Ending Algebraic Misconceptions: Building Correct Knowledge by Showing Incorrect Examples

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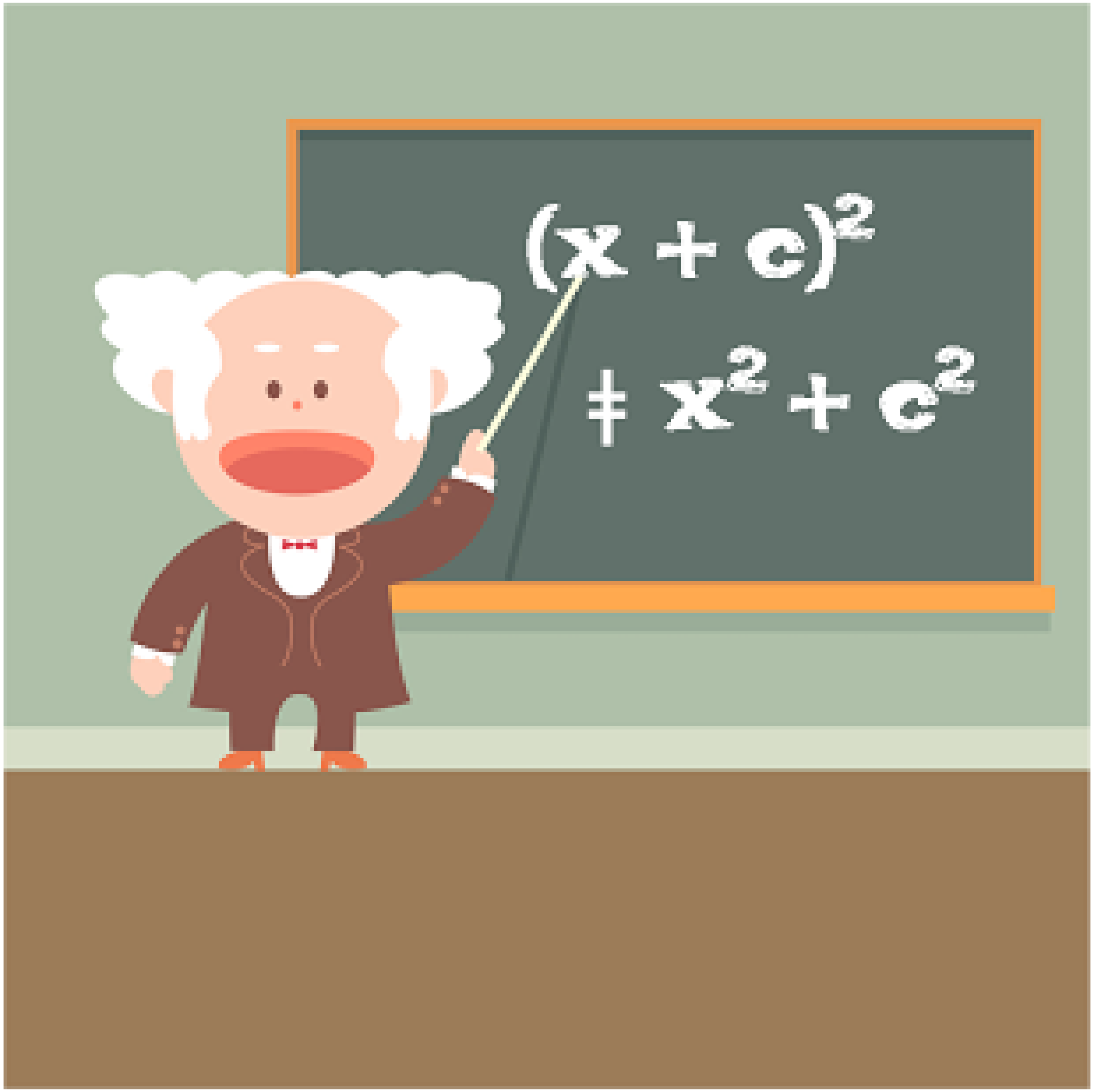

