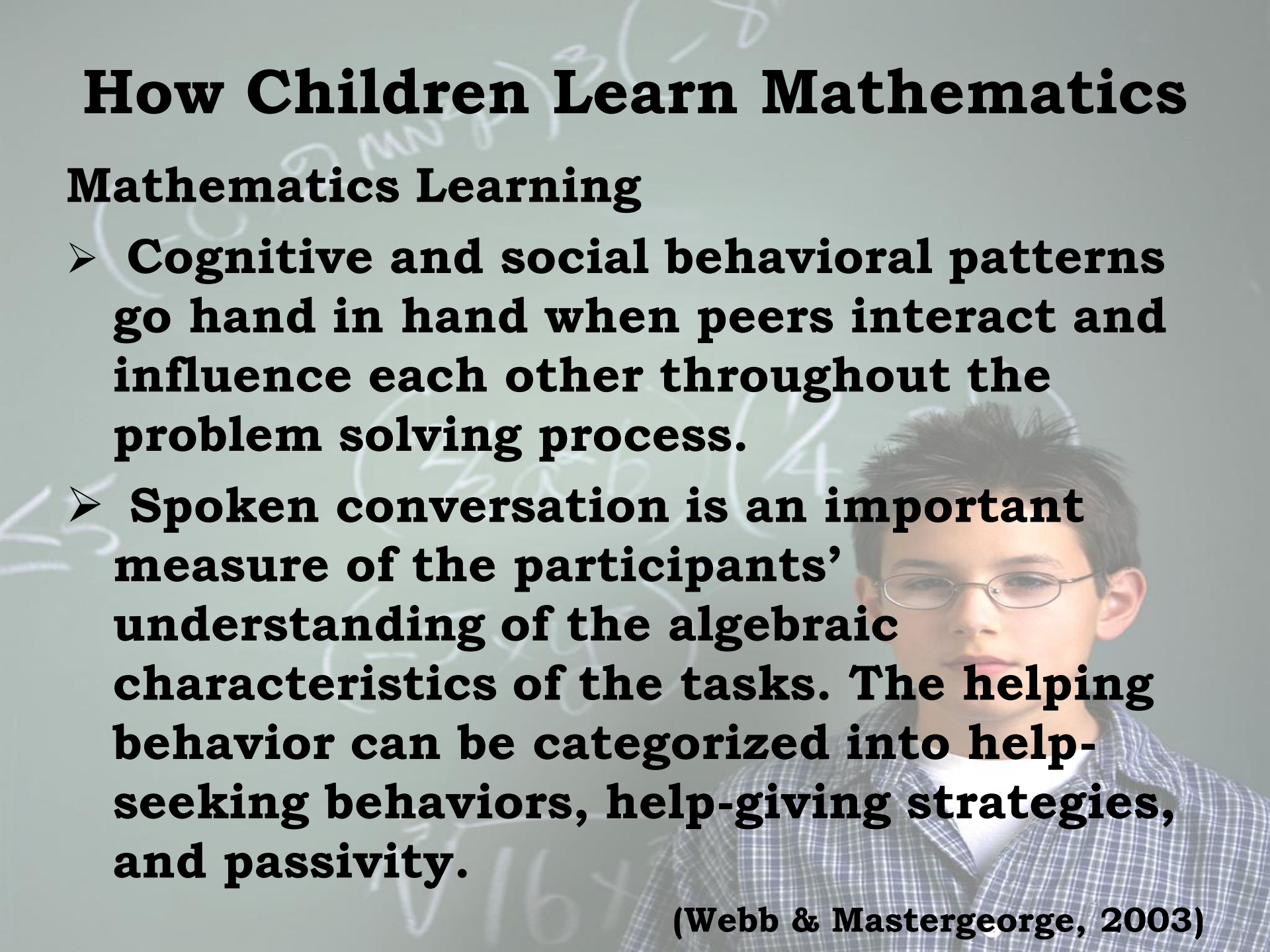
(Lesh and Zawojewski, 2007)

How Children Learn Mathematics

Mathematics Learning

- **Cognitive and social behavioral patterns go hand in hand when peers interact and influence each other throughout the problem solving process.**
- **Spoken conversation is an important measure of the participants' understanding of the algebraic characteristics of the tasks. The helping behavior can be categorized into help-seeking behaviors, help-giving strategies, and passivity.**

(Webb & Mastergeorge, 2003)



How Children Learn Mathematics

Mathematics Learning

- **Another interpretation model of interactive and communicative activities suggests the categories *key activities*, *mental activities*, and *regulating activities* which occur in several cycles throughout the problem-solving session**

(Dekker & Elshout-Mohr, 1998)

- **All aspects combined, metacognitive processes, personal beliefs and depositions, and learning of new problem-solving strategies, determine the "productive problem-solving persona" a student develops.**

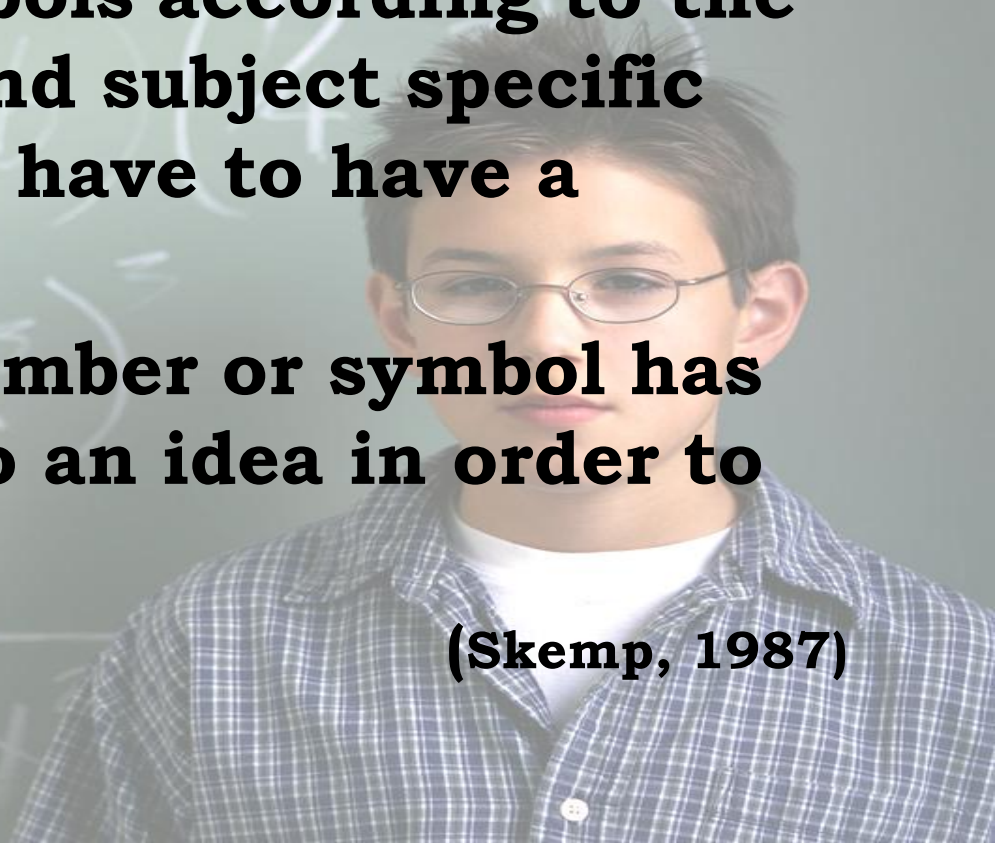
(Lesh and Zawojewski, 2007)

Children's Mathematics Communication

Numbers and Symbols

- **Before children are able to use numbers and symbols according to the historical social and subject specific conventions, they have to have a reference.**
- **Functions of a number or symbol has to be connected to an idea in order to receive meaning.**

(Skemp, 1987)

