# Using Real-World Settings to Foster Mathematical Learning

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# What?

- A framework to help you think about how to use problems set in real-world contexts to **develop** understanding of mathematics concepts.

(...as opposed to giving problems that ask students to **apply** what they’ve already learned.)

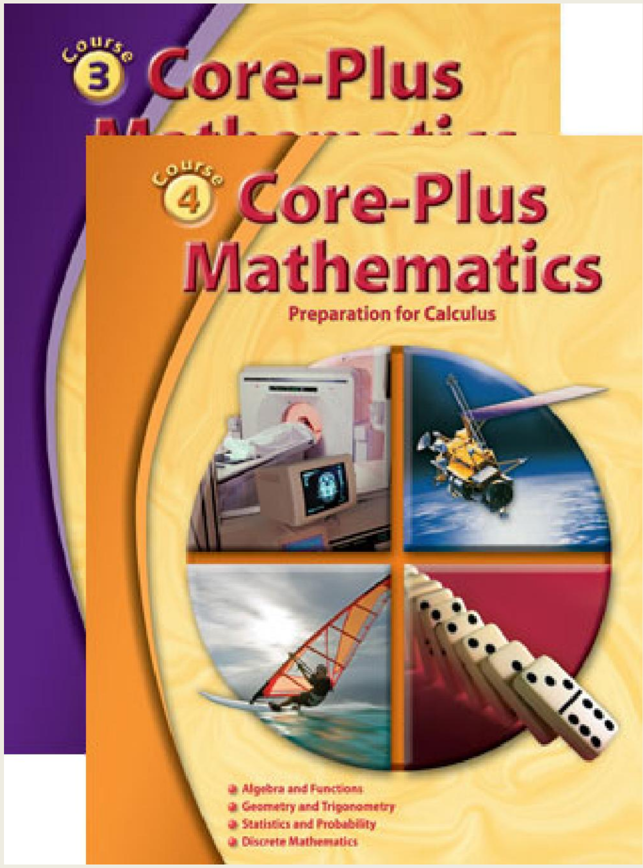
# Agenda

- Introductions
- Instructional example
- Supplementing the written curriculum
- A framework for thinking about using real-world contexts in math class
- Using the framework to redesign adaptations
- A possible planning sequence to design this type of lesson

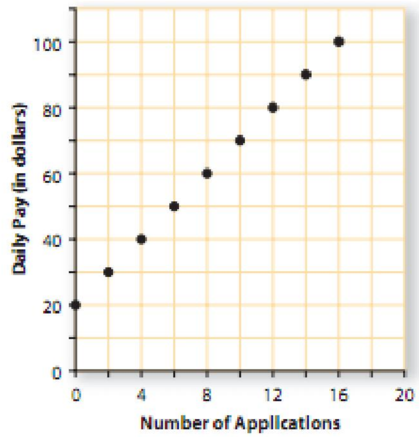
# Introductions

- Who we are
- How we met
- Our partnership

# Core Plus Mathematics Project



Pay for Soliciting Credit Card Customers




## Think About This Situation

**Think about the connections among graphs, data patterns, function rules, and problem conditions for linear relationships.**

**a** How does Barry's daily pay change as the number of applications he collects increases? How is that pattern of change shown in the graph?

**b** If the linear pattern shown by the graph holds for other (*number of applications, daily pay*) pairs, how much would you expect Barry to earn for a day during which he collects just 1 application? For a day he collects 13 applications? For a day he collects 25 applications?

**c** What information from the graph might you use to write a rule showing how to calculate daily pay for any number of applications?

**Selling Credit Cards** Companies that offer credit cards pay the people who collect applications for those cards and the people who contact current cardholders to sell them additional financial services.

1. For collecting credit card applications, Barry's daily pay $B$ is related to the number of applications he collects $n$ by the rule $B = 20 + 5n$.

   a. Use the function rule to complete this table of sample $(n, B)$ values:

| Number of Applications | 0 | 1 | 2 | 3 | 4 | 5 | 10 | 20 | 50 |
|---|---|---|---|---|---|---|---|---|---|
| Daily Pay (in dollars) | | | | | | | | | |

   b. Compare the pattern of change shown in your table with that shown in the graph on the preceding page.

   c. How much will Barry earn on a day when he does not collect any credit card applications? How can this information be seen in the rule $B = 20 + 5n$? In the table of sample $(n, B)$ values? In the graph on the preceding page?

   d. How much additional money does Barry earn for each application he collects? How can this information be seen in the rule $B = 20 + 5n$? In the table? In the graph?

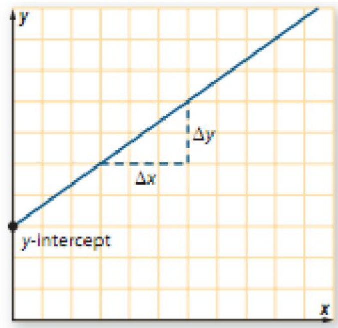
   e. Use the words *NOW* and *NEXT* to write a rule showing how Barry's daily pay changes with each new credit card application he collects.

**Linear Functions Without Contexts** When studying linear functions, it helps to think about real contexts. However, the connections among graphs, tables, and symbolic rules are the same for linear functions relating *any* two variables.

You've probably noticed by now that the rate of change of a linear function is constant and that the rate of change corresponds to the direction and steepness of the graph, or the *slope* of the graph.

You can determine the **rate of change** of $y$ as $x$ increases, or the **slope** of the graph between two points, using the ratio:

$$\frac{\text{change in } y}{\text{change in } x} \text{ or } \frac{\Delta y}{\Delta x}.$$




($\Delta$ is the Greek letter "delta," which is used to represent "difference" or "change.")

Another key feature of a linear function is the **y-intercept** of its graph, the point where the graph intersects the $y$-axis.

**6** Draw a graph for each function on a separate set of coordinate axes.

**a.** $y = 1 + \frac{2}{3}x$ **b.** $y = 2x$

**c.** $y = 2x - 3$ **d.** $y = 2 - \frac{1}{2}x$

Then analyze each function rule and its graph as described below.

**i.** Label the coordinates of three points $A$, $B$, and $C$ on each graph. Calculate the slopes of the segments between points $A$ and $B$, between points $B$ and $C$, and between points $A$ and $C$.

**ii.** Label the coordinates of the $y$-intercept on each graph.

**iii.** Explain how the numbers in the symbolic rule relate to the graph.

# Analyzing classroom data

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A key feature of any function is the way the value of the dependent variable changes as the value of the independent variable changes. Notice that as the number of services Cheri sells increases from 30 to 40, her pay increases from \$100 to \$120. This is an increase of \$20 in pay for an increase of 10 in the number of services sold, or an average of \$2 per sale. Her pay increases at a *rate* of \$2 per service sold.

**c.** Using your table from Part b, study the *rate of change* in Cheri's daily pay as the number of services she sells increases by completing entries in a table like the one below.

| **Change in Sales** | **Change in Pay** (in \$) | **Rate of Change** (in \$ per sale) |
|---|---|---|
| 10 to 20 | | |
| 20 to 25 | | |
| 25 to 40 | | |
| 50 to 100 | | |

# Analyzing classroom data

- Students had trouble translating from the rule to the table.
- Students had trouble calculating the “rate of change”.
- Students had a great deal of difficulty when they were no longer working in context.