$(x + c)^2$
$\neq x^2 + c^2$

Agenda

- Common Algebraic Misconceptions
- One Solution Strategy: Worked Examples
- Alignment with Common Core
- Classroom Use
- How to Create Worked Examples
- Small Group Activity
- Discussion

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→**Common Algebraic Misconceptions**

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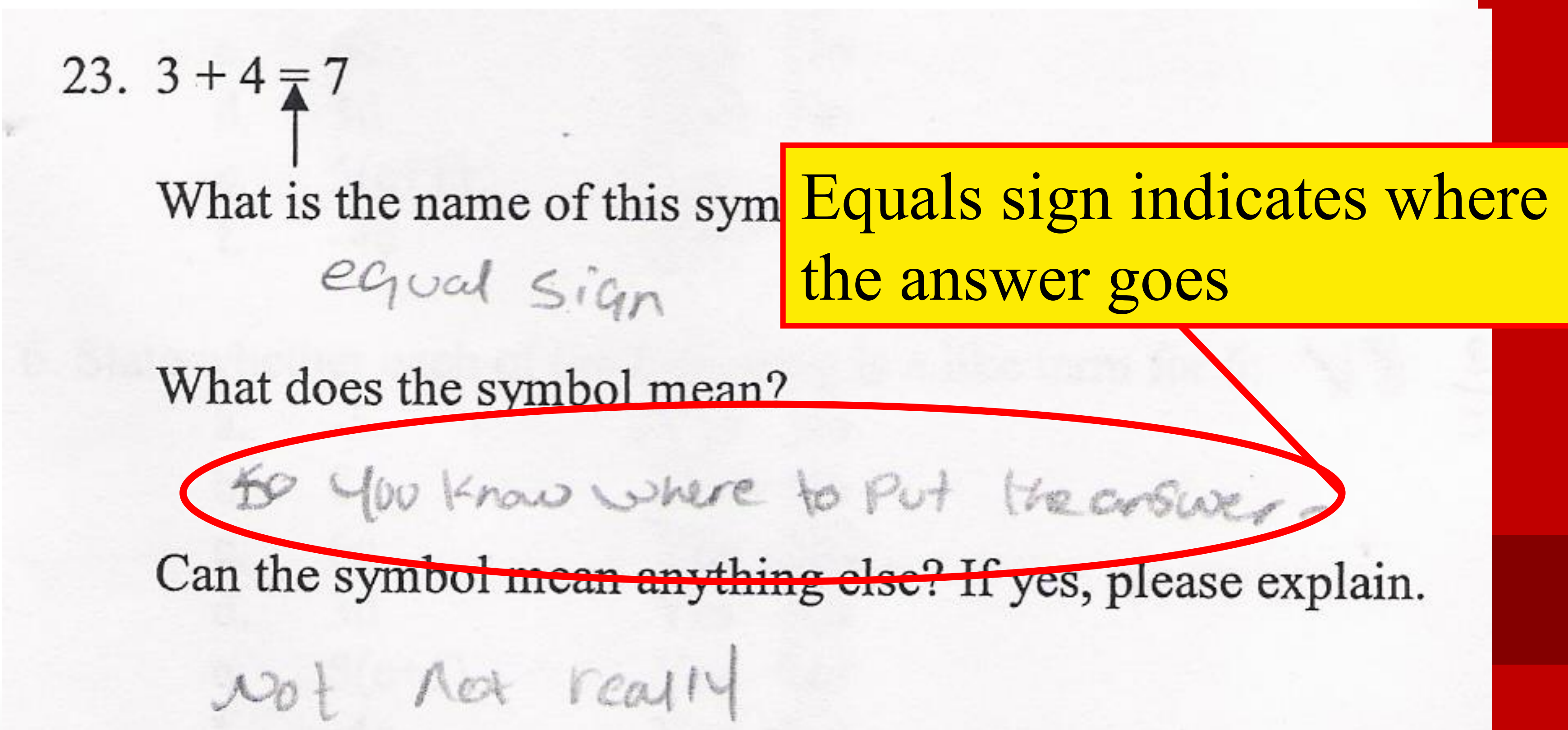
Common Algebraic Misconceptions:

- The Equals Sign
- Negative Symbol
- Combining Like Terms

Common Algebraic Misconceptions: The Equals Sign

- The equals sign is an indicator of an operation to be performed.