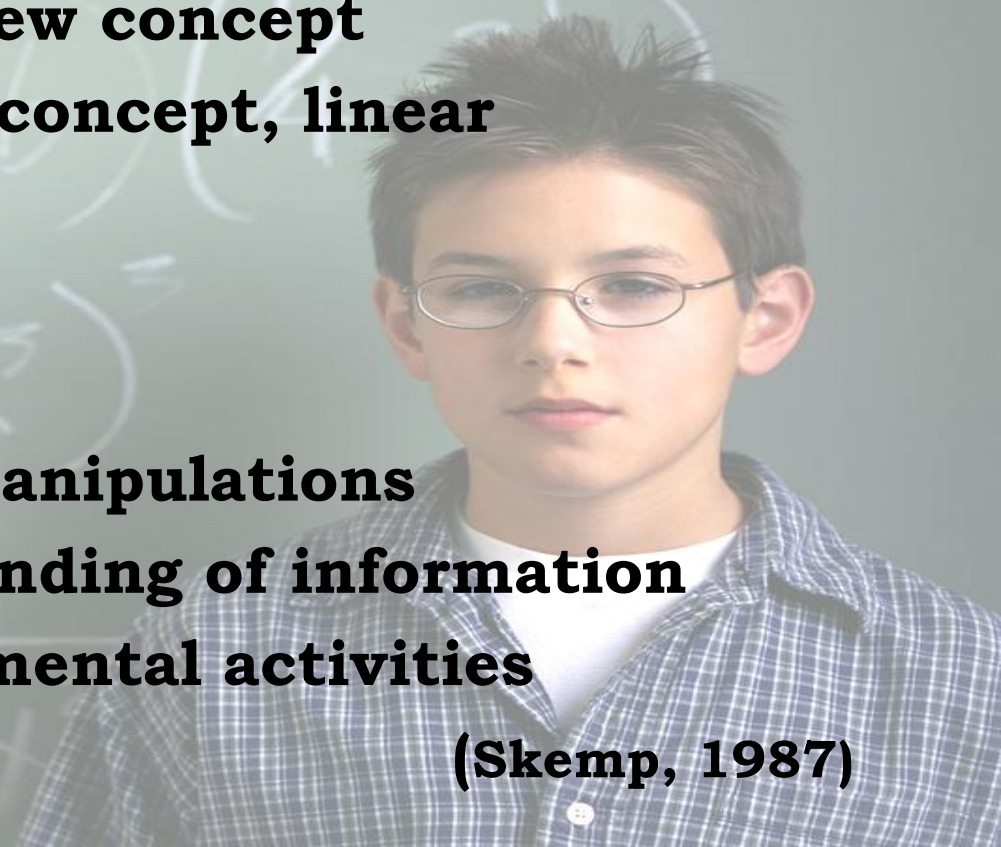


Children's Mathematics Communication

Functions of Symbols

(a) Communication
(b) Recording knowledge
(c) Communicating of new concept
(d) Classifying multiple concept, linear
(e) Explaining
(f) Reflecting activities
(g) Structuring
(h) Atomizing routine manipulations
(i) Recovering understanding of information
(j) Creating individual mental activities

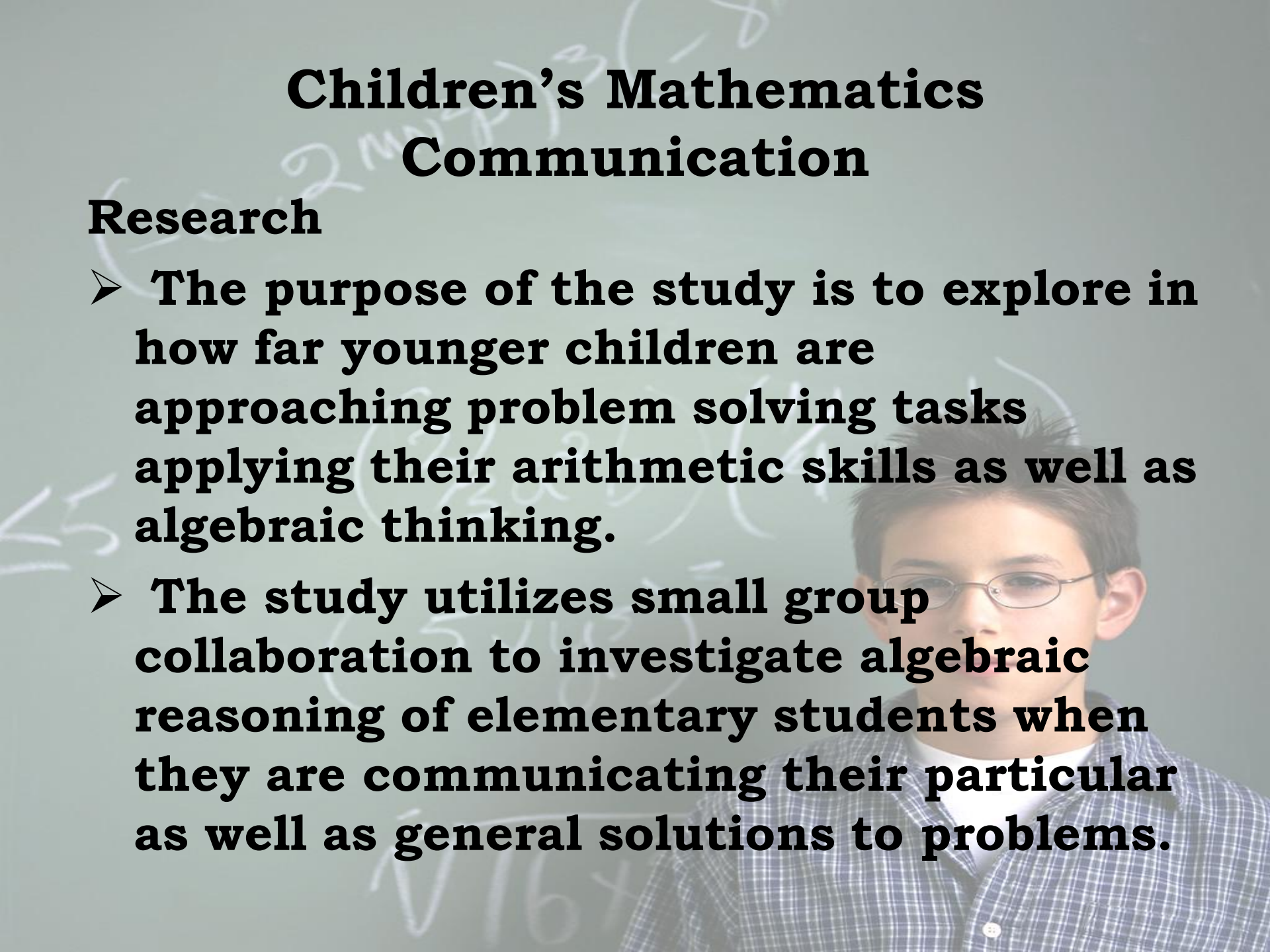
(Skemp, 1987)