**6** Draw a graph for each function on a separate set of coordinate axes.

**a.** $y = 1 + \frac{2}{3}x$ **b.** $y = 2x$

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# Interview data

- “I think that they're not... making the connection between looking at the rule now and identifying the slope and the y intercept as clearly as I thought they did last week when it was in context.”
- “.. I feel like I'm going back and I'm having to grab and pull those pieces together “

# Interventions

# Addressing literacy demands

Directions: Please have partner 1 read the following passage aloud while partner 2 monitors and corrects. Once you have finished, complete the magnet summary in the box below.

When studying linear functions, it helps to think about real contexts. However, the **connections among graph, tables, and symbolic** rules are the same for linear functions relating any two variables.

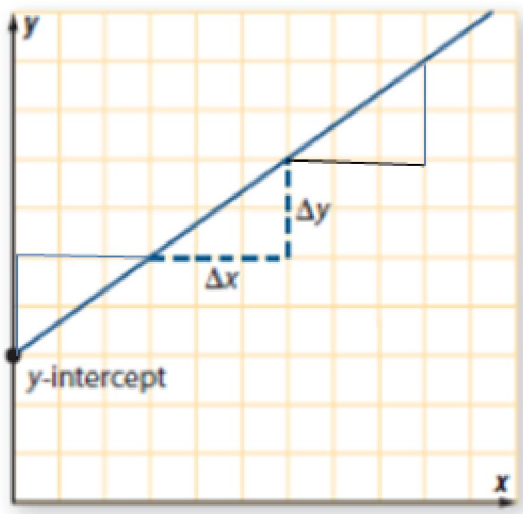
You've probably noticed by now that the **rate of change of a linear function is constant** and **that the rate of change corresponds to the direction and steepness** of the graph or the slope of the graph.

You can determine the **rate of change of y as x increases** or the **slope** of the graph between two points using the ratio:

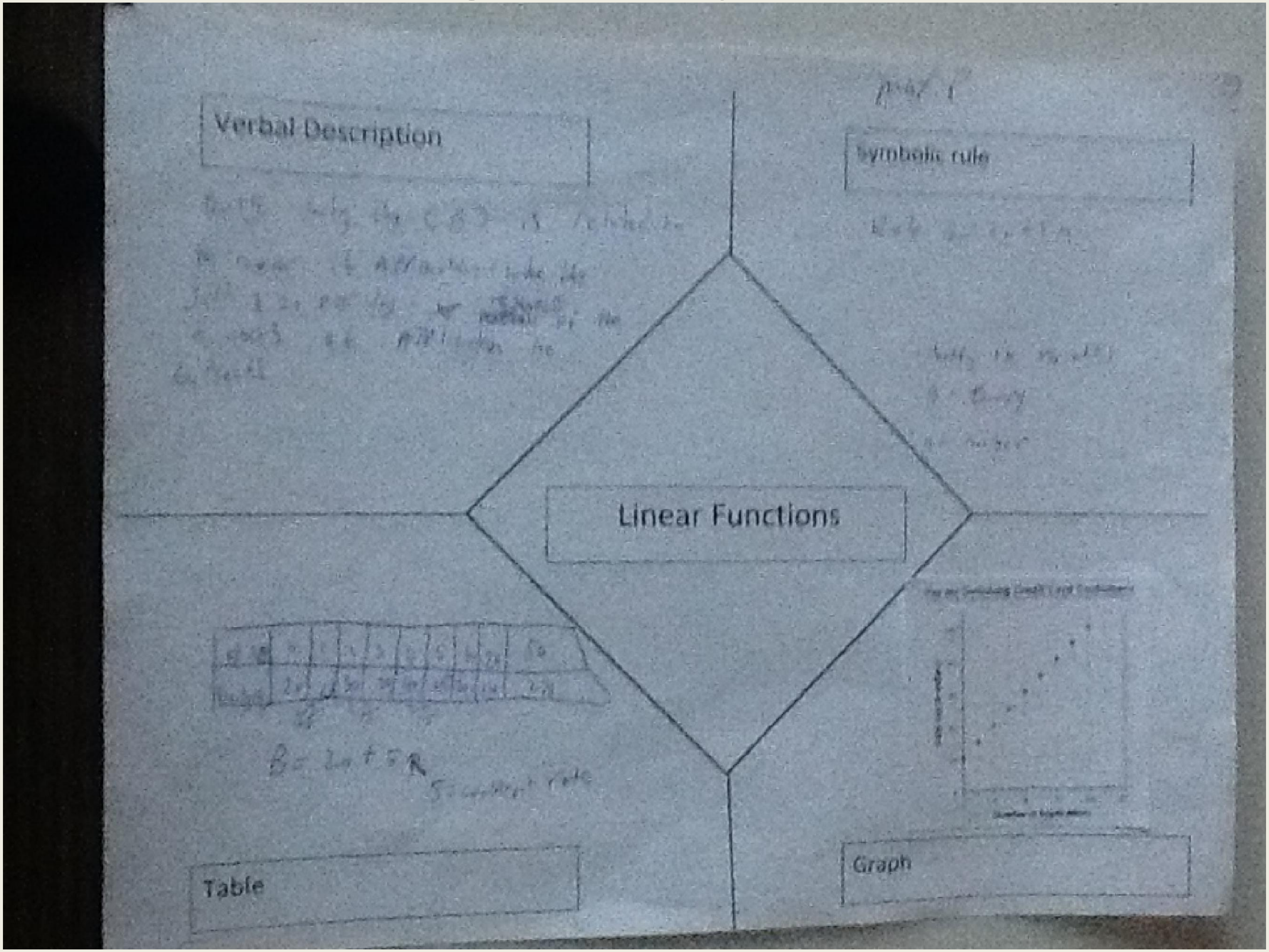
$$\frac{change\ in\ y}{change\ in\ x} = \frac{\Delta y}{\Delta x}$$

$\Delta$ is the Greek letter "**delta**" which is used to represent "**difference**" or "**change**".

The change in y ($\Delta y$) and the change in x ($\Delta x$) is shown on the graph below. Study the graph and talk with your partner about ways you can find the change in y and the change in x, by examining the graph. Put your thoughts in the space next to the graph.