23. $3 + 4 = 7$

What is the name of this sym

equal sign

What does the symbol mean?

So you know where to put the answer.

Can the symbol mean anything else? If yes, please explain.

not not really

Equals sign indicates where the answer goes

Common Algebraic Misconceptions: Negatives

- Negative signs are separate from terms and can move independently.

Circle the variable terms:

1. $5b + 4 + 2b = 3 - 9b$
2. $2 + 3y = 7y - 8$

Separate from term

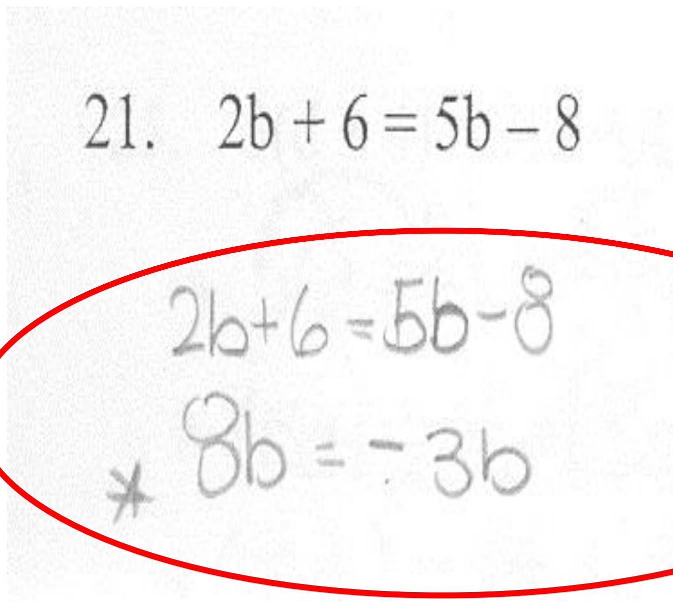
Can move independently

13. State whether each of the following is equal to -4x + 3:

a.	4x + 3	Yes	No
b.	3 – 4x	Yes	No
c.	4x – 3	Yes	No
d.	3 + (-4x)	Yes	No
e.	3 + 4x	Yes	No

Common Algebraic Misconceptions: Combining Like Terms

- You can combine variables and constant terms.



21. $2b + 6 = 5b - 8$

$2b+6=5b-8$

$* \quad 8b = -3b$

Combines variable and constant terms

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Ending Algebraic Misconceptions: Worked Examples

"[A] worked example is a step-by-step demonstration of how to perform a task or how to solve a problem"
(Clark, Nguyen & Sweller, 2006, p. 190).

- Leads to the same amount of learning in less time

OR

- Increased learning or transfer of knowledge in the same amount of time

Ending Algebraic Misconceptions: Self Explanation

"Explaining information to oneself as you read or study"

(Chi, 2000)

- Facilitates replacement of faulty knowledge and integration of new information
- Forces the learner to make their new knowledge explicit.

→Combining worked examples and self-explanation has improved conceptual understanding in prior classroom studies (Booth et al., 2013).

1 - Two-Step Linear Equations
2 - Correct Example AC1

Table of Contents | Lesson | Problems

Jermain was asked to solve the following equation for x, and he made a GOOD first step to solve the problem. Look at his first step.

$$3 = 2x - 7$$
$$10 = 2x$$

What did Jermain do?

On the left side he...

Choose one... | Choose one...

Add

On the right side he...

Choose one... | Choose one...

- Choose one...
- Added
- Subtracted
- Multiplied
- Divided
- Did nothing

Add

Why is that a GOOD step for Jermain to take?
Please answer both why it was a VALID step and a HELPFUL step.

