

# thinking outside the box

A  
CHOCOLATEY  
OPTIMIZATION  
PROBLEM

NCTM 2013 Annual  
Meeting & Exposition  
Denver  
April 20, 2013



Taken from [http://a.tgcdn.net/images/products/zoom/f158\\_chocolate\\_gaming\\_dice\\_set.jpg](http://a.tgcdn.net/images/products/zoom/f158_chocolate_gaming_dice_set.jpg)

## 3-8 Gallery Workshop

|                      |  |
|----------------------|--|
| CCSSM Areas:         | Measurement and Data<br>Geometry<br>Functions  |
| Grade Band/Audience: | 3 to 5; 6 to 8   |
| Focus on Math:       | This hands-on activity allows students to explore a real-world optimization problem. In order to perform the task, students need to identify patterns, determine relationships, use appropriate tools, and multiple problem solving strategies. They will have the opportunity to collaborate with classmates and construct viable arguments to support their mathematical reasoning. This student-centered task provides teachers with the opportunity to assess students' mathematical thinking and understanding. |

Presenters:

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## 3rd & 4th Grade

**How do packaging engineers use what they know about the properties of a product and the needs of the consumer to design packages?**

### CCSS for Mathematics

Measurement and Data (MD)

3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Measurement and Data (MD)

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

## Performance Task

It is almost Mother's Day in the United States. Godiva, a manufacturer of premium chocolates and related products, needs to add a new gift box to their Mother's Day Collection to launch their new kind of chocolate truffle, shaped like a cube.

Since you are the packaging engineer at the Research and Development department at Godiva, it is your task to design and construct a prototype of the gift box. You have to take into consideration the requirements from the Board of Directors.

- The base of each chocolate truffle is 1" x 1" square
- The gift box has to have a rectangular shape.
- Must hold 12 chocolate truffles.
- A ribbon must be wrapped around the box.
- You will have to use the least amount of ribbon.

Your task as the packaging engineer is to design a gift box, out of a rectangular sheet of material, that meets the requirements from the Board of Directors.

For this task, you will have to come up with different designs that hold 12 chocolate truffles. Think about all the different ways you could fit 12 chocolate truffles onto a rectangle.

You will record the dimensions of each design in a table. You can only use whole-number side lengths.

- Fill in the chart for the length and width of each of your designs.
- Find the area and perimeter of each of your rectangles.
- Decide how many chocolate truffles would fit in the box.
- Decide how long the ribbon needs to be.

Now, choose the design that uses the least amount of ribbon. Draw the net of the box and construct the prototype gift box. You will be able to use tape to bind the edges.

You will have to present your prototype and a proposal to the Board of Directors. Make sure that the proposal includes the advantages of the design and why they should adopt and start producing it.

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## 4th Grade Extension

The ribbon that goes around the middle of the box is very expensive, so your board of directors has told you that you can only use 18" of ribbon for each box. Think about all the possible designs you could use to create this box. Fill in the table to show all the possible combinations for a rectangle with a perimeter of 18". Which design will hold the most candies? Choose a box that holds the most candies. This is the box you will construct as your prototype gift box. You will be able to use tape to bind the edges.

You will have to present your prototype and a proposal to the Board of Directors. Make sure that the proposal includes the advantages of the design and why they should adopt and start producing it.

### Performance Task

It is almost Mother's Day in the United States. Godiva, a manufacturer of premium chocolates and related products, needs to add a new gift box to their Mother's Day Collection to launch their new kind of chocolate truffle shaped like a cube.

Since you are the packaging engineer at the Research and Development department at Godiva, it is your task to design and construct a prototype of the gift box. You have to take into consideration the requirements from the Board of Directors.

- The dimensions of each cubed chocolate truffle are 1 in x 1 in x 1 in.
- The gift box has to have a rectangular shape.
- The gift box must be constructed from a rectangular sheet of material with dimensions 12 in x 18 in.
- The net of the gift box must be drawn on the rectangular sheet of material. The new machines will stamp out the net of the box and production line workers will assemble the box and carefully place the truffles inside it.

Your task as the packaging engineer is to design and construct a packaging box made out of the rectangular sheet of material that will hold the most truffles.

As part of the engineering process, you will have to come up with different designs before you construct the final one. You will record the dimensions of each design in the following table. You can only use whole-number side lengths.

After you analyze the data of the different dimensions and choose the final design, you will have to draw the net of the box and construct the prototype gift box. You will be able to use tape to bind the edges.

You will have to present your prototype and a proposal to the Board of Directors. Make sure that the proposal includes the advantages of the design and why they should adopt and start producing it.

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### 6th Grade Extension

As part of the engineering process, you will have to come up with lots of different designs before you construct the final one. You will record the dimensions of each design in the following table. You will have to use whole-number side lengths, as well as fractional side lengths.

You will have to present your prototype and a proposal to the Board of Directors. Make sure that the proposal includes the advantages of the design and why they should adopt and start producing it.

## 5th & 6th Grade

**How do packaging engineers use what they know about the properties of a product and the needs of the consumer to design packages?**

### CCSS for Mathematics

#### Measurement and Data (MD)

5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

- Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
- Apply the formulas  $V = lwxh$  and  $V = bwh$  for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems.