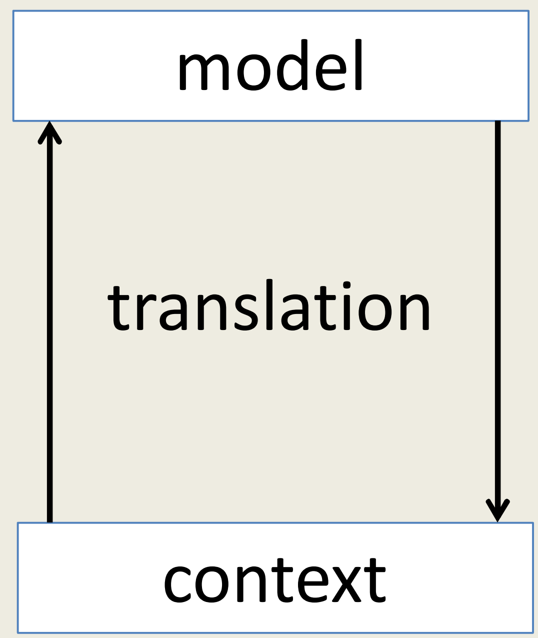


# Connecting the Representations

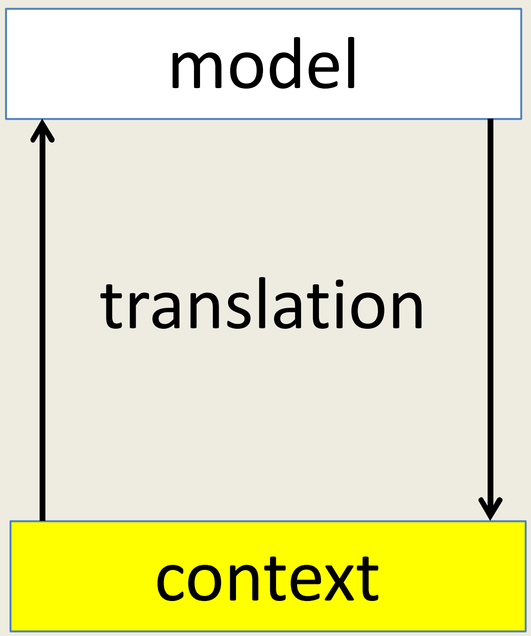


# A framework for categorizing questions and tasks

# Working with contextualized examples



# Questions focused on context



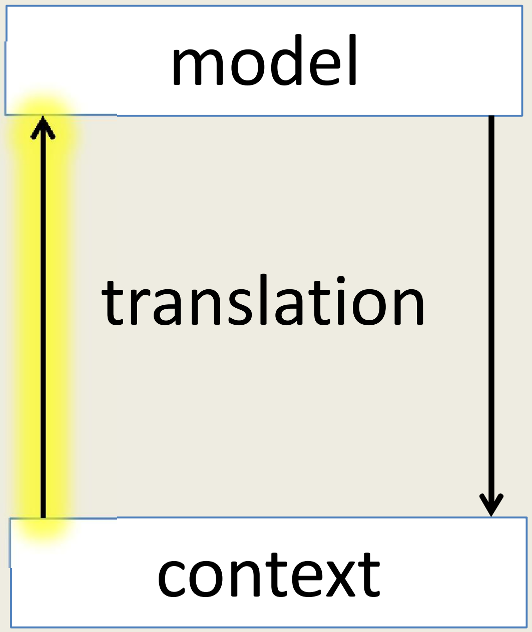

Does anyone know anyone who works in “sales”? How is their salary determined?

Have you ever been out somewhere and had someone ask you to sign up for a credit card and offered you a free gift?

Do you think they get paid a fixed hourly wage or do you think they make more money if they collect more applications?

Jackson, K. J., Shahan, E. C., Gibbons, L. K., & Cobb, P. A. (2012). Launching Complex Tasks. *Mathematics Teaching in the Middle School*, *18*(1), 24–29.

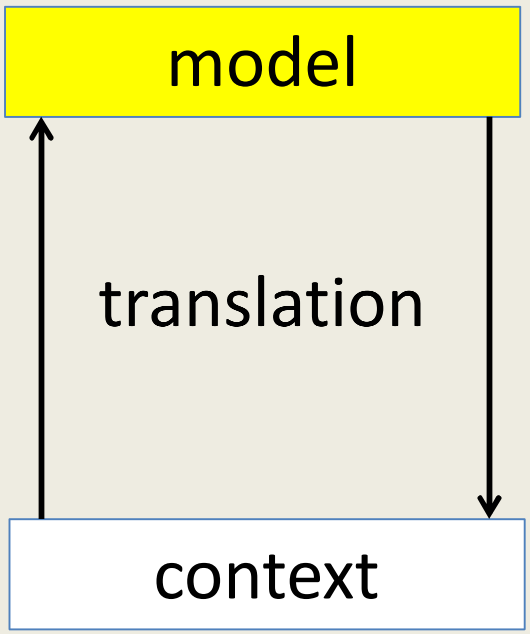
# Producing a model




You earn $8 an hour working for a restaurant.

Write a rule for how much money you would make after working for h hours.

# Questions focused on mathematical models

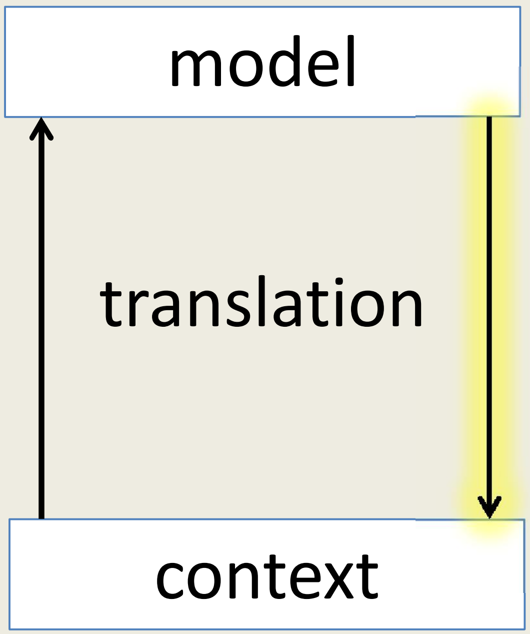


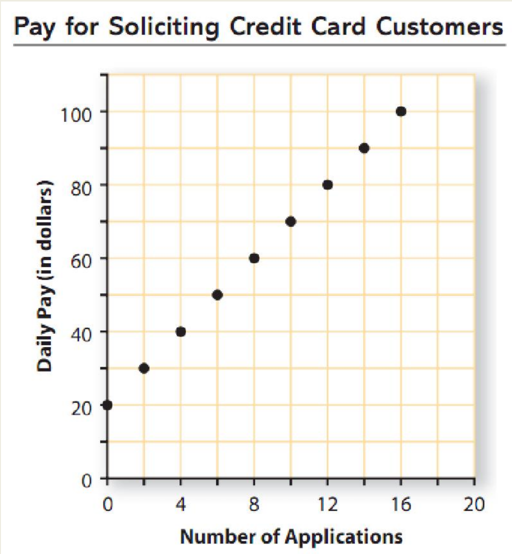

1. For collecting credit card applications, Barry's daily pay $B$ is related to the number of applications he collects $n$ by the rule
   $B = 20 + 5n$

a. Use the function rule to complete this table of sample (n, B) values:

b. Compare the pattern of change shown in your table with that shown in the graph on the preceding page.

# Answering contextual questions using a mathematical model




a. How does Barry's daily pay change as the number of applications he collects increases?

| | | | | |
|---|---|---|---|---|
| Number of applications | 0 | 1 | 2 | 3 |
| Daily Pay (in dollars) | 20 | 25 | 30 | 35 |

1.d How much money does Barry get for every application he collects?

# Answering contextual questions using a mathematical model

A key feature of any function is the way the value of the dependent variable changes as the value of the independent variable changes. Notice that as the number of services Cheri sells increases from 30 to 40, her pay increases from \$100 to \$120. This is an increase of \$20 in pay for an increase of 10 in the number of services sold, or an average of \$2 per sale. Her pay increases at a *rate* of \$2 per service sold.