It is valid because | Choose one...

Add

It is helpful because | Choose one...

Add

Eliza solved this problem **correctly.** Here is her work:

$6 - k = -3$

$$6 - k = -3$$
$$-6 \qquad\qquad -6$$
$$-k = -9$$
$$\div -1 \qquad \div -1$$
$$k = 9$$

Why did Eliza subtract 6 FROM BOTH SIDES of the equation?

Why did Eliza divide by -1?

Incorrect Examples

- Help students recognize incorrect procedures
- Increases conceptual and procedural knowledge gains
- Teachers' fear that exposing students to incorrect examples may *increase* misconceptions is unfounded (Booth et al., 2013).

1 - Two-Step Linear Equations
2 - Incorrect Example AI1

Table of Contents | Lesson | Problems

Chante was asked to solve the following equation for x, and she made a WRONG first step to solve the problem. Look at her first step.

$-5x + 3 = 7$
$-5x = 10$

What did Chante do?

On the left side she...

Choose one... | Choose one...

Add

On the right side she...

Choose one... | Choose one...

Add

Why is that a WRONG step for Chante to take?

Choose one... | Choose one...

Ashley **didn't** simplify this expression correctly. Here is her work:

4^{-2}

4^{-2}

-16

How does a negative exponent affect its base?

What is the correct answer to this problem?

Agenda

- Common Algebraic Misconceptions
- One Solution Strategy: Worked Examples

→**Alignment with Common Core**

- Classroom Use
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Alignment with Common Core: Content Standards

- Can align with any content area!

Example: **High School Algebra, Reasoning with Equations & Inequalities Standard A.1**

Katelyn was asked to solve the following equation for x, and she made a GOOD first step to solve the problem. Look at her first step.

7 = 2x
7/2 = x

What did Katelyn do?

Solving a simple equation

Why is that a GOOD step for Katelyn to take?
Please answer both why it was a VALID step and a HELPFUL step.

Justify a solution method

Alignment with Common Core: Practice Standards

MP1: **Make sense of problems** and persevere in solving them.

SET 4 Write an expression or equation to represent the situation. **You <u>do not</u> need to solve any equations.**

Melinda wrote this equation correctly. Here is what she wrote:

Stephan works as a waiter and makes $40 in tips each night he worked this week. He gave $25 of his week's tips to the serving assistant and had $215 left for himself. How many days did he work this week?

$40x - 25 = 215$

What does the *x* stand for in Melinda's equation?