#### Geometry (G)

6.G.2 Apply the formulas  $V=lwh$  and  $V=bh$  to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real world and mathematical problems.

## 7th & 8th Grade

**How do packaging engineers use what they know about the properties of a product and the needs of the consumer to design packages?**

### CCSS for Mathematics

#### Geometry (G)

7.G.6 Solve real world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons and right prisms.

#### Functions (F)

8.F.5 Describe quantitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear).

## Performance Task

It is almost Mother's Day in the United States. Godiva, a manufacturer of premium chocolates and related products, needs to add a new gift box to their Mother's Day Collection to launch their new kind of chocolate truffle shaped like a cube.

Since you are the packaging engineer at the Research and Development department at Godiva, it is your task to design and construct a prototype of the gift box. You have to take into consideration the requirements from the Board of Directors.

- The dimensions of each cubed chocolate truffle are 1in x 1in x 1in.
- The gift box has to have a rectangular shape.
- The gift box must be constructed from a rectangular sheet of material with dimensions 12in x 18in.
- The net of the gift box must be drawn on the rectangular sheet of material. The new machines will stamp out the net of the box and production line workers will assemble the box and carefully place the truffles inside it.
- The cost of a squared inch of packaging material is \$0.21.
- The cost of producing 1 chocolate truffle is \$0.68.
- The sale price of 1 chocolate truffle is \$1.42.

Your task as the packaging engineer is to design and construct a packaging box made out of the rectangular sheet of material that will generate the most profit.

As part of the engineering process, you will have to come up with lots of different designs before you construct the final one. You will record your data in the following table.

After you analyze the data of the different dimensions and choose the final design, you will have to draw the net of the box and construct the prototype gift box. You will be able to use tape to bind the edges.

You will have to present your prototype and a proposal to the Board of Directors. Make sure that the proposal includes the advantages of the design and why they should adopt and start producing it.

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## 8th Grade Extension

Analyze the data by analyzing the relationships between different quantities. Determine if the relation is a function. If it is a function, determine if it is increasing or decreasing, and linear or nonlinear.

- Height vs Volume
- Volume vs Profit
- Surface Area vs Profit
- Height vs Profit

After you analyze the data of the different dimensions and choose the final design, you will have to draw the net of the box and construct the prototype gift box. You will be able to use tape to bind the edges.

You will have to present your prototype and a proposal to the Board of Directors. Make sure that the proposal includes the advantages of the design and why they should adopt and start producing it.

# 3rd Grade

## thinking outside the box

### A CHOCOLATEY OPTIMIZATION PROBLEM

| Box Design No. | Length | Width | Area of the Base | Number of Chocolate Truffles | Perimeter of the Base | Length of Ribbon Used |
|----------------|--------|-------|------------------|------------------------------|-----------------------|-----------------------|
|                |        |       |                  |                              |                       |                       |
|                |        |       |                  |                              |                       |                       |
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| Box Design No. | Length | Width | Area of the Base | Number of Chocolate Truffles | Perimeter of the Base | Length of Ribbon Used |
|----------------|--------|-------|------------------|------------------------------|-----------------------|-----------------------|
|                |        |       |                  |                              |                       |                       |
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| Box Design No. | Length | Width | Height | Volume | Number of Chocolate Truffles |
|----------------|--------|-------|--------|--------|------------------------------|
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| Box Design No. | Length | Width | Height | Volume | Number of Chocolate Truffles |
|----------------|--------|-------|--------|--------|------------------------------|
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# 7th Grade

## thinking outside the box

### A CHOCOLATEY OPTIMIZATION PROBLEM

| Box Design No. | Volume | Surface Area | Cost of Packaging Material | Cost of Producing Chocolate Truffles | Sales | Profit |
|----------------|--------|--------------|----------------------------|--------------------------------------|-------|--------|
|                |        |              |                            |                                      |       |        |
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# 8th Grade