Children's Mathematics Communication

Research

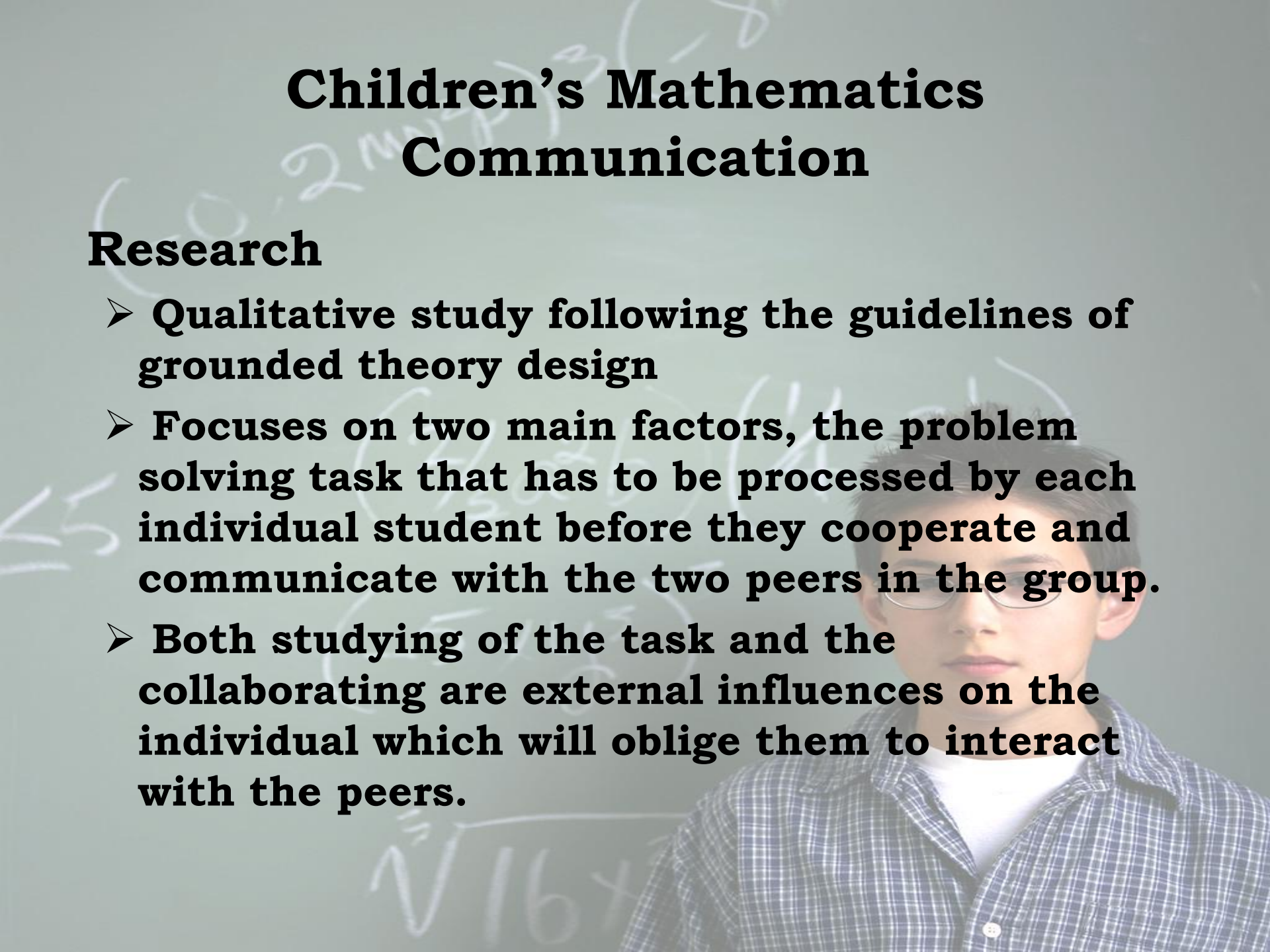
- **The purpose of the study is to explore in how far younger children are approaching problem solving tasks applying their arithmetic skills as well as algebraic thinking.**
- **The study utilizes small group collaboration to investigate algebraic reasoning of elementary students when they are communicating their particular as well as general solutions to problems.**



Children's Mathematics Communication

Research

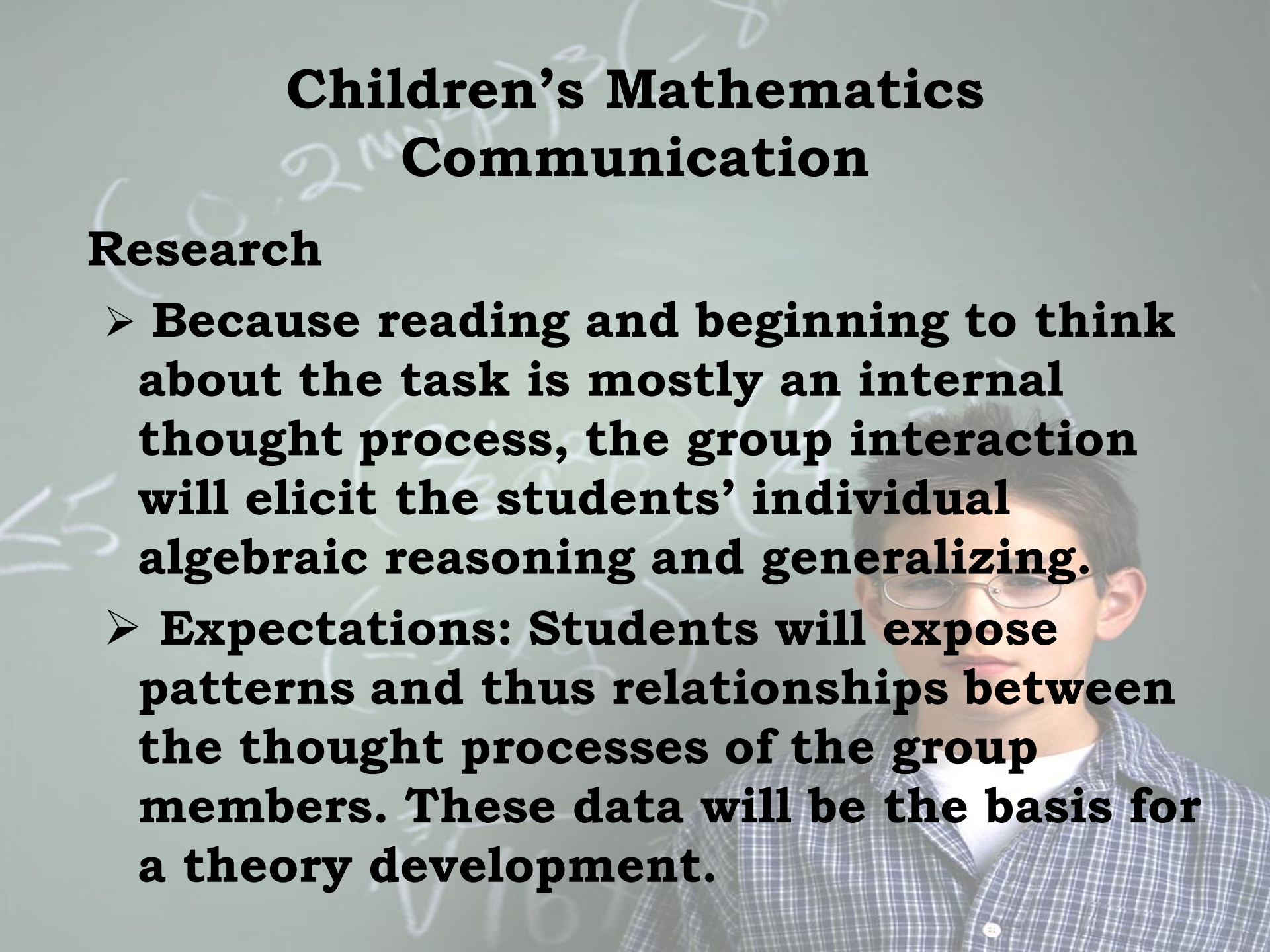
- Qualitative study following the guidelines of grounded theory design
- Focuses on two main factors, the problem solving task that has to be processed by each individual student before they cooperate and communicate with the two peers in the group.
- Both studying of the task and the collaborating are external influences on the individual which will oblige them to interact with the peers.