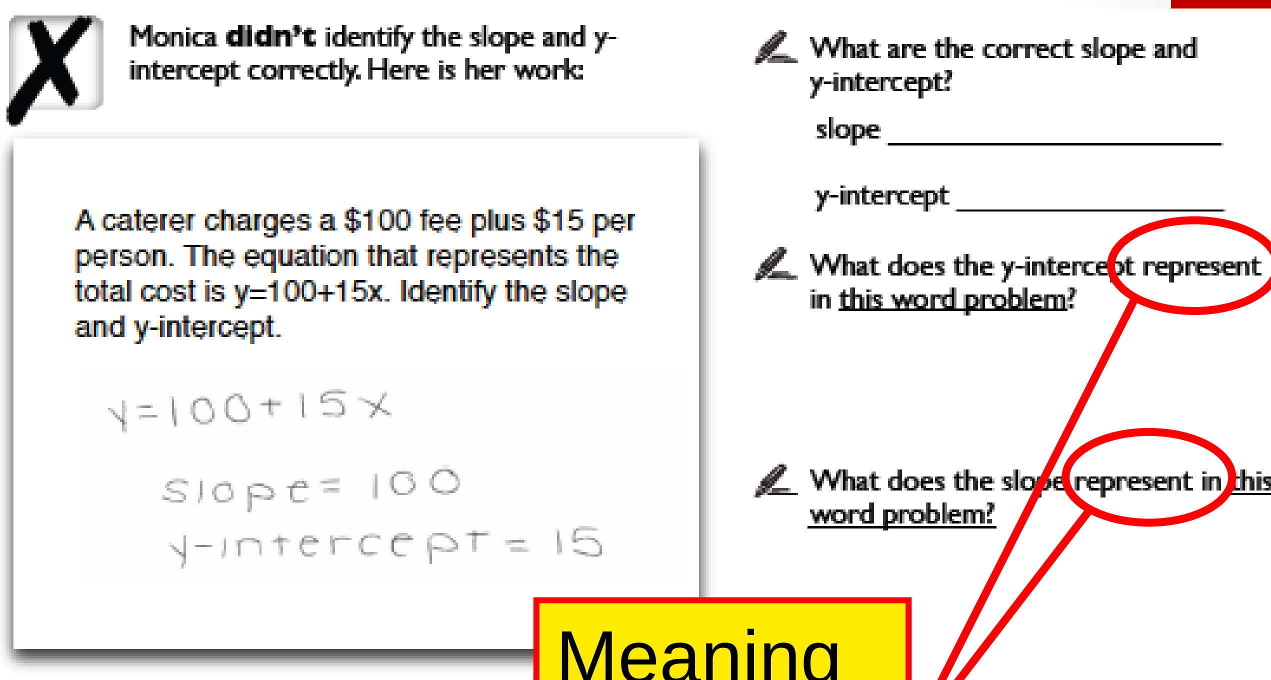
Make sense of problem

How did she know to

Alignment with Common Core: Practice Standards

MP2: Reason abstractly and quantitatively.

"Attending to the **meaning of quantities**, not just how to compute them."



Monica **didn't** identify the slope and y-intercept correctly. Here is her work:

A caterer charges a $100 fee plus $15 per person. The equation that represents the total cost is y=100+15x. Identify the slope and y-intercept.

y=100+15x

slope= 100

y-intercept = 15

What are the correct slope and y-intercept?

slope ____________

y-intercept ____________

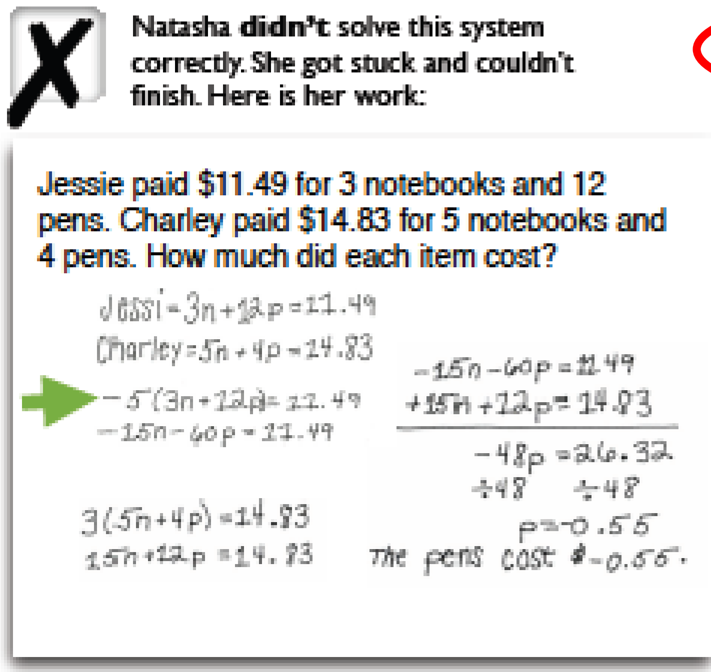
What does the y-intercept represent in this word problem?

What does the slope represent in this word problem?

Meaning of the quantities

Alignment with Common Core: Practice Standards

MP3: Construct **viable arguments** and **critique the reasoning** of others.



Natasha **didn't** solve this system correctly. She got stuck and couldn't finish. Here is her work:

Jessie paid $11.49 for 3 notebooks and 12 pens. Charley paid $14.83 for 5 notebooks and 4 pens. How much did each item cost?

