EXPLORING THE COMMON CORE PRACTICES IN SECONDARY CLASSROOMS

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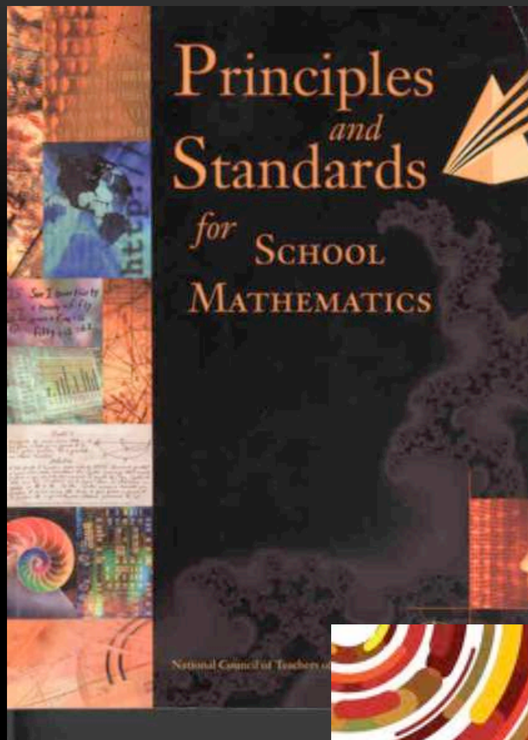
Principles
and
Standards
for School
Mathematics


ADDING
IT
UP


COMMON CORE
STATE STANDARDS INITIATIVE


Connecting the NCTM Process Standards & the CCSSM Practices
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NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

IN THIS SESSION…

- Explore connections between the CCSSM practices and Practice 3 (Constructing & Critiquing Arguments)
- Workshop modifying tasks to increase potential for engagement in Practice 3

REASONING AND PROVING: A UNIFYING FRAMEWORK

- Represented primarily in Practice 3, but relates to the other practices as well.
- A key part of “doing mathematics”

DOING MATHEMATICS TASKS….

- Require complex and nonalgorithmic thinking—not predictable or well-rehearsed approaches.
- Require students to explore and understand mathematical concepts, processes, or relationships.
- Demand self-monitoring or self-regulation of one's own thinking.
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- Require students to analyze the task and actively examine task constraints that may limit possible solutions.
- Require considerable cognitive effort and may involve some level of anxiety because of the unpredictable nature of the solution process.

Smith & Stein (1998)

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REASONING-AND-PROVING TASKS

- Reasoning-and-proving encompasses a wide range of activities involved with mathematical argumentation, from generalizing a mathematical relationship to evaluating a mathematical argument. *(Stylianides, 2009)*

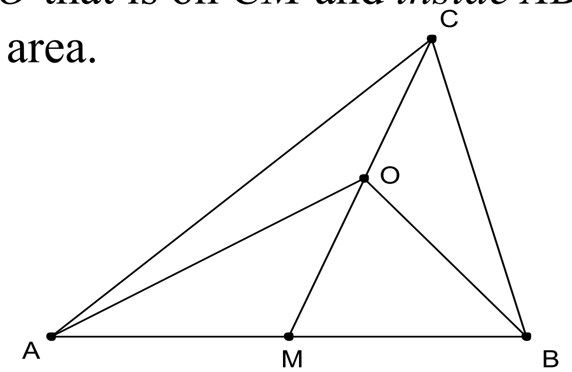
- Recognizing what you know to be true (i.e., assumptions) and clarifying definitions are reasoning-and-proving activities, too.

IMPORTANCE OF DISCUSSING ASSUMPTIONS

Task:

Given: Triangle *ABC*, median *CM*, and a point *O* that is on *CM* and *inside* *ABC*.

Show: Triangles *AOC* and *BOC* have the same area.



Argument:

By the definition of a median, *MB* and *MA* are congruent, so triangles *AMC* and *BMC* have equal bases. *AMC* and *BMC* share the altitude *C*, so they also have equal heights. To find the area of *AMC* and *BMC* we use the formula for area of a triangle: $A = ½bh$. Because the bases are equal and the heights are equal the area will be the same when we plug the base and height into the formula.

Similarly, triangles *AOM* and *BOM* have the same area.

Finally, we know that:

Area(*AOC*) = Area(*ACM*) – Area(*AOM*)
= Area(*BCM*) – Area(*BOM*)
= Area(*BOC*).

MODIFYING TASKS TO BUILD STUDENTS' REASONING-AND-PROVING SKILLS

Some that are obvious…

① Ask students to consider what happens in the *n*th case

② Ask students to generate an argument to justify a conjecture

③ Reduce scaffolding in a problem so that students must select a solution strategy and reason to find an answer

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Some not so obvious…

④ Include instructions for students to identify what prior knowledge or properties they used in solving a problem

⑤ Ask students to define key mathematical objects before they solve problems

⑥ Ask students to model the situation through physical manipulatives, drawings, etc.