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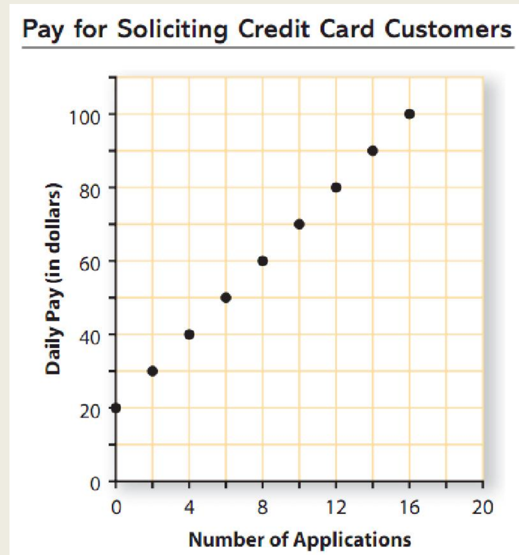


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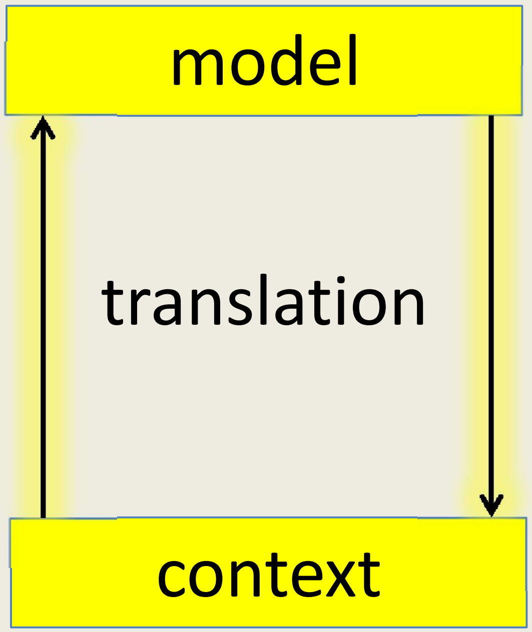
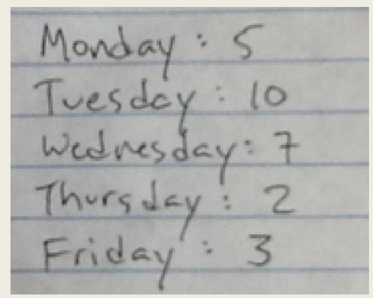
# Working with contextualized examples




At Barry's job, he is paid \$20 a day, and \$5 for every credit card application he collects.

In one week, he recorded how many applications he collected:




OneBank
4234 Financial Ave
New York, NY 10025

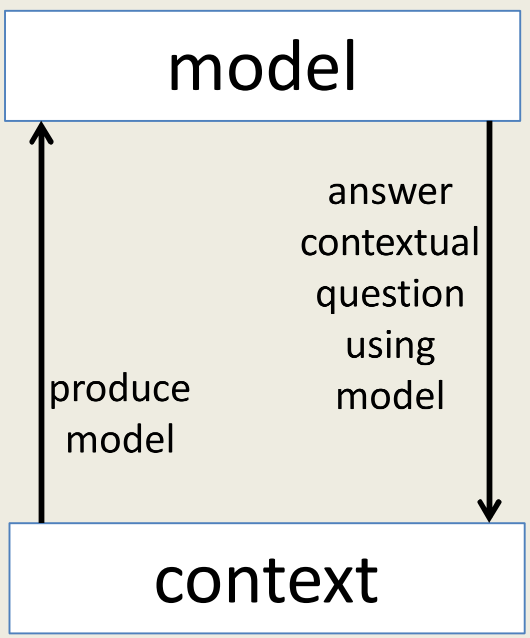
Check Number: 1025
April 24, 2013

Pay: Two Hundred Twenty dollars and 00 cents \$218.00

To the order of:
Barry Winstead
1423 23rd St
New York, NY 10025

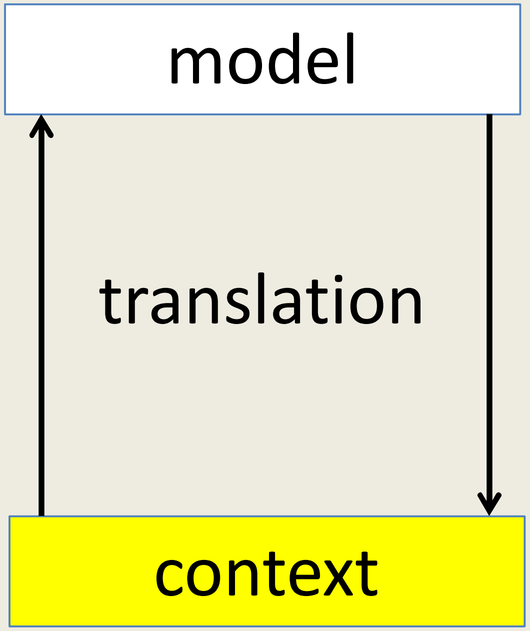
# Classifying questions

| | | | |
|---|---|---|---|
| texts | 50 | 52 | 54 |
| monthly bill | $55 | $55.20 | $55.40 |




1. Plot the values in the table on a graph describing the cell phone plan.
2. How much would it cost if you don't text at all that month?
3. How many people have a phone plan with unlimited texts? How many have to pay for texts?
4. Write a rule for a plan where you pay $30 a month and $.15 per text.

# What’s the point?




1. Make sure students understand the situation and the important quantities and relationships.

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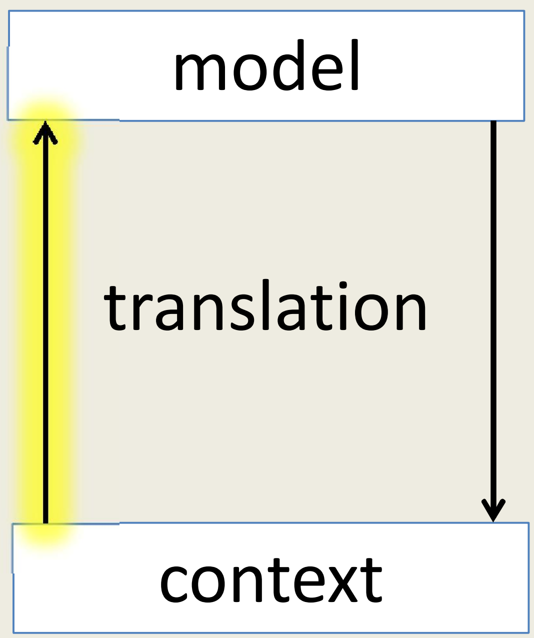


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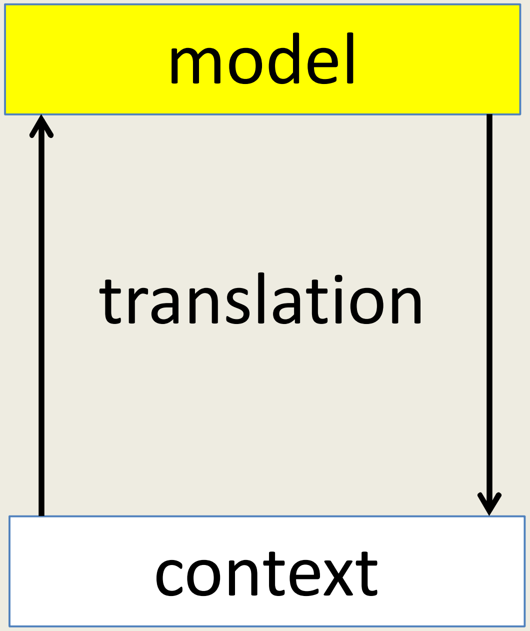