# thinking outside the box

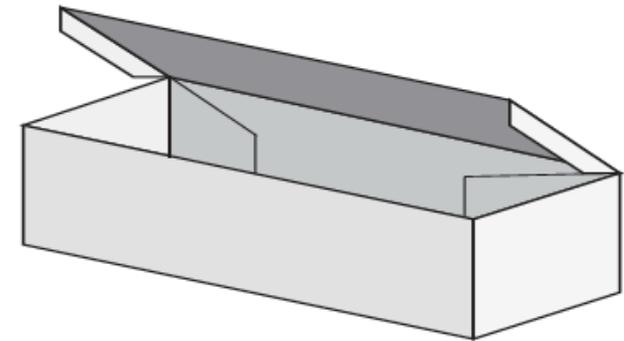
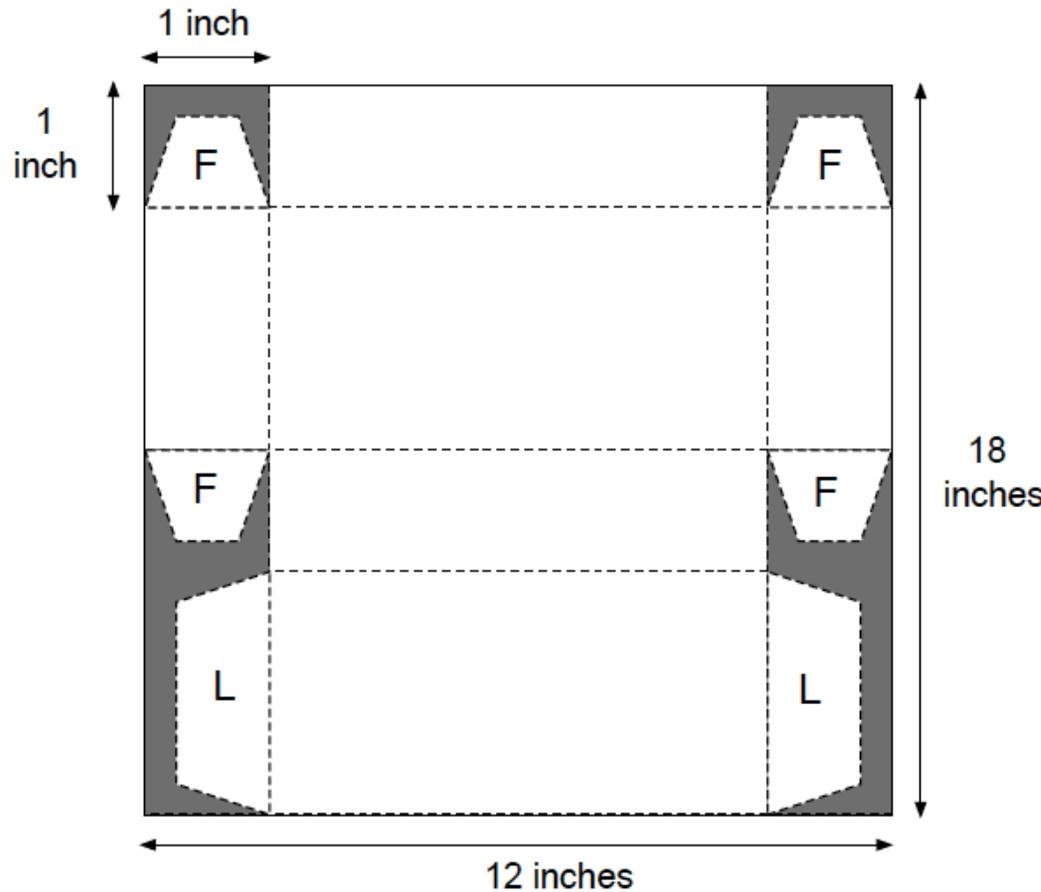
## A CHOCOLATEY OPTIMIZATION PROBLEM

| Box Design No. | Volume | Surface Area | Cost of Packaging Material | Cost of Producing Chocolate Truffles | Sales | Profit |
|----------------|--------|--------------|----------------------------|--------------------------------------|-------|--------|
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# thinking outside the box

A CHOCOLATEY OPTIMIZATION PROBLEM

## Gift Box Design



**(Diagrams not drawn to scale.)**

- The shaded areas are cut away and the rest is folded along the dashed lines.
- The sides are folded up and stuck together using the four flaps marked F.
- The lid has two flaps, marked L, which are not glued.

Image taken from Mathematics Assessment Resource Service (MARS) <http://map.mathshell.org/materials/index.php>

3<sup>rd</sup> Grade Rubric

| Criteria        | 4 | 3   | 2  | 1  |
|-----------------|---|---|--|--|
| Problem Solving |   | <p>Uses the attributes of shapes in order to determine areas and perimeters to design, develop, and produce packages.</p> <p>Finds the area of a rectangle with whole-number sides by tiling it with unit squares and showing that the area is the same as would be found by multiplying the side lengths.</p> <p>Finds rectangles with the same perimeter and different areas and with the same area and different perimeters.</p> | <p>Uses some of the attributes of shapes in order to determine areas and perimeters to design, develop, and produce packages.</p> <p>Finds the area of a rectangle with whole-number sides by either tiling it with unit squares, or multiplying the side lengths, but not both.</p> <p>Finds rectangles with either the same perimeter or different areas or with the same area and different perimeters, but not both.</p> | <p>Is not able to use the attributes of shapes to design, develop, and produce packages.</p> <p>Is not able to find the area of a rectangle by tiling with unit squares or multiplying the lengths of the sides.</p> <p>Is not able to find rectangles with the same perimeter and different areas or with the same area and different perimeters.</p> |

3<sup>rd</sup> Grade Rubric

| Criteria                                       | 4 | 3   | 2   | 1  |
|--|---|---|---|--|
| <b>Data Collection</b>                         |   | Collects data and organizes it in a table. More than 4 sets of dimensions are represented in the table.   | Collects data and organizes it in a table. Between 2-3 sets of dimensions are represented in the table.   | Does not collect sufficient data. Only 1 set of dimensions is represented in the table.  |
| <b>Gift Box Prototype</b>                      |   | Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.<br><br>Draws the net and labels the dimensions on the 12in x 18in sheet of paper.<br><br>Constructs the prototype of the gift box by using the net. | Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.<br><br>Draws the net but does not label the dimensions correctly.<br><br>Constructs the prototype of the box. | Does not represent three-dimensional figures using nets made up of rectangles.<br><br>Constructs the prototype of the box without using a net.<br><br>Is not able to construct a prototype of the box. |
| <b>Communication and Reasoning &amp; Proof</b> |   | Drafts a proposal in persuasive language that is addressed to the Board of Directors. The proposal includes: the advantages of the design, why it should be adopted and produced.   | Drafts a proposal. The proposal includes one of the following: the advantages of the design, why it should be adopted and produced.   | Does not draft a proposal or the proposal drafted includes none of the following: the advantages of the design, why it should be adopted and produced.   |