Children's Mathematics Communication

Research

- **Because reading and beginning to think about the task is mostly an internal thought process, the group interaction will elicit the students' individual algebraic reasoning and generalizing.**
- **Expectations: Students will expose patterns and thus relationships between the thought processes of the group members. These data will be the basis for a theory development.**

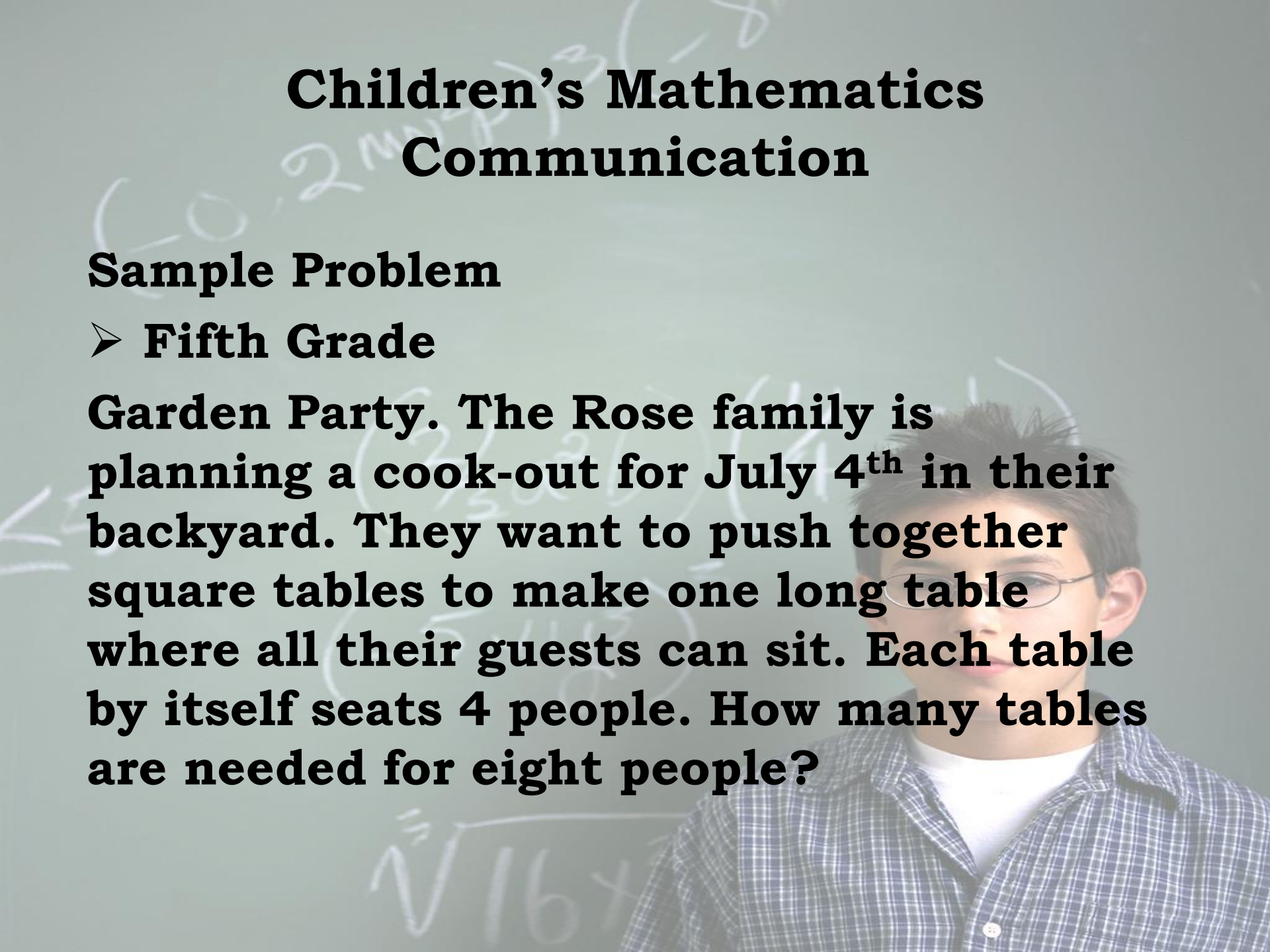


Children's Mathematics Communication

Sample Problem

- **Fifth Grade**

Garden Party. The Rose family is planning a cook-out for July 4th in their backyard. They want to push together square tables to make one long table where all their guests can sit. Each table by itself seats 4 people. How many tables are needed for eight people?



Children's Mathematics Communication

Findings: Excerpts from Video Transcript

N and K together: You can have 4 people on each side.

K: Yeah.

D: Yeah, yeah. This is … an easier way to think about this is, you always have to remember that when you move them together, you always will have 7 on the outside…

N: …and those on the inside, in the middle, will have 6.

K: Yeah.

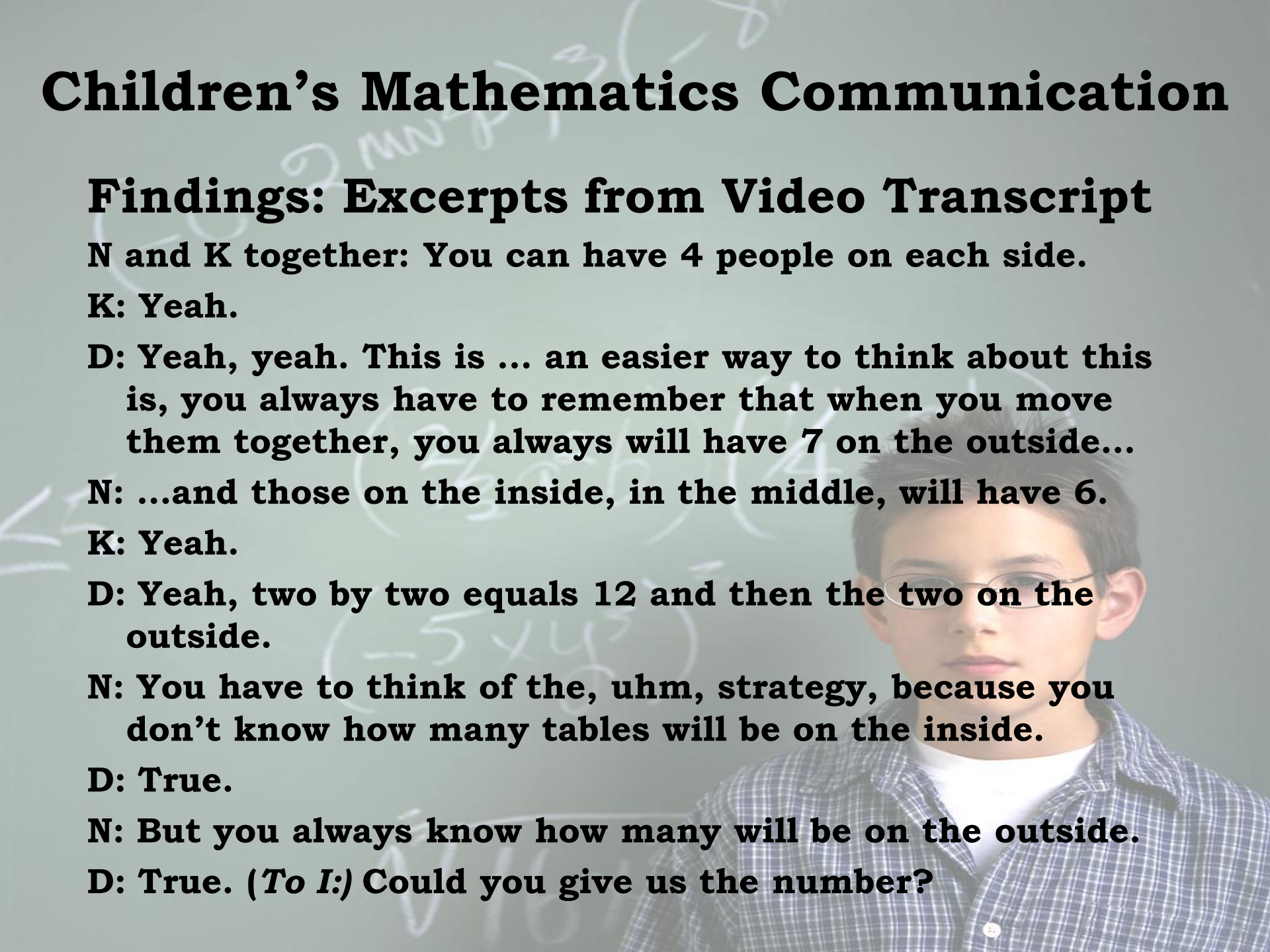
D: Yeah, two by two equals 12 and then the two on the outside.