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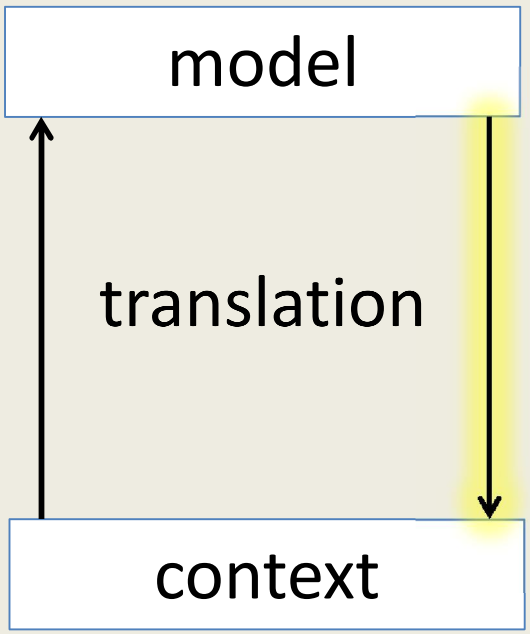
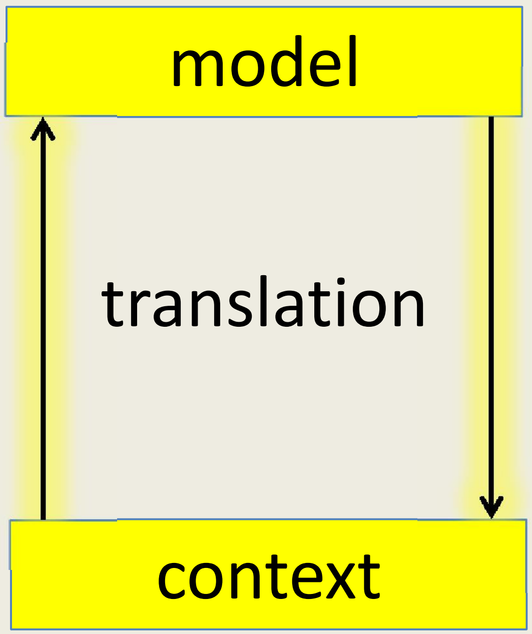


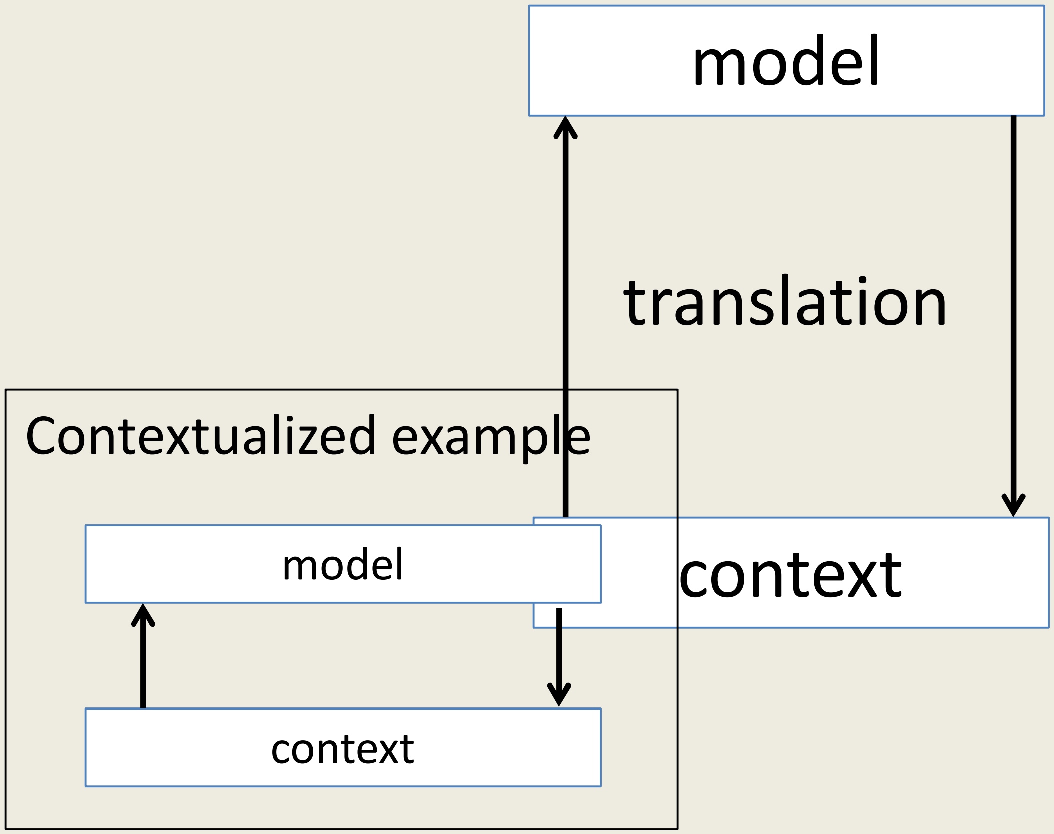

1. Make sure students understand the situation and the important quantities and relationships.
2. If students build the model, they’re more likely to understand it.
3. Use these questions to build connections between representations.
4. Use these questions to get students to invent strategies that can later be formalized. Explicitly connect students informal reasoning to formal conventions.

# What’s the point?

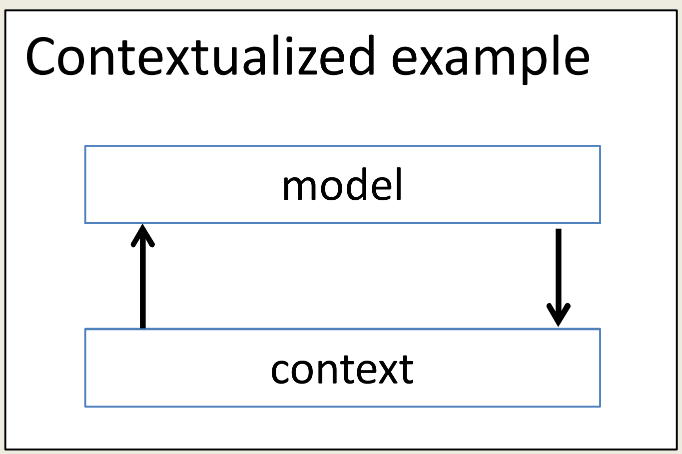



5. Students need practice with going through the entire modeling process themselves.

# Working with contextualized examples



# Contextualized Instruction Framework




Non-contextualized example

Requires students to work or reflect on a “naked” mathematical situation that does not contain any extra-mathematical elements.

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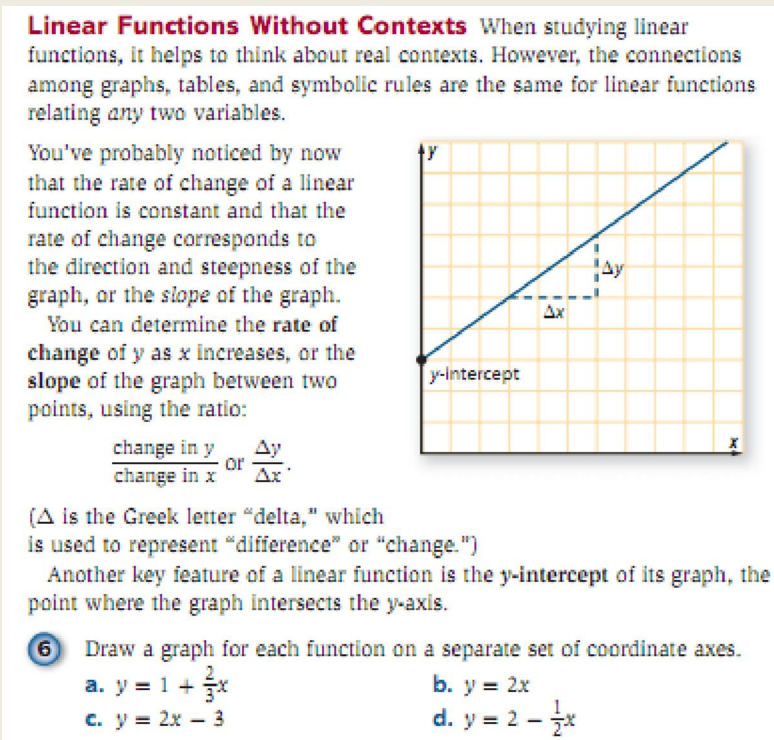
Examples