4<sup>th</sup> Grade Rubric

| Criteria               | 4 | 3  | 2  | 1  |
|------------------------|---|--|--|--|
| <b>Problem Solving</b> |   | <p>Uses the attributes of shapes in order to determine areas and perimeters to design, develop, and produce packages.</p> <p>Applies the area and perimeter formulas for rectangles in real world and mathematical problems.</p> <p>Expresses measurements in a larger unit in a smaller unit.</p> | <p>Uses some of the attributes of shapes in order to determine areas and perimeters to design, develop, and produce packages.</p> <p>Applies the area and perimeter formulas for rectangles in real world and mathematical problems with some errors.</p> <p>Expresses measurements in a larger unit in a smaller unit with some errors.</p> | <p>Is not able to use the attributes of shapes to design, develop, and produce packages.</p> <p>Is not able to apply the area and perimeter formulas for rectangles in real world and mathematical problems.</p> <p>Is not able to demonstrate how to express measurements in a larger unit in a smaller unit.</p> |
| <b>Data Collection</b> |   | <p>Collects data and organizes it in a table. More than 4 sets of dimensions are represented in the table.</p>   | <p>Collects data and organizes it in a table. Between 2-3 sets of dimensions are represented in the table.</p>   | <p>Does not collect sufficient data. Only 1 set of dimensions is represented in the table.</p>   |

4<sup>th</sup> Grade Rubric

| Criteria                                       | 4 | 3  | 2  | 1   |
|--|---|--|--|---|
| <b>Gift Box Prototype</b>                      |   | <p>Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.</p> <p>Draws the net and labels the dimensions on the 12in x 18in sheet of paper.</p> <p>Constructs the prototype of the gift box by using the net.</p> | <p>Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.</p> <p>Draws the net but does not label the dimensions correctly.</p> <p>Constructs the prototype of the box.</p> | <p>Does not represent three-dimensional figures using nets made up of rectangles.</p> <p>Constructs the prototype of the box without using a net.</p> <p>Is not able to construct a prototype of the box.</p> |
| <b>Communication and Reasoning &amp; Proof</b> |   | <p>Drafts a proposal in persuasive language that is addressed to the Board of Directors. The proposal includes: the advantages of the design, why it should be adopted and produced.</p>   | <p>Drafts a proposal. The proposal includes one of the following: the advantages of the design, why it should be adopted and produced.</p>   | <p>Does not draft a proposal or the proposal drafted includes none of the following: the advantages of the design, why it should be adopted and produced.</p>   |

5<sup>th</sup> Grade Rubric

| Criteria               | 4 | 3  | 2  | 1  |
|------------------------|---|--|--|--|
| <b>Problem Solving</b> |   | <p>Uses the attributes of shapes in order to determine volumes to design, develop, and produce packages.</p> <p>Finds the volume of right rectangular prisms with whole-number side lengths by: packing it with unit cubes and applying the formulas <math>V = l \times w \times h</math> and <math>V = b \times h</math>.</p> <p>Decomposes three-dimensional shapes and finds volumes of right rectangular prisms by viewing them as decomposed into layers of array of cubes.</p> | <p>Uses some of the attributes of shapes in order to determine volumes to design, develop, and produce packages.</p> <p>Finds the volume of right rectangular prisms by either packing it with unit cubes or applying the formulas <math>V = l \times w \times h</math> and <math>V = b \times h</math>, but not both.</p> | <p>Is not able to use the attributes of shapes to design, develop, and produce packages.</p> <p>Is not able to apply the volume formulas correctly and does not determine the volume of a right rectangular prism by packing it with unit cubes.</p> |
| <b>Data Collection</b> |   | <p>Collects data and organizes it in a table. More than 4 sets of dimensions are represented in the table.</p>   | <p>Collects data and organizes it in a table. Between 2-3 sets of dimensions are represented in the table.</p>   | <p>Does not collect sufficient data. Only 1 set of dimensions is represented in the table.</p>   |

5<sup>th</sup> Grade Rubric

| Criteria                                       | 4 | 3  | 2  | 1   |
|--|---|--|--|---|
| <b>Gift Box Prototype</b>                      |   | <p>Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.</p> <p>Draws the net and labels the dimensions on the 12in x 18in sheet of paper.</p> <p>Constructs the prototype of the gift box by using the net.</p> | <p>Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.</p> <p>Draws the net but does not label the dimensions correctly.</p> <p>Constructs the prototype of the box.</p> | <p>Does not represent three-dimensional figures using nets made up of rectangles.</p> <p>Constructs the prototype of the box without using a net.</p>         |
| <b>Communication and Reasoning &amp; Proof</b> |   | <p>Drafts a proposal in persuasive language that is addressed to the Board of Directors. The proposal includes: the advantages of the design, why it should be adopted and produced.</p>   | <p>Drafts a proposal. The proposal includes one of the following: the advantages of the design, why it should be adopted and produced.</p>   | <p>Does not draft a proposal or the proposal drafted includes none of the following: the advantages of the design, why it should be adopted and produced.</p> |