N: You have to think of the, uhm, strategy, because you don't know how many tables will be on the inside.

D: True.

N: But you always know how many will be on the outside.

D: True. (*To I:*) Could you give us the number?



Children's Mathematics Communication

Findings: Excerpts from Video Transcript

I: Hm?

D: Could you give us the number?

I: I don't know the number either…

N: (*laughs*)

D: Oh. Hm, just a number?

N: How about …

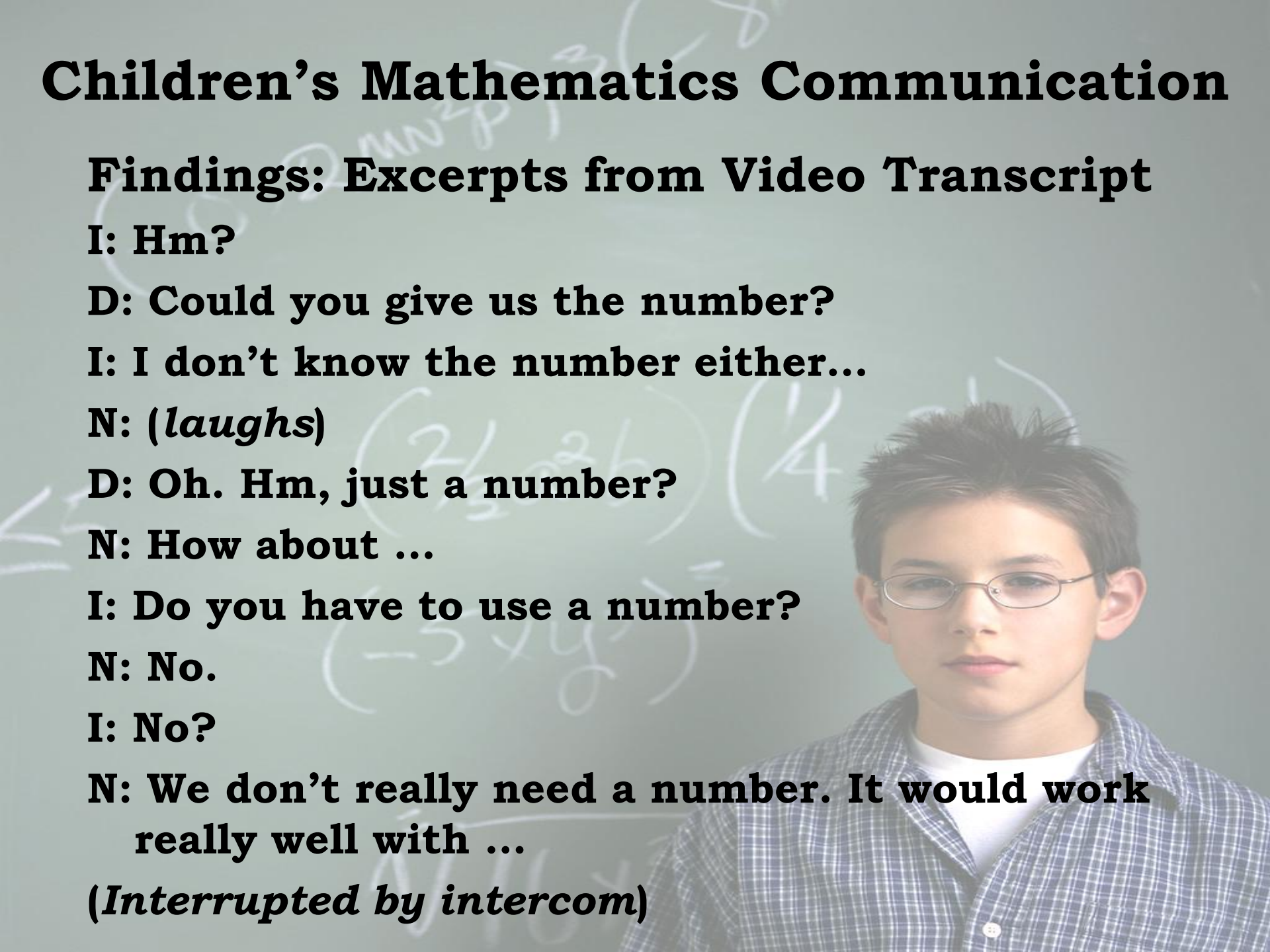
I: Do you have to use a number?

N: No.

I: No?

N: We don't really need a number. It would work really well with …

(*Interrupted by intercom*)



Children's Mathematics Communication

Findings: Excerpts from Video Transcript

D: So it would really work with the other one.

N: Because you could just add the eight together and take away …

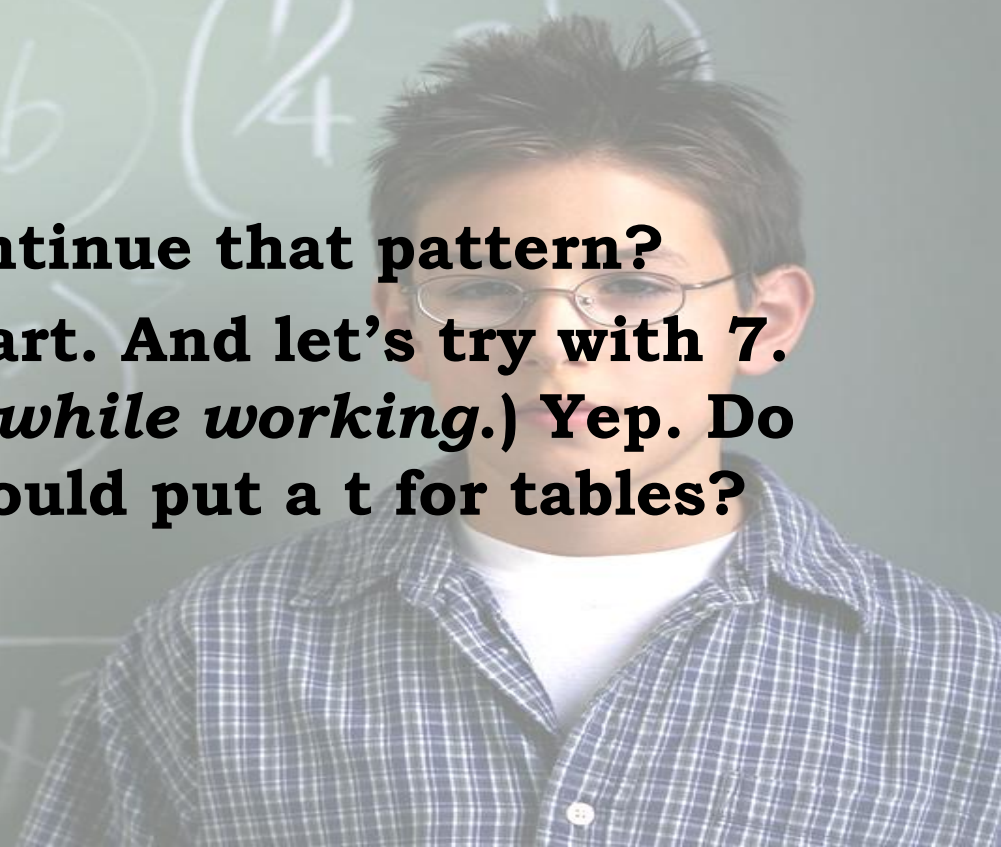
D: …two.

N: … two.

I: Okay. And can you continue that pattern?

D: Okay, let's try the chart. And let's try with 7. (*Whispers to himself while working.*) Yep. Do you guys think we should put a t for tables? Just a t?

K: Yeah.