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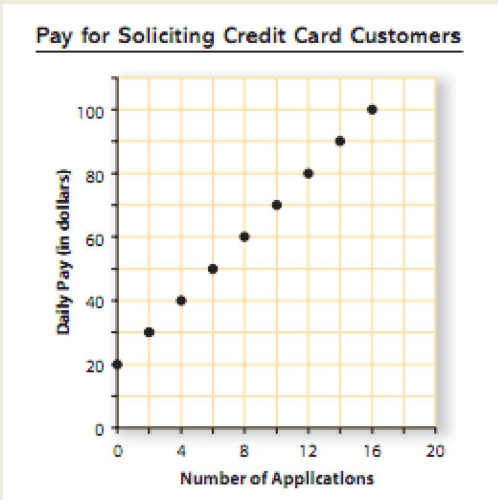
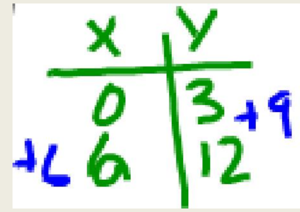
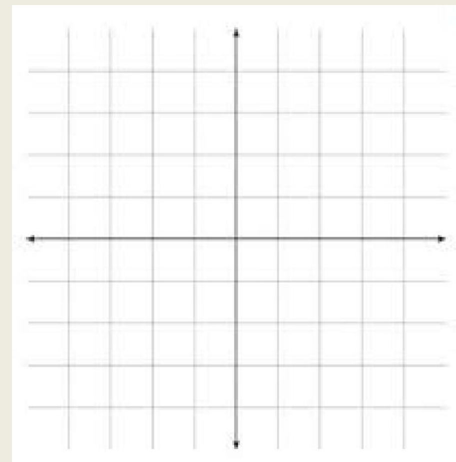
Non-contextualized example

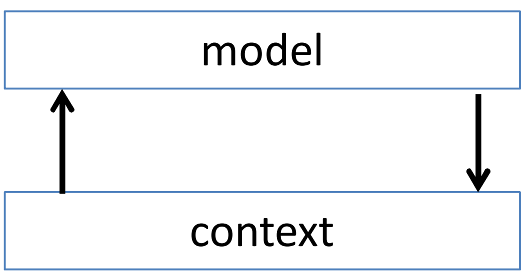
| Number of Services Sold | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|
| Daily Pay (in dollars) | 60 | 80 | 100 | 120 | 140 |



Contextualized example

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**b.** Compare the pattern of change shown in your table with that shown in the graph on the preceding page.

**c.** How much will Barry earn on a day when he does not collect any credit card applications? How can this information be seen in the rule $B = 20 + 5n$? In the table of sample $(n, B)$ values? In the graph on the preceding page?

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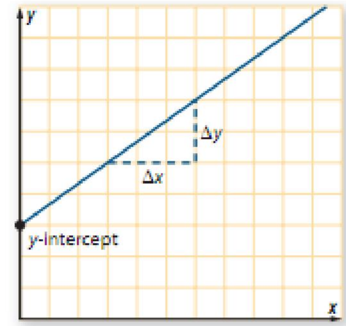


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**b.** $y = 2x$

**c.** $y = 2x - 3$

**d.** $y = 2 - \frac{1}{2}x$

## Contextualized example

model

↑ ↓

context

## Non-contextualized example

# Rethinking for next year

- Have students work through the Barry problem
- Organize the different representations of Barry using the graphic organizer
- Focus on connecting their work with Barry to the non-contextualized, formal description of slope

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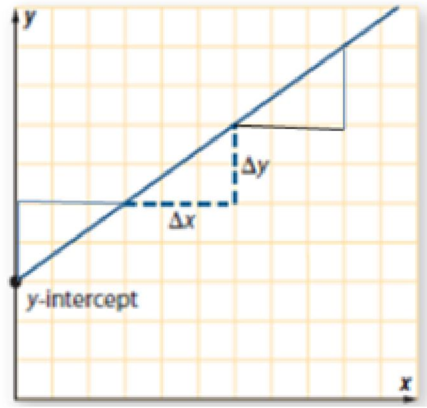
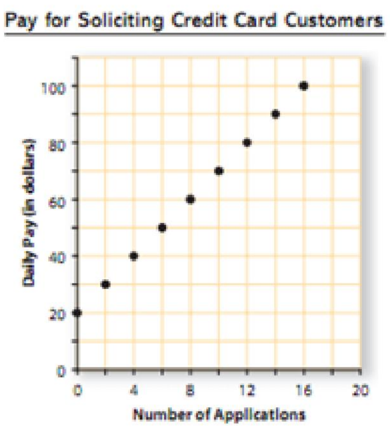



What is y in Barry's situation?________________________________________

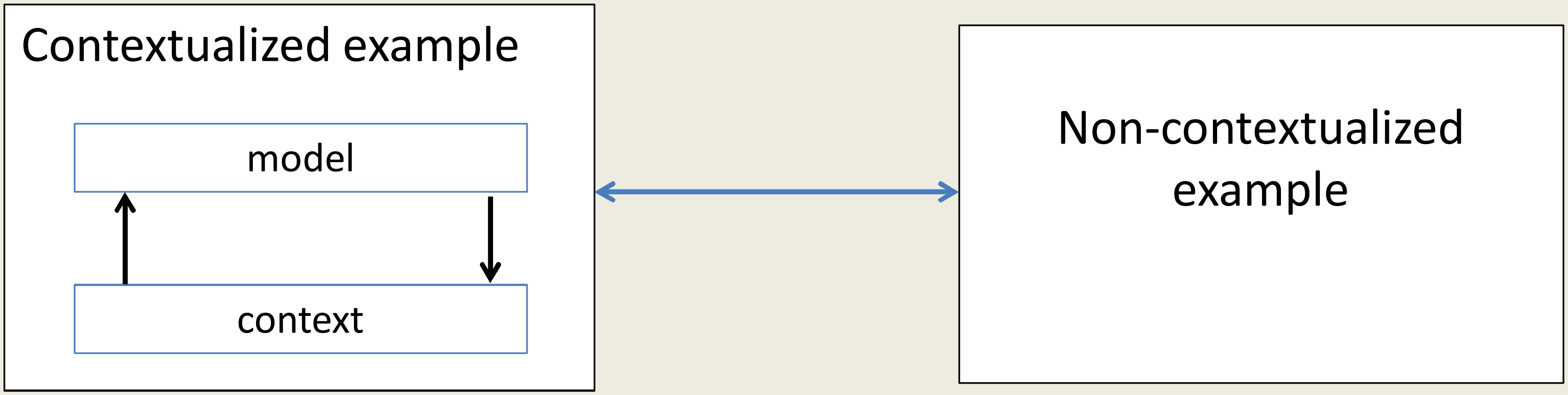
What is x in Barry's situation?________________________________________