6<sup>th</sup> Grade Rubric

| Criteria               | 4 | 3  | 2  | 1   |
|------------------------|---|--|--|---|
| <b>Problem Solving</b> |   | <p>Uses the attributes of shapes in order to determine volumes to design, develop, and produce packages.</p> <p>Finds the volume of right rectangular prisms with whole-number side lengths and fractional side lengths by applying the formulas <math>V = l \times w \times h</math> and <math>V = b \times h</math>.</p> | <p>Uses some of the attributes of shapes in order to determine volumes to design, develop, and produce packages.</p> <p>Finds the volume of right rectangular prisms only using whole-number side lengths by applying the formulas <math>V = l \times w \times h</math> and <math>V = b \times h</math>.</p> | <p>Is not able to use the attributes of shapes to design, develop, and produce packages.</p> <p>Is not able to apply the volume formulas correctly.</p> |
| <b>Data Collection</b> |   | <p>Collects data and organizes it in a table. More than 4 sets of dimensions are represented in the table.</p>   | <p>Collects data and organizes it in a table. Between 2-3 sets of dimensions are represented in the table.</p>   | <p>Does not collect sufficient data. Only 1 set of dimensions is represented in the table.</p>  |

6<sup>th</sup> Grade Rubric

| Criteria                                       | 4 | 3  | 2  | 1   |
|--|---|--|--|---|
| <b>Gift Box Prototype</b>                      |   | <p>Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.</p> <p>Draws the net and labels the dimensions on the 12in x 18in sheet of paper.</p> <p>Constructs the prototype of the gift box by using the net.</p> | <p>Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.</p> <p>Draws the net but does not label the dimensions correctly.</p> <p>Constructs the prototype of the box.</p> | <p>Does not represent three-dimensional figures using nets made up of rectangles.</p> <p>Constructs the prototype of the box without using a net.</p>         |
| <b>Communication and Reasoning &amp; Proof</b> |   | <p>Drafts a proposal in persuasive language that is addressed to the Board of Directors. The proposal includes: the advantages of the design, why it should be adopted and produced.</p>   | <p>Drafts a proposal. The proposal includes one of the following: the advantages of the design, why it should be adopted and produced.</p>   | <p>Does not draft a proposal or the proposal drafted includes none of the following: the advantages of the design, why it should be adopted and produced.</p> |

7<sup>th</sup> Grade Rubric

| Criteria               | 4 | 3  | 2  | 1   |
|------------------------|---|--|--|---|
| <b>Problem Solving</b> |   | <p>Uses the attributes of shapes in order to determine volumes to design, develop, and produce packages.</p> <p>Finds the volume of right rectangular prisms with whole-number side lengths and fractional side lengths by applying the formulas <math>V = l \times w \times h</math> and <math>V = b \times h</math>.</p> <p>Finds surface areas of prisms by decomposing them into pieces whose area they can determine or by using the net.</p> | <p>Uses some of the attributes of shapes in order to determine volumes to design, develop, and produce packages.</p> <p>Finds the volume of right rectangular prisms only using whole-number side lengths by applying the formulas <math>V = l \times w \times h</math> and <math>V = b \times h</math>.</p> <p>Does not find correctly the surface area of the prism.</p> | <p>Is not able to use the attributes of shapes to design, develop, and produce packages.</p> <p>Is not able to apply the volume formulas correctly.</p> <p>Is not able to find the surface area of the prism.</p> |
| <b>Data Collection</b> |   | <p>Collects data and organizes it in a table. More than 4 sets of dimensions are represented in the table.</p>   | <p>Collects data and organizes it in a table. Between 2-3 sets of dimensions are represented in the table.</p>   | <p>Does not collect sufficient data. Only 1 set of dimensions is represented in the table.</p>  |

7<sup>th</sup> Grade Rubric

| Criteria                                       | 4 | 3  | 2  | 1   |
|--|---|--|--|---|
| <b>Gift Box Prototype</b>                      |   | <p>Represents three-dimensional figures using nets made up of rectangles and uses the nets to find the surface area and construct the prototype box.</p> <p>Draws the net and labels the dimensions on the 12in x 18in sheet of paper.</p> <p>Constructs the prototype of the gift box by using the net.</p> | <p>Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.</p> <p>Draws the net but does not label the dimensions correctly.</p> <p>Constructs the prototype of the box.</p> | <p>Does not represent three-dimensional figures using nets made up of rectangles.</p> <p>Constructs the prototype of the box without using a net.</p>         |
| <b>Communication and Reasoning &amp; Proof</b> |   | <p>Drafts a proposal in persuasive language that is addressed to the Board of Directors. The proposal includes: the advantages of the design, why it should be adopted and produced.</p>   | <p>Drafts a proposal. The proposal includes one of the following: the advantages of the design, why it should be adopted and produced.</p>   | <p>Does not draft a proposal or the proposal drafted includes none of the following: the advantages of the design, why it should be adopted and produced.</p> |