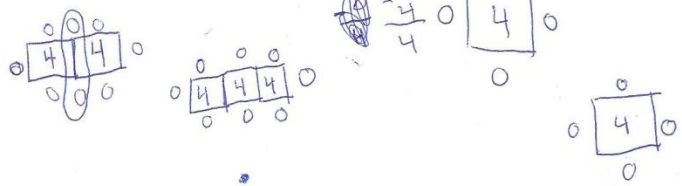


Work Sample

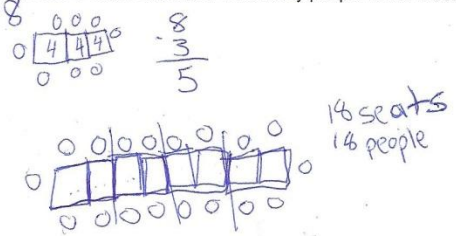
Garden Party

The Rose family is planning a garden party in their beautiful large backyard. They want to use folding tables to build a long table where all guests can sit. Each table has the shape of a square and seats 4 people.

1. How many tables do the Rose's need for 8 people?



2. They only have 8 tables available. How many people at the most can attend the party?



3. Mr. and Mrs. Rose decide to start their own party planning business. Can you come up with a rule that will help them to quickly figure out how many tables are needed for all the different numbers of guests their customers will have for their parties? (They still want to use the square tables for 4 persons.)

Guest ÷ Seats → How many tables

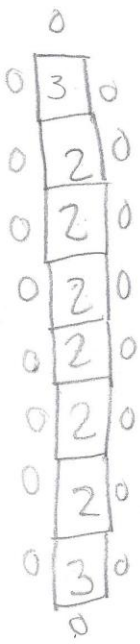
Slide the number of tables they need together
Ask how many people that are going to the party

People 16 / Tables 7

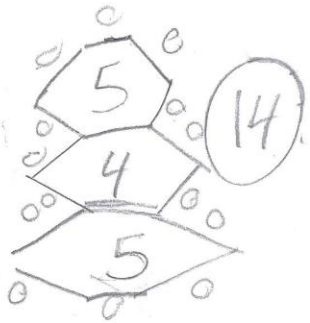
16 ÷ 7 = 2 R.2 (02 R.2)

16 ÷ 4 = 4

number of tables ÷ sides on the table = how many tables



the # of guest ÷ two minus (1 or 3) (odd or even)



Kids: 4)23 R.3 (05 R.3) -0↓ 23 -20 3

Adults: 2)23 R.1 -2↓ 03 -2 1

23 ÷ 2 - 3 = 11 - 1 = 9

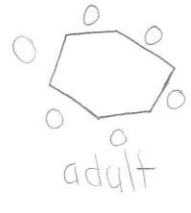
Work Sample

$(5 \times 2) + 8 = 18$

10

Extensions:

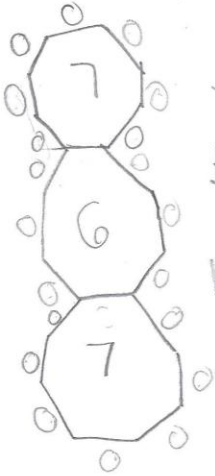
4. What would change, if the Roses wanted to use hexagonal tables instead?



adult

kids

5. What would change with octagonal tables?



7 + 6 = 13

13 + 7 = 20

3 octagonal tables seat 20 people

When you put 3 tables together the table or tables will be missing 2 seats and the tables on the outside have 5 seats

The number of guest divided by four minus two plus four

$(23 \div 4 - 2 + 4)$

I would seat more people because a hexagon has more sides, 5 people could be seated at each end.

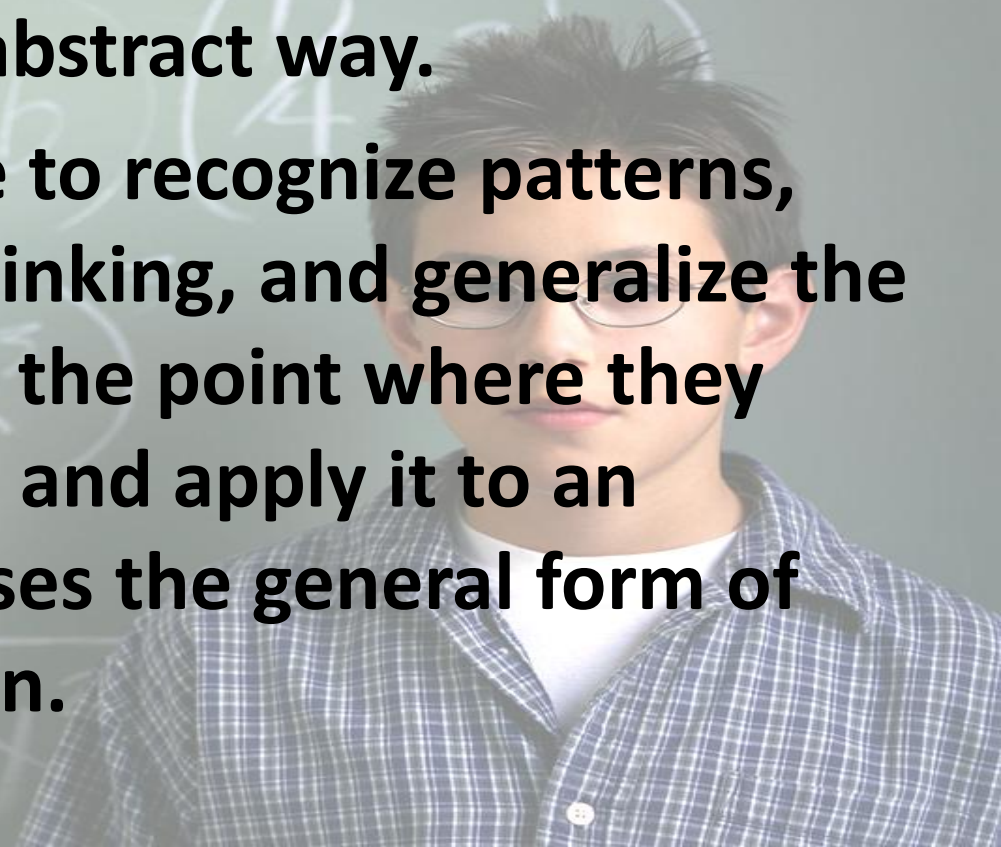
Number of Tables	Number of guest
1	8
2	14
3	20
4	26
5	32
6	38
7	44
8	50
9	56
10	62

if you have so many tables and add on a table that table on the outside will change to a six and it will go on and on and on.