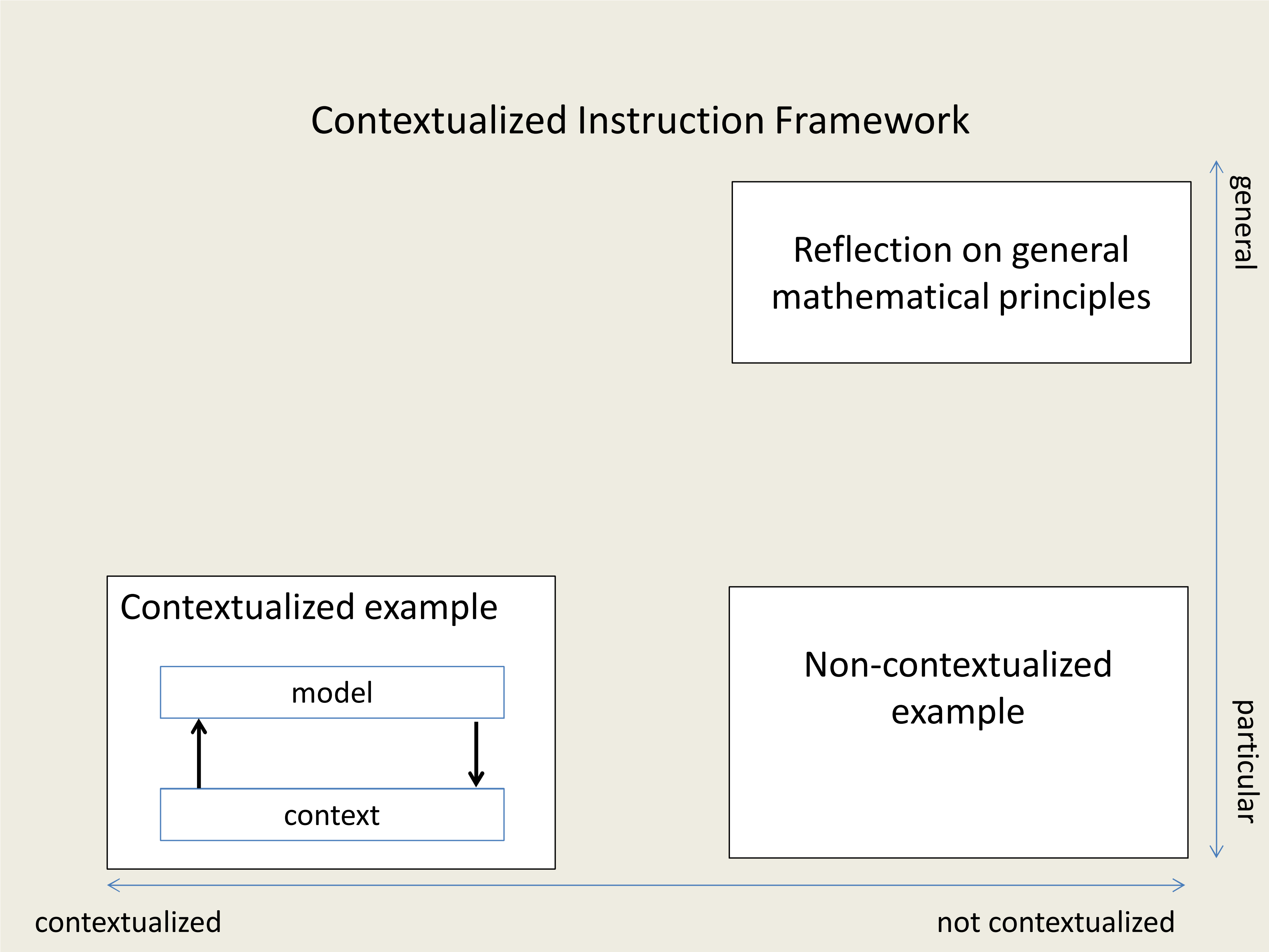
How did you find out how much Barry earned for each application in #1.d?

How is what you did in 1d related to $\frac{change\ in\ y}{change\ in\ x}$?

# What’s the point?

- Connect students’ activity in context to their work with non-contextualized examples.





Contextualized Instruction Framework
Reflection on general mathematical principles
general
Contextualized example
model
context
Non-contextualized example
particular
contextualized
not contextualized

Reflection on general mathematical principles

Includes activity requiring students to describe procedures or algorithms, explain the connections between different types of representations, or explain the meaning of abstract mathematical concepts *not referring to one specific example*.

## Summarize the Mathematics

**Linear functions relating two variables $x$ and $y$ can be represented using tables, graphs, symbolic rules, or verbal descriptions. Key features of a linear function can be seen in each representation.**

a. How can you determine whether a function is linear by inspecting a:
   - **i.** table of $(x, y)$ values?
   - **ii.** graph of the function?
   - **iii.** symbolic rule relating $y$ to $x$?
   - **iv.** *NOW-NEXT* rule?

b. How can the rate of change and the slope of the graph for a linear function be found from a:
   - **i.** table of $(x, y)$ values?
   - **ii.** graph of the function?
   - **iii.** symbolic rule relating $y$ to $x$?
   - **iv.** *NOW-NEXT* rule?

c. How can the $y$-intercept of the graph of a function be seen in a:
   - **i.** table of $(x, y)$ values?
   - **ii.** graph of the function?
   - **iii.** symbolic rule relating $y$ to $x$?

***Be prepared to share your ideas and reasoning with the class.***

# How can you connect the three types of activity?

Reflection on general mathematical principles

Contextualized example

model

context

Non-contextualized example

Look back at what you did in #1c, 2c, 4, 6.

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**Linear functions relating two variables $x$ and $y$ can be represented using tables, graphs, symbolic rules, or verbal descriptions. Key features of a linear function can be seen in each representation.**

**a** How can you determine whether a function is linear by inspecting a:

- **i.** table of $(x, y)$ values?
- **ii.** graph of the function?
- **iii.** symbolic rule relating $y$ to $x$?
- **iv.** *NOW-NEXT* rule?