8<sup>th</sup> Grade Rubric

| Criteria        | 4 | 3  | 2  | 1  |
|-----------------|---|--|--|--|
| Problem Solving |   | <p>Uses the attributes of shapes in order to determine volumes to design, develop, and produce packages.</p> <p>Finds the volume of right rectangular prisms with whole-number side lengths and fractional side lengths by applying the formulas <math>V = l \times w \times h</math> and <math>V = b \times h</math>.</p> <p>Finds surface areas of prisms by decomposing them into pieces whose area they can determine.</p> <p>Analyzes the data by analyzing the relationships between different quantities. Determines if the relation is a function. If it is a function, determines if it is increasing or decreasing, and linear or nonlinear.</p> | <p>Uses some of the attributes of shapes in order to determine volumes to design, develop, and produce packages.</p> <p>Finds the volume of right rectangular prisms only using whole-number side lengths by applying the formulas <math>V = l \times w \times h</math> and <math>V = b \times h</math>.</p> <p>Does not find correctly the surface area of the prism.</p> <p>Analyzes some of the relationships between quantities.</p> | <p>Is not able to use the attributes of shapes to design, develop, and produce packages.</p> <p>Is not able to apply the volume formulas correctly.</p> <p>Is not able to find the surface area of the prism.</p> <p>Is not able to determine the relationship between quantities.</p> |

8<sup>th</sup> Grade Rubric

| Criteria                                       | 4 | 3   | 2   | 1  |
|--|---|---|---|--|
| <b>Data Collection</b>                         |   | Collects data and organizes it in a table. More than 4 sets of dimensions are represented in the table.   | Collects data and organizes it in a table. Between 2-3 sets of dimensions are represented in the table.   | Does not collect sufficient data. Only 1 set of dimensions is represented in the table.  |
| <b>Gift Box Prototype</b>                      |   | Represents three-dimensional figures using nets made up of rectangles and uses the nets to find the surface area and construct the prototype box.<br><br>Draws the net and labels the dimensions on the 12in x 18in sheet of paper.<br><br>Constructs the prototype of the gift box by using the net. | Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.<br><br>Draws the net but does not label the dimensions correctly.<br><br>Constructs the prototype of the box. | Does not represent three-dimensional figures using nets made up of rectangles.<br><br>Constructs the prototype of the box without using a net.         |
| <b>Communication and Reasoning &amp; Proof</b> |   | Drafts a proposal in persuasive language that is addressed to the Board of Directors. The proposal includes: the advantages of the design, why it should be adopted and produced.   | Drafts a proposal. The proposal includes one of the following: the advantages of the design, why it should be adopted and produced.   | Does not draft a proposal or the proposal drafted includes none of the following: the advantages of the design, why it should be adopted and produced. |

**Understanding by Design (UbD)  
Template**

**Name of Lesson:** Thinking Outside the Box: A Chocolatey Optimization Problem  
**Date:** April 2013

**DESIRED RESULTS**

**Established Goals:**

Common Core Standards for Mathematics

**Measurement and Data (MD)**

*Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.*

3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

**Understandings:**

- The needs of products and consumers impact package design.
- The functions of a package depend on the properties of the product inside the package and the consumer of the product.
- Package engineering is an interdisciplinary field in which scientific and engineering principles are applied to design, develop, and produce packages.

**Essential Questions:**

How do packaging engineers use what they know about the properties of a product and the needs of the consumer to design packages?

**Students will know:**

- Area is an attribute of two-dimensional regions.<sup>1</sup>
- The area of a shape is measured by finding the total number of same-sized units of area required to cover the shape without gaps or overlaps.<sup>1</sup>
- A 1-unit by 1-unit square is the standard unit for measuring area.<sup>1</sup>
- Rectangular arrays can be decomposed into identical rows or into identical columns.<sup>1</sup>
- The formula used to calculate the area of a rectangle is  $A=l \times w$ .
- Perimeter is an attribute of a plane figure

<sup>1</sup>Taken from the CCSS for Mathematics Grade 3, p.21.

**Students will be able to do:**

- CCSS for Mathematical Practice  
4. Model with mathematics
- Measure area by counting unit squares
- Find the area of a rectangle by:
  - Tiling it with unit squares
  - And showing that the area is the same as would be found by multiplying the side lengths
- Find the perimeter of a rectangle
- Find rectangles with the same perimeter and different areas or with the same area and different perimeters.
- Use attributes of shapes in order to determine areas and perimeters to design, develop, and produce packages

**Understanding by Design (UbD)  
Template**

**Name of Lesson:** Thinking Outside the Box: A Chocolatey Optimization Problem  
**Date:** April 2013

**DESIRED RESULTS**

**Established Goals:**

Common Core Standards for Mathematics

**Measurement and Data (MD)**

*Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.*

4.MD.1 Know relative sizes of measurement units within one system of units; express measurements in a larger unit in terms of a smaller unit.