Jessi = $3n + 12p = 11.49$
Charley = $5n + 4p = 14.83$
$-5(3n + 12p) = 11.49$
$-15n - 60p = 11.49$

$3(5n + 4p) = 14.83$
$15n + 12p = 14.83$

$-15n - 60p = 11.49$
$+15n + 12p = 14.83$
$-48p = 26.32$
$\div 48 \quad \div 48$
$p = -0.55$
The pens cost $-0.55.

Critique the reasoning of others

Does Natasha's price for a pen seem reasonable? Why or why not?

In the step marked with an arrow, Natasha multiplied the left side of the equation by -5. What should she have done to keep the equation equivalent?

Construct viable arguments

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→**Classroom Use**

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- Small Group Activity
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Classroom Use: AlgebraByExample

- Strategic Education Research Partnership (SERP)
- Research carried out between school districts and researchers
- Resulted in creation of AlgebraByExample

Classroom Use: AlgebraByExample

- Teachers give 4 worked example assignments per unit.
- Assignments are given as warm-ups, in-class practice (guided or independent), or review.
- **Several thousand students** in 7 districts across the East Coast and Midwest participated in research about these assignments.

AlgebraByExample Results

- Students who worked with example-based problems **improved more on conceptual knowledge** measures and learned **just as much procedural knowledge** when compared to students who did traditional problems only.

- **Minority students** in the example-based group **improved more** on conceptual knowledge measures than minority students in the control group.

Classroom Use: The Cognitive Tutor

- The Cognitive Tutor is an “intelligent” mathematics software program.
- Researchers integrated worked examples and self-explanation into the two-step equation unit.
- Over **500 8th grade** students in 6 studies participated in this research.

Cognitive Tutor Results: Incorrect Examples Matter

	Correct Only	Both Correct & Incorrect	Incorrect Only
Overall Gains		✔	
Conceptual Knowledge		✔	✔

Cognitive Tutor Results: Results Differ by Prior Knowledge

	Correct Only	Both Correct & Incorrect	Incorrect Only
Low Ability			✔
Average		✔	
High Ability		✔	✔

Recap

- →Common algebraic misconceptions exist.
- →Using worked examples paired with self-explanation prompts is one strategy that specifically targets these misconceptions.
- →Incorrect examples are important!
- →These problem types align with the common core…
- →And have positively increased student understanding in classroom-based research!

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- Common Algebraic Misconceptions
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- Classroom Use

→**How to Create Worked Examples**

- Small Group Activity
- Discussion

How to Create Incorrect Examples

- Steps:
  1. Write the objective.
  2. List a few common misconceptions associated with this objective.
  3. Choose <u>1 misconception </u>for each example.
  4. Create the incorrect worked example using that misconception.
     A) Clearly mark the problem as incorrect.
     B) Use a student name – remember to be gender and culturally diverse.
  5. Write the self-explanation prompt focusing on the misconception.

Self-Explanation Prompt Tips

- Do…
  - Ask the “why” questions.
  - Have students explain their reasoning.
  - Call students attention to the features of the problem you think are important.
- Don’t just …
  - Ask questions that instruct students to state the procedure.
  - Ask “what is wrong with the example”, “what mistake was made,” or “what is the correct answer.”

Let’s Practice

- Step 1: Write the objective.

SWBAT simplify an expression
by combining like terms.

Let's Practice

- Step 2: List a few common misconceptions associated with this objective.
- Step 3: Choose <u>1 misconception </u>for each example.