$2t \times 6 + 2 = 14$

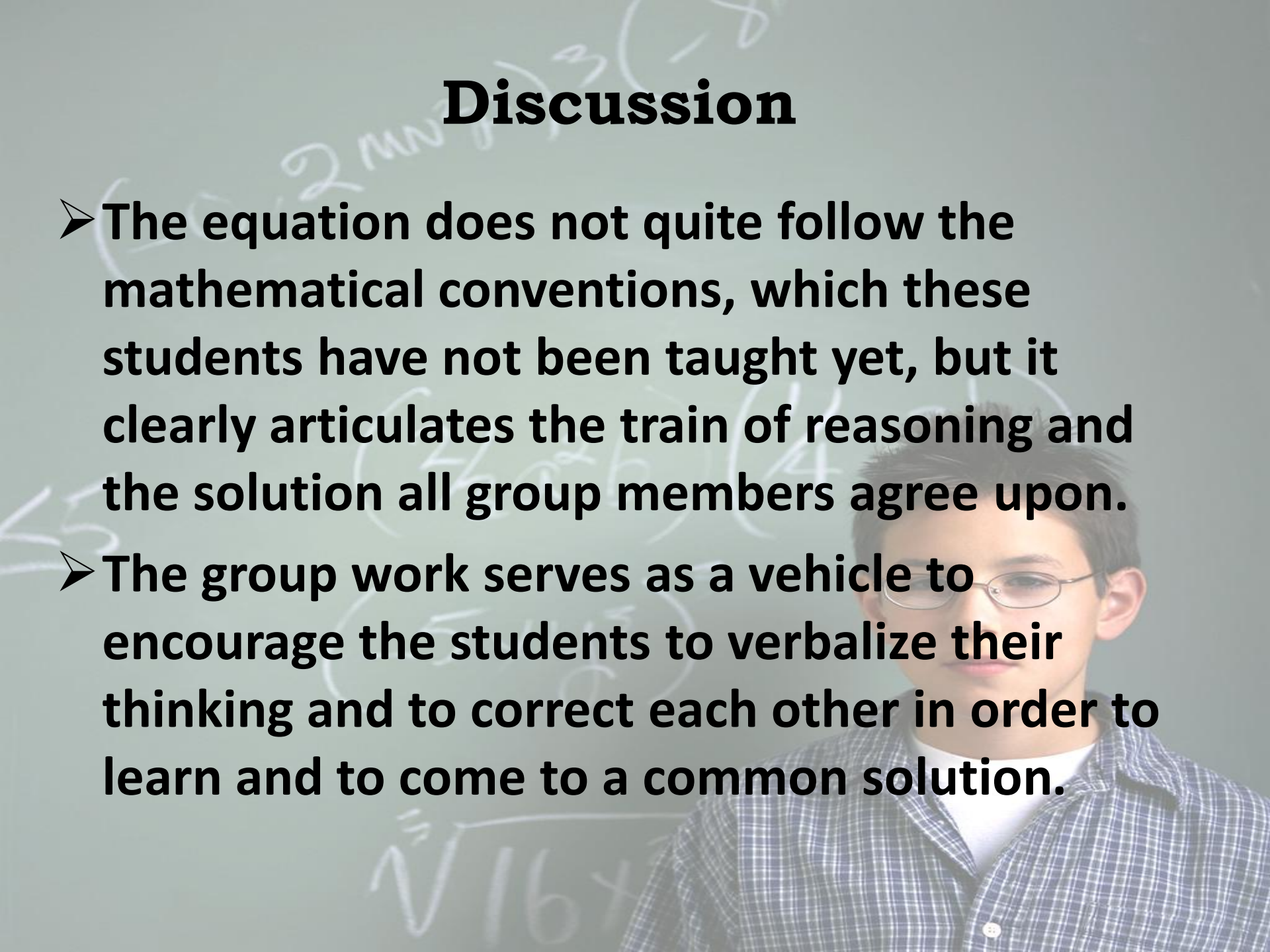
Discussion

- The findings of this pilot study reveal evidence that students younger than 12 years are conceivably capable to develop algebraic reasoning in a more abstract way.
- The students are able to recognize patterns, reason about their thinking, and generalize the solutions. They reach the point where they implement a variable and apply it to an equation that expresses the general form of their problem solution.



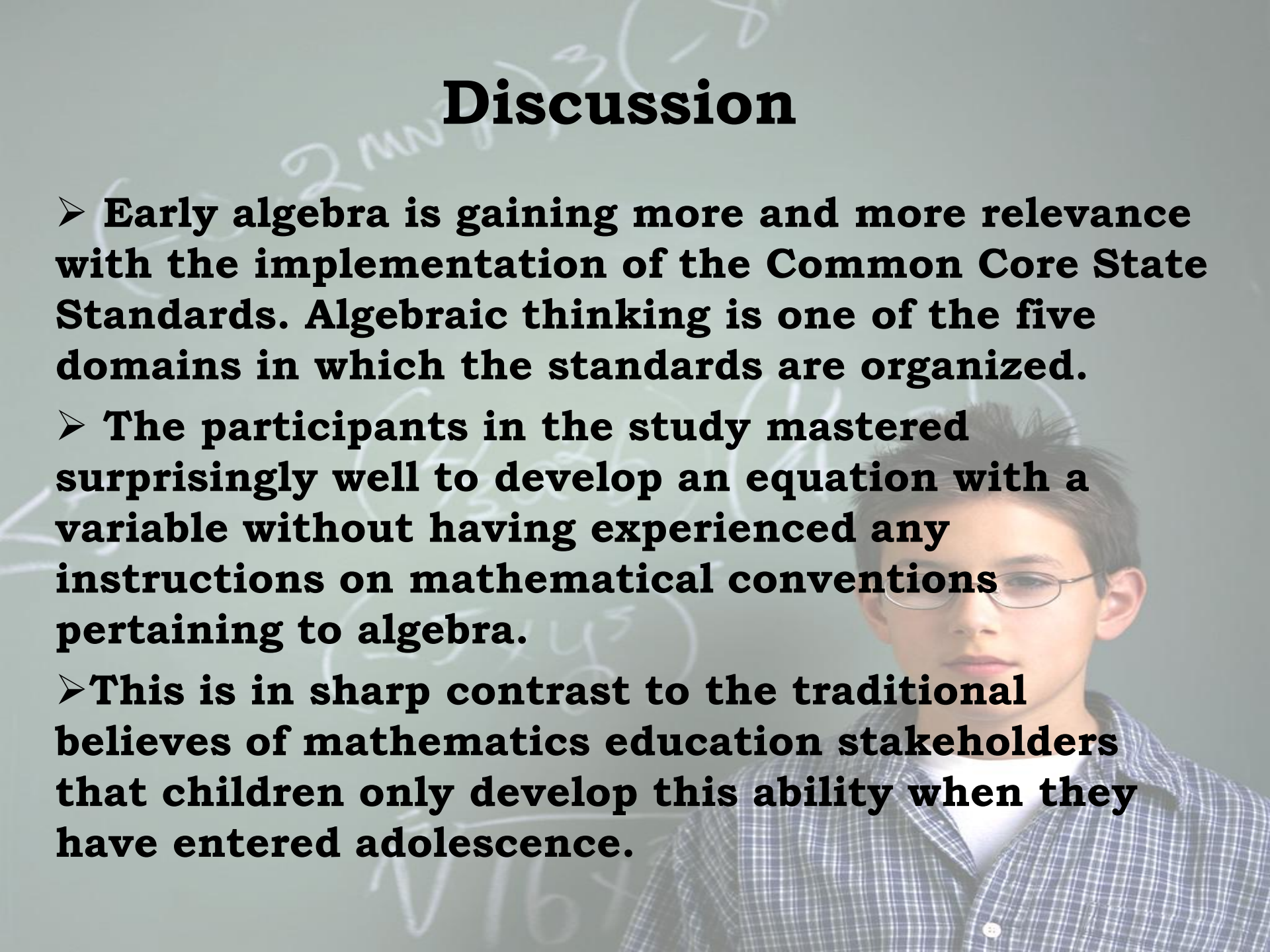
Discussion

- The equation does not quite follow the mathematical conventions, which these students have not been taught yet, but it clearly articulates the train of reasoning and the solution all group members agree upon.
- The group work serves as a vehicle to encourage the students to verbalize their thinking and to correct each other in order to learn and to come to a common solution.