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

**Understandings:**

- The needs of products and consumers impact package design.
- The functions of a package depend on the needs of the product inside the package and the consumer of the product.
- Package engineering is an interdisciplinary field in which scientific and engineering principles are applied to design, develop, and produce packages.

**Essential Questions:**

How do packaging engineers use what they know about the properties of a product and the needs of the consumer to design packages?

**Students will know:**

- Area is an attribute of two-dimensional regions.<sup>1</sup>
- The area of a shape is measured by finding the total number of same-sized units of area required to cover the shape without gaps or overlaps.<sup>1</sup>
- A 1-unit by 1-unit square is the standard unit for measuring area.<sup>1</sup>
- Rectangular arrays can be decomposed into identical rows or into identical columns.<sup>1</sup>
- The formula used to calculate the area of a rectangle is  $A=l \times w$ .
- Perimeter is an attribute of a plane figure
- Properties of two-dimensional shapes and how to use them to solve problems involving symmetry<sup>2</sup>

<sup>1</sup>Taken from the CCSS for Mathematics Grade 3, p.21.

<sup>2</sup>Taken from the CCSS for Mathematics Grade 4, p.27.

**Students will be able to do:**

- CCSS for Mathematical Practice  
4. Model with mathematics
- Measure area by counting unit squares
- Find the area of a rectangle by:
  - Tiling it with unit squares
  - And showing that the area is the same as would be found by multiplying the side lengths
- Find rectangles with the same perimeter and different areas or with the same area and different perimeters.
- Express measurements in a larger unit in a smaller unit (unit conversion).
- Apply the area and perimeter formulas for rectangles in real world and mathematical problems.
- Use attributes of shapes in order to determine areas and perimeters to design, develop, and produce packages

**EVIDENCE/ASSESSMENTS**

Backward Stages: 1. Identify desired results. 2. Determine acceptable evidence. 3. Plan learning experiences and instruction.

Adapted from Wiggins & McTighe (2005) *Understanding by Design (UbD)*

**Understanding by Design (UbD)  
Template**

**Name of Lesson: Thinking Outside the Box: A Chocolatey Optimization Problem**

**Date: April 2013**

**DESIRED RESULTS**

**Established Goals:**

Common Core Standards for Mathematics

**Measurement and Data (MD)**

*Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.*

5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

- a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
- b. Apply the formulas  $V = l \times w \times h$  and  $V = b \times h$  for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems.

**Understandings:**

- The needs of products and consumers impact package design.
- The functions of a package depend on the needs of the product inside the package and the consumer of the product.
- Package engineering is an interdisciplinary field in which scientific and engineering principles are applied to design, develop, and produce packages.

**Essential Questions:**

How do packaging engineers use what they know about the properties of a product and the needs of the consumer to design packages?

**Students will know:**

- Volume is the amount of space enclosed in a three-dimensional shape.
- Volume is an attribute of three-dimensional spaces.<sup>1</sup>
- Volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps.<sup>1</sup>
- A 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume.<sup>1</sup>
- The formulas used to calculate the volume of a rectangular prism are  $V = l \times w \times h$  or  $V = b \times h$ .
- Net is a two-dimensional pattern of a three-dimensional figure that can be folded to form the figure.

<sup>1</sup>Taken from the CCSS for Mathematics Grade 5, p.33.

**Students will be able to do:**

- CCSS for Mathematical Practice  
4. Model with mathematics
- Use the attributes of shapes in order to determine volumes to design, develop, and produce packages.
- Find the volume of right rectangular prisms with whole-number side lengths by:
  - Packing it with unit cubes
  - Applying the formulas  $V = l \times w \times h$  and  $V = b \times h$
- Decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of array of cubes.
- Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.

**Understanding by Design (UbD)  
Template**

**Name of Lesson: Thinking Outside the Box: A Chocolatey Optimization Problem**      **Date: April 2013**

**DESIRED RESULTS**

**Established Goals:**

Common Core Standards for Mathematics

**Geometry (G)**

*Solve real-world and mathematical problems involving area, surface area, and volume.*

6.G.2 Apply the formulas  $V=lwh$  and  $V=bh$  to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real world and mathematical problems.

**Understandings:**

- The needs of products and consumers impact package design.
- The functions of a package depend on the needs of the product inside the package and the consumer of the product.
- Package engineering is an interdisciplinary field in which scientific and engineering principles are applied to design, develop, and produce packages.

**Essential Questions:**

How do packaging engineers use what they know about the properties of a product and the needs of the consumer to design packages?

**Students will know:**

- Volume is the amount of space enclosed in a three-dimensional shape.
- Volume is an attribute of three-dimensional spaces.<sup>1</sup>
- A 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume.<sup>1</sup>
- The formulas used to calculate the volume of a rectangular prism are  $V=lwxh$  or  $V=bxh$ .
- Net is a two-dimensional pattern of a three-dimensional figure that can be folded to form the figure.
- <sup>1</sup>Taken from the CCSS for Mathematics Grade 5, p.33.