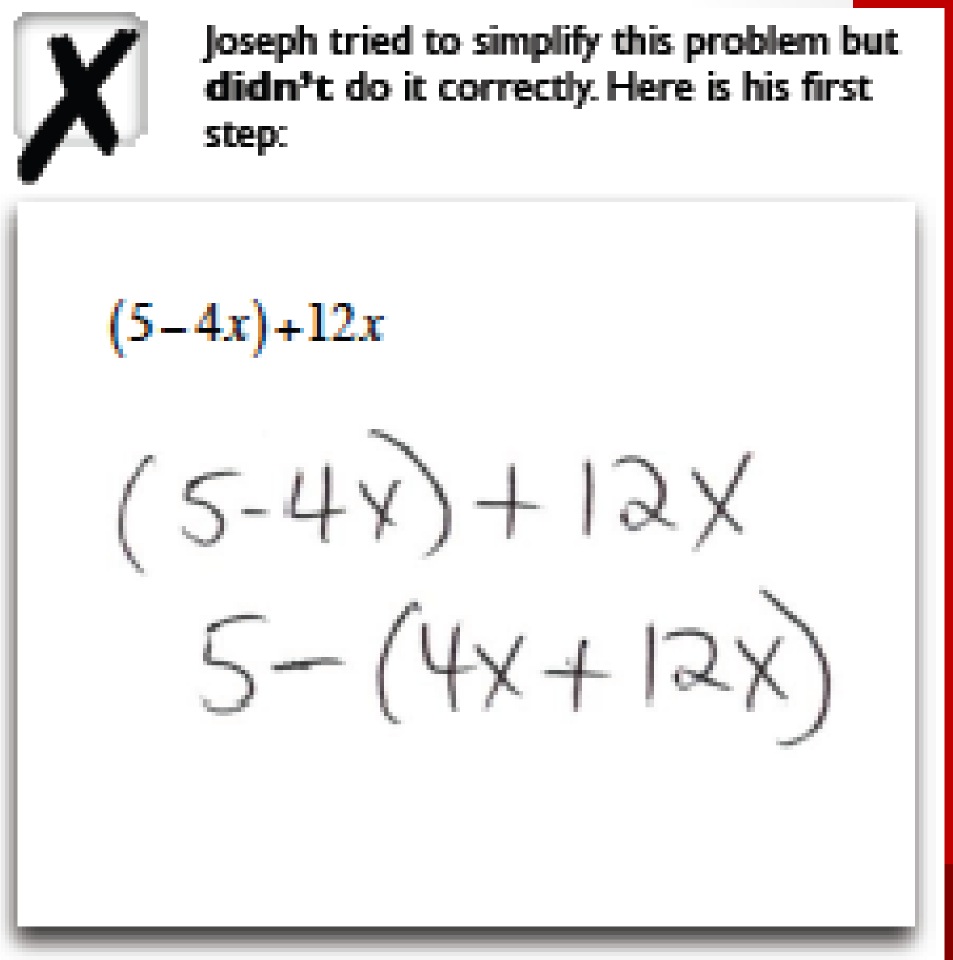
Students do not move negative sign with the term.

Let’s Practice

Step 4: Create the incorrect worked example using that misconception.

A) Clearly mark the problem as incorrect.

B) Use a student name – remember to be gender and culturally diverse.



Let's Practice

- Step 5: Write the self-explanation prompt focusing on the misconception.

Joseph tried to simplify this problem but **didn't** do it correctly. Here is his first step:

$(5-4x)+12x$

$(5-4x)+12x$

$5-(4x+12x)$

Why can't Joseph just move the parentheses in this problem?

What should Joseph have written in order to write the answer in simplest form?

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→**Small Group Activity**

- Discussion

Small Group Activity

Work in small groups to create an incorrect worked-example with a self-explanation prompt.

- Things to remember…
  - ✓Remember to focus on a specific misconception.
  - ✓Ask students to explain their reasoning.
  - ✓Avoid simply asking students what is wrong with the example.

$3 + 6x = 4 - 5x$

- Instructions:

  On the first piece of paper…

  1. Write the objective.
  2. List a few common misconceptions associated with this objective.

  On the second piece of paper…

  1. Choose <u>1 misconception </u>for each example.
  2. Create the incorrect worked example using that misconception.
     - A) Clearly mark the problem as incorrect.
     - B) Use a student name – remember to be gender and culturally diverse.
  3. Write the self-explanation prompt focusing on the misconception.