Discussion

- **Early algebra is gaining more and more relevance with the implementation of the Common Core State Standards. Algebraic thinking is one of the five domains in which the standards are organized.**
- **The participants in the study mastered surprisingly well to develop an equation with a variable without having experienced any instructions on mathematical conventions pertaining to algebra.**
- **This is in sharp contrast to the traditional believes of mathematics education stakeholders that children only develop this ability when they have entered adolescence.**



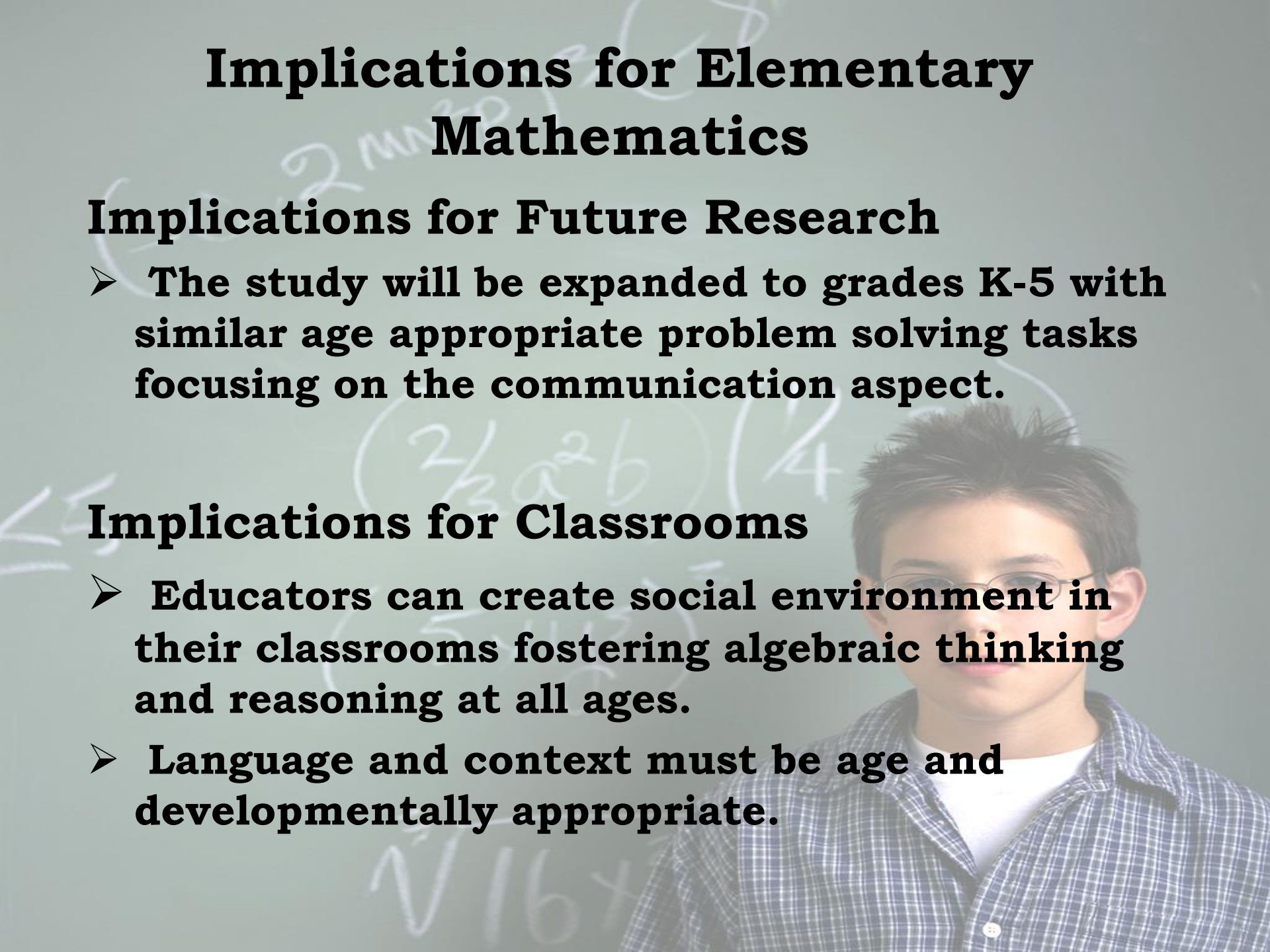
Implications for Elementary Mathematics

Implications for Future Research

- The study will be expanded to grades K-5 with similar age appropriate problem solving tasks focusing on the communication aspect.

Implications for Classrooms

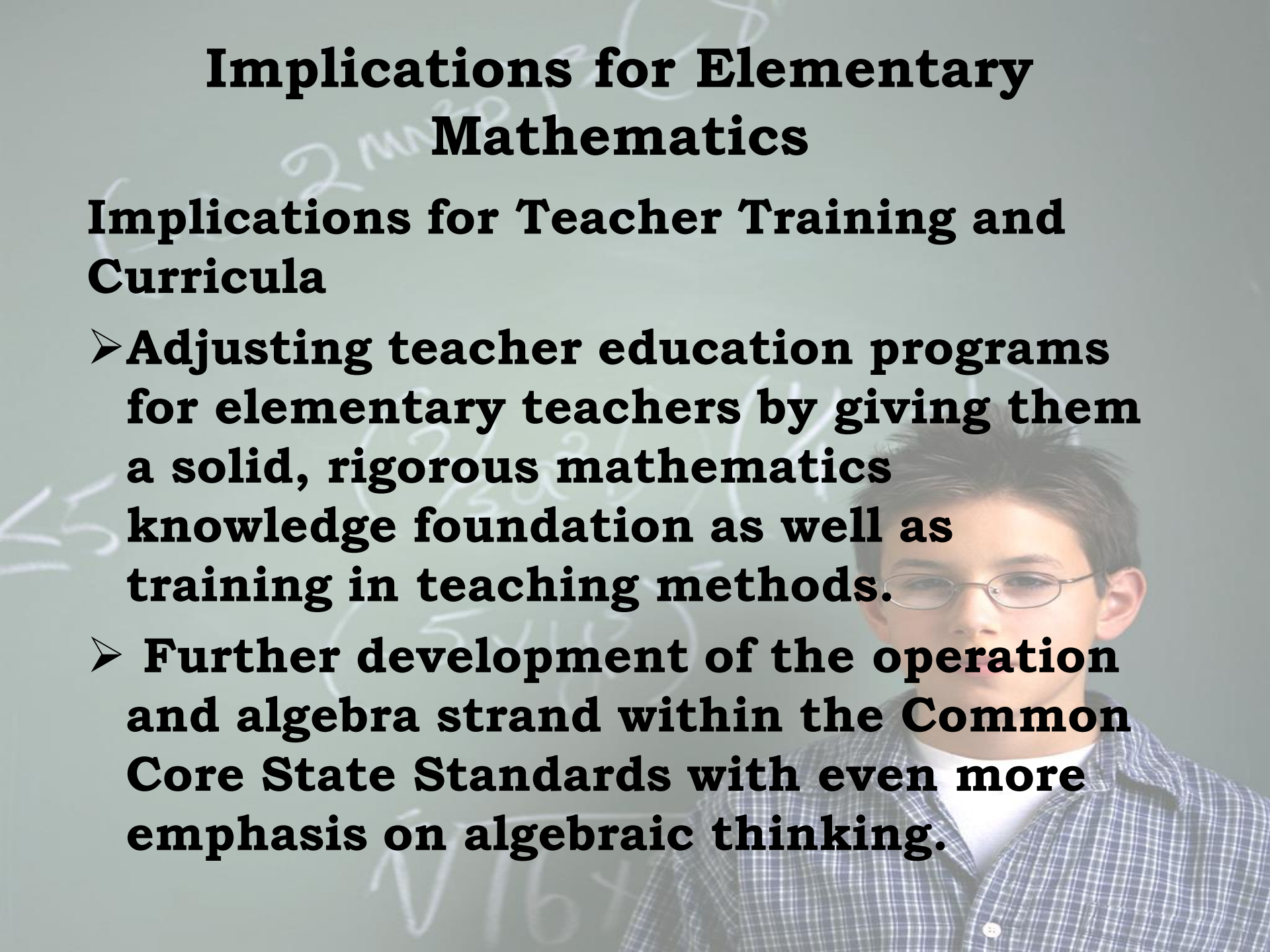
- Educators can create social environment in their classrooms fostering algebraic thinking and reasoning at all ages.
- Language and context must be age and developmentally appropriate.



Implications for Elementary Mathematics

Implications for Teacher Training and Curricula

- Adjusting teacher education programs for elementary teachers by giving them a solid, rigorous mathematics knowledge foundation as well as training in teaching methods.
- Further development of the operation and algebra strand within the Common Core State Standards with even more emphasis on algebraic thinking.



Implications for Elementary Mathematics

Implications for Teacher Training and Curricula

- If the level of mathematics learning could be raised during early school years, middle school, high school, and college mathematics had a better foundation and could excel further than these programs are currently able to do.

Thank you

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