**b** How can the rate of change and the slope of the graph for a linear function be found from a:

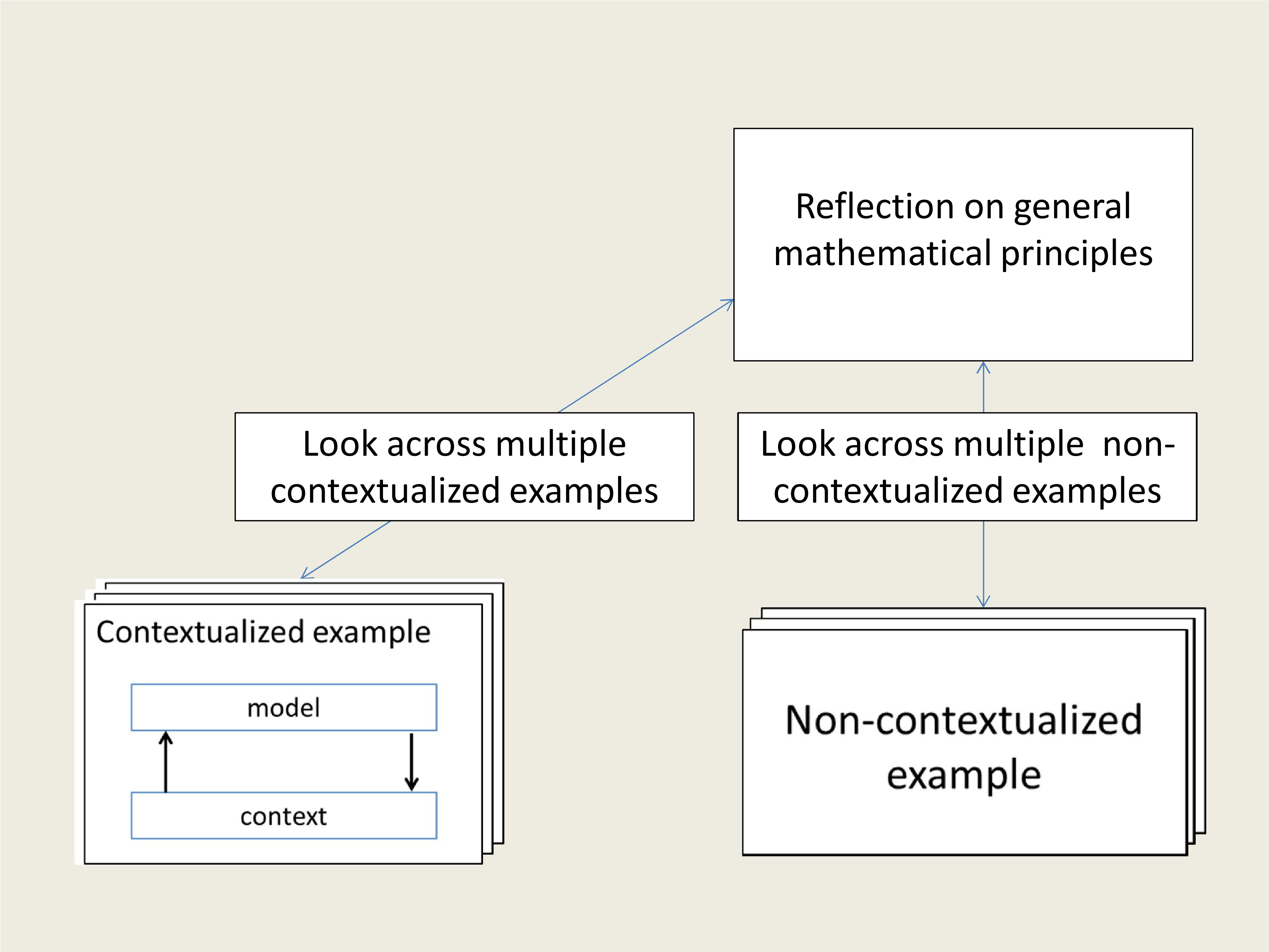
did

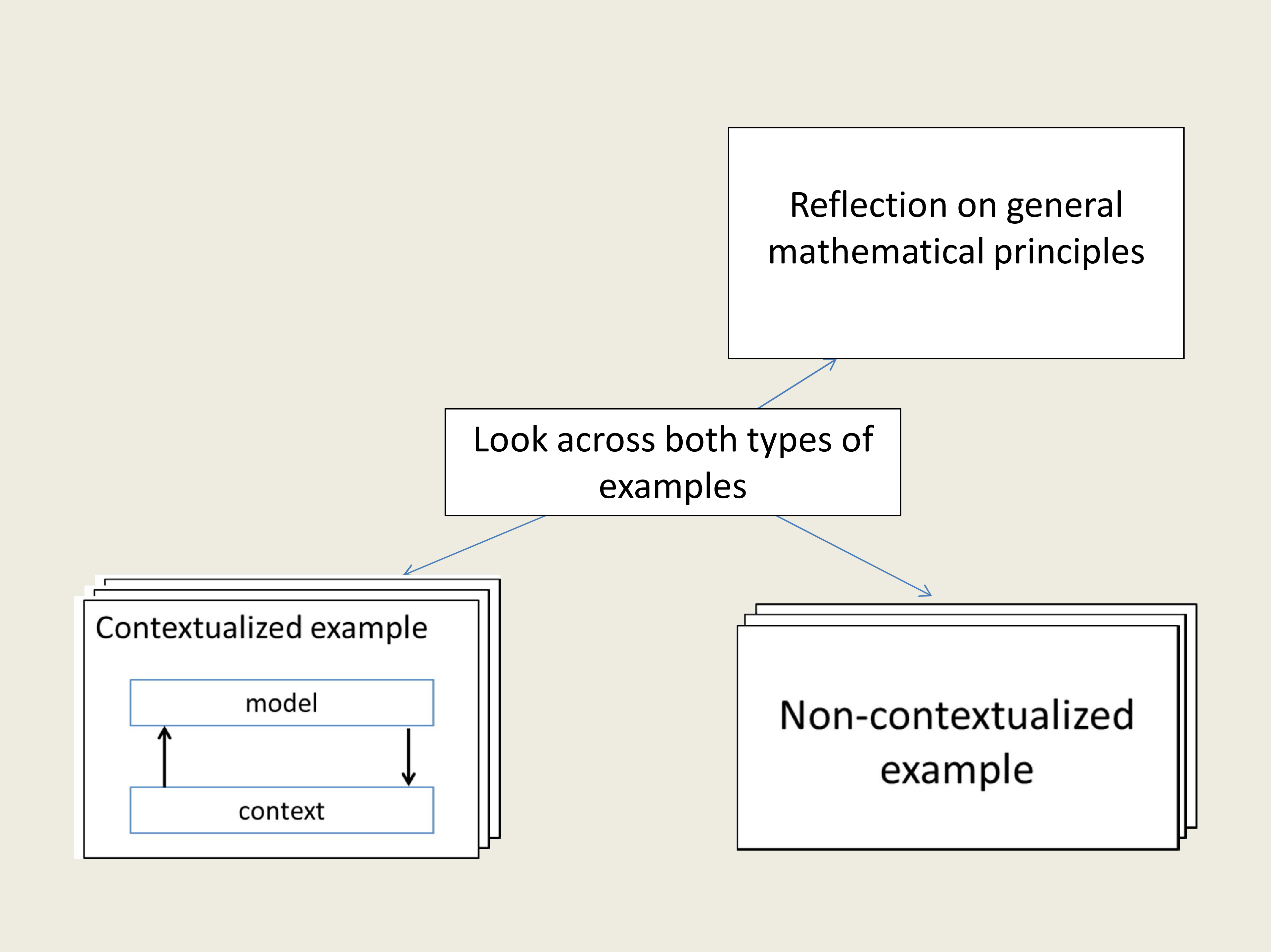
- **i.** table of $(x, y)$ values?
- **ii.** graph of the function?
- **iii.** symbolic rule relating $y$ to $x$?
- **iv.** *NOW-NEXT* rule?

**c** How can the $y$-intercept of the graph of a function be seen in a:

- **i.** table of $(x, y)$ values?
- **ii.** graph of the function?
- **iii.** symbolic rule relating $y$ to $x$?

***Be prepared to share your ideas and reasoning with the class.***


Reflection on general mathematical principles
Look across multiple contextualized examples
Look across multiple non-contextualized examples
Contextualized example
model
context
Non-contextualized example


Reflection on general mathematical principles
Look across both types of examples
Contextualized example
model
context
Non-contextualized example

# Suggested sequence for planning

- Then, for each math objective, think of the contextual question that will get students to perform the required task .
- Determine what tools/models students could use to perform the mathematical task that is desired (e.g. providing a table to calculate the rate of change).
- Think of questions that ask students to reflect back on what they did.
- Two ways to get them generalize. Tell them, or give them another real-world example and get them to look across and say what they did.
- Think of non-contextualized examples to connect to and how to connect them.
- Think of how to connect to big idea questions.

# Suggested sequence for planning

- Determine a real-life situation where the structure lends itself to a question lends itself to informally doing a process that you're going for (e.g. calculate rate of change).
- Then, for each math objective, think of the contextual question that will get students to perform the required task .
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- Determine what tools/models students could use to perform the mathematical task that is desired (e.g. providing a table to calculate the rate of change).
- Think of questions that ask students to reflect back on what they did; connect this to conventional representations/procedures.
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Thank you!!
www.nctm.org/confapp

Feedback?

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