Our Example

Jackson **didn't** solve this problem correctly.
Here is his first step:

$3 + 6x = 4 - 5x$

$$3 + 6x = 4 - 5x$$
$$9x = 4 - 5x$$

Which terms did Jackson incorrectly combine to get $9x$?

Give an example of two terms that would correctly add to $9x$.

Gallery Walk

- Walk around the room and look at other examples.
- Maybe take a picture to use as an example later!
- Take note of :
  - The range of misconceptions
  - The diversity of self-explanation prompts (even for the same misconception!)

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→**Discussion**

Discussion Questions

- *How you can apply worked examples with self-explanation to your classroom?*
- *What can you see as the benefits for your students?*
- *What are some of the challenges that may arise?*
- *How could you get past those challenges?*

Teacher Perspectives

*The incorrect examples are actually sometimes the ones that **really are better** for showing students. The incorrect examples are often the **best learning tool**. What the AlgebraByExample writers have done is put in [students'] very common mistakes, common misconceptions, and what they've done by putting in these incorrect examples and telling the students this is incorrect [is] force them to say, "**well that's what I do, what should I do then?** If I'm doing the same thing as this boy in this problem, what's wrong with that?*

-Ellen, 8th grade algebra teacher
Ann Arbor Public Schools
Ann Arbor, MI

Teacher Perspectives

*I find that either correct examples or incorrect examples help kids **identify themselves** with somebody else easily. [With misconceptions], kids can be really stubborn, and they really don't believe you that it's wrong. To see a kid look at an incorrect example and say, 'no, but this is correct,' and kind of have that moment of 'oh, I really was off, and **now I understand it more**.' I think they are allowed to engage with that problem more so than if it was just a standard practice problem.*

-Katie, 8th grade algebra teacher
Evanston-Skokie District 65 | Evanston, IL

Teacher Perspectives

*Looking at wrong answers prompts students to **focus their attention** specifically on the feature that's most difficult or that might cause a problem in the future.*

-Kevin, 9th grade algebra teacher
Arlington Public Schools
Arlington, VA

Thank you…

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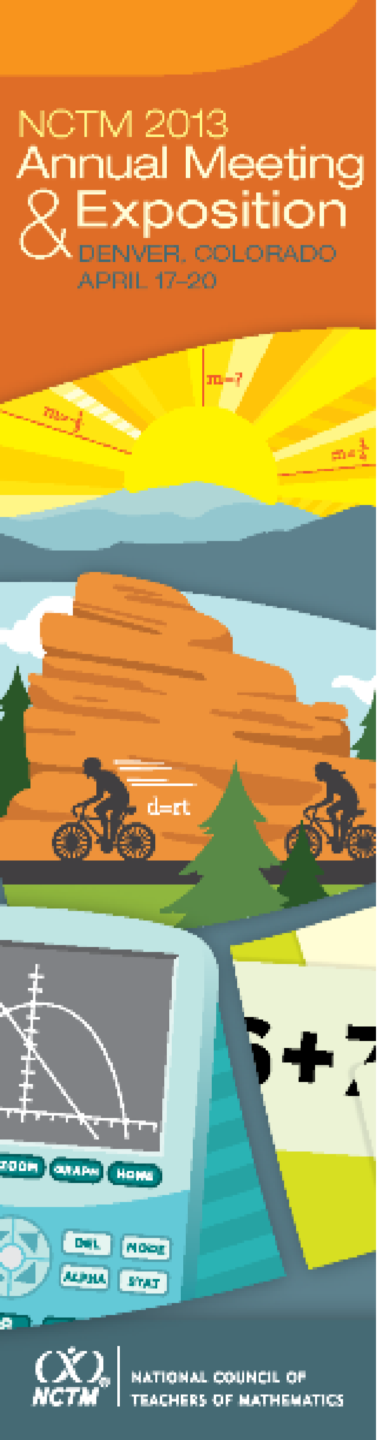

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