**Students will be able to do:**

- CCSS for Mathematical Practice  
4. Model with mathematics
- Use the attributes of shapes in order to determine volumes to design, develop, and produce packages.
- Find the volume of right rectangular prisms with fractional side lengths by:
  - Applying the formulas  $V=lwxh$  and  $V=bxh$
- Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.

**Understanding by Design (UbD)  
Template**

**Name of Lesson: Thinking Outside the Box: A Chocolatey Optimization Problem**      **Date: April 2013**

|   |  |
|---|--|
| <b>DESIRED RESULTS</b>  |  |
| <b>Established Goals:</b>   |  |
| <p>Common Core Standards for Mathematics<br/> <b>Geometry (G)</b><br/> <i>Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.</i><br/>                     7.G.6 Solve real world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons and right prisms.</p>   |  |
| <p><b>Understandings:</b></p> <ul style="list-style-type: none"> <li>• The needs of products and consumers impact package design.</li> <li>• The functions of a package depend on the needs of the product inside the package and the consumer of the product.</li> <li>• Package engineering is an interdisciplinary field in which scientific and engineering principles are applied to design, develop, and produce packages.</li> </ul>   | <p><b>Essential Questions:</b></p> <p>How do packaging engineers use what they know about the properties of a product and the needs of the consumer to design packages?</p>  |
| <p><b>Students will know:</b></p> <ul style="list-style-type: none"> <li>• Volume is the amount of space enclosed in a three-dimensional shape.</li> <li>• Volume is an attribute of three-dimensional spaces.<sup>1</sup></li> <li>• A 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume.<sup>1</sup></li> <li>• The formulas used to calculate the volume of a rectangular prism are <math>V=l \times w \times h</math> or <math>V=b \times h</math>.</li> <li>• Surface area is the number of square units that will exactly cover the surface of a three-dimensional shape.</li> <li>• Net is a two-dimensional pattern of a three-dimensional figure that can be folded to form the figure.</li> <li>• <sup>1</sup>Taken from the CCSS for Mathematics Grade 5, p.33.</li> </ul> | <p><b>Students will be able to do:</b></p> <ul style="list-style-type: none"> <li>• CCSS for Mathematical Practice<br/>4. Model with mathematics</li> <li>• Use the attributes of shapes in order to determine volumes and surface areas to design, develop, and produce packages.</li> <li>• Find the volume of right rectangular prisms with whole-number side lengths or fractional side lengths by:                         <ul style="list-style-type: none"> <li>○ Applying the formulas <math>V=l \times w \times h</math> and <math>V=b \times h</math></li> </ul> </li> <li>• Find surface areas of prisms by decomposing them into pieces whose area they can determine or by using the net.</li> <li>• Represents three-dimensional figures using nets made up of rectangles and uses the nets to find the surface area and construct the prototype box.</li> </ul> |

Backward Stages: 1. Identify desired results. 2. Determine acceptable evidence. 3. Plan learning experiences and instruction.  
 Adapted from Wiggins & McTighe (2005) *Understanding by Design (UbD)*

**Understanding by Design (UbD)  
Template**

**Name of Lesson: Thinking Outside the Box: A Chocolatey Optimization Problem**      **Date: April 2013**

**DESIRED RESULTS**

**Established Goals:**

Common Core Standards for Mathematics

**Geometry (G)**

*Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.*

7.G.6 Solve real world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons and right prisms.

**Functions (F)**

*Use functions to model relationships between quantities.*

8.F.5 Describe quantitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear).

**Understandings:**

- The needs of products and consumers impact package design.
- The functions of a package depend on the needs of the product inside the package and the consumer of the product.
- Package engineering is an interdisciplinary field in which scientific and engineering principles are applied to design, develop, and produce packages.

**Essential Questions:**

How do packaging engineers use what they know about the properties of a product and the needs of the consumer to design packages?

**Students will know:**

- Volume is the amount of space enclosed in a three-dimensional shape.
- Volume is an attribute of three-dimensional spaces.<sup>1</sup>
- A 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume.<sup>1</sup>
- The formulas used to calculate the volume of a rectangular prism are  $V=l \times w \times h$  or  $V=b \times h$ .
- Surface area is the number of square units that will exactly cover the surface of a three-dimensional shape.
- A function is a rule that assigns to each input exactly one output.<sup>2</sup>
- Functions describe situations where one quantity determines another.<sup>2</sup>
- Net is a two-dimensional pattern of a three-dimensional figure that can be folded to form the figure.

**Students will be able to do:**

- CCSS for Mathematical Practice  
4. Model with mathematics
- Use the attributes of shapes in order to determine volumes and surface areas to design, develop, and produce packages.
- Find the volume of right rectangular prisms with whole-number side lengths or fractional side lengths by:
  - Applying the formulas  $V=l \times w \times h$  and  $V=b \times h$
- Find surface areas of prisms by decomposing them into pieces whose area they can determine or by using the net.
- Represents three-dimensional figures using nets made up of rectangles and uses the nets to find the surface area and construct the prototype box.
- Analyzes the data by analyzing the relationships between different quantities. Determines if the relation is a function. If it is a function, determines if it is increasing or decreasing, and linear or nonlinear.

<sup>1</sup>Taken from the CCSS for Mathematics Grade 5, p.33.