Ellis, Bieda & Knuth, 2012

INTEGRATING THE PRACTICES

In a small group (3-4), modify each task to increase the potential that students will engage in reasoning-and-proving processes.

Keep track of which task modifications you use (#1-6), as well as any others.

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INTEGRATING THE PRACTICES

(A) Make sense of problems and persevere in solving them

(B) Reason abstractly and quantitatively

Construct viable arguments and critique the reasoning of others

(C) Model with mathematics

Use appropriate tools strategically

(D) Attend to precision

(E) Look for and make use of structure

Look for and express regularity in repeated reasoning

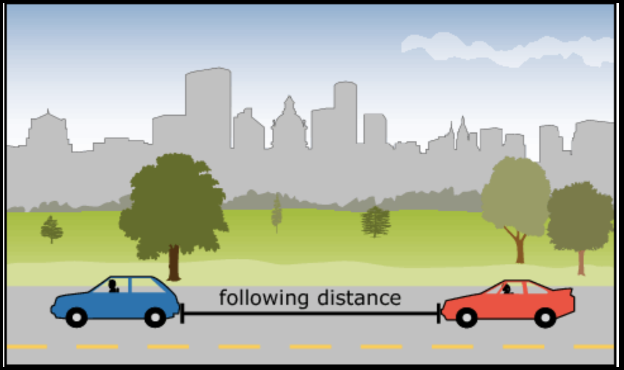
PRIMARY PRACTICE: MAKE SENSE OF PROBLEMS AND PERSEVERE IN SOLVING THEM

Jamal is filling bags with sand. All of the bags are the same size. Each bag must weigh less than 50 pounds. One sand bag weighs 57 pounds and another sand bag weighs 41 pounds.

Explain whether Jamal can pour sand from one bag into the other so that the weight of each bag is less than 50 pounds.

PRIMARY PRACTICE: REASON ABSTRACTLY AND QUANTITATIVELY

The “two-second rule” is used by a driver who wants to maintain a safe following distance at any speed. A driver must count two seconds from when the car in front of him or her passes a fixed point, such as a tree, until the driver passes the same fixed point. Drivers use this rule to determine the minimum distance to follow a car traveling at the same speed. A diagram representing this distance is shown.



As the speed of the cars increases, the minimum following distance also increases. Explain how the “two-second rule” leads to a greater minimum following distance as the speed of the cars increases. As part of your explanation, include the minimum following distances, **in feet**, for cars traveling at 30 miles per hour and 60 miles per hour.

PRIMARY PRACTICE: MODEL WITH MATHEMATICS

A rabbit population can increase at a rapid rate if left unchecked. Assume that 10 rabbits are put in an enclosed wildlife ranch and the rabbit population triples each year for the next 5 years, as shown in the table.

Year	Rabbit population
0	10
1	30
2	90
3	270
4	810
5	2430

Let **y** represent the number of rabbits after **x** years. Drag the tiles to the appropriate slots to build a function rule that could be used to model **y** as a function of **x**, where **x** is a non-negative integer.

$3x$	3^x	x^3	x	0	3	10	30	•	+

$y =$ ☐ ☐ ☐

Submit Answer

PRIMARY PRACTICE: ATTEND TO PRECISION

Shari has been graphing rational functions such as the following:

$$y=\frac{2x^2+1}{5} \qquad y=\frac{-x^2+3x+4}{8x} \qquad y=\frac{x^3-9}{x^2-1}$$

She notices that the second two functions have vertical asymptotes, whereas the first does not. Shari says, “Whenever there’s a variable in the denominator, the function has a vertical asymptote.”

Do you agree or disagree with Shari’s claim?

PRIMARY PRACTICE: LOOK FOR AND MAKE USE OF STRUCTURE

You have a balance scale, and you are trying to weigh 40 packages of meat ranging in weight from 1 kb to 40 kg. You have only four weights with which to work: a 1 kg, 3 kg, 9 kg, and 27 kg. How can you weigh each package of meat with just these four weights?

Driscoll, 1999, p. 75

PRIMARY PRACTICE: LOOK FOR AND EXPRESS REGULARITY IN REPEATED REASONING

Complete the tables below. What do you notice about the 3rd columns?

x	$y = 2x + 5$	Δy
1	7	--
2	9	7 – 9 = 2

x	$y = ax + b$	Δy
1	$a(1) + b$	--
2	$a(2) + b$	

Let $y = ax + b$. Show that if x_0 is increased by 1, the corresponding Δy is constant. What is this constant?

DISCUSSION

Describe your task modification

- How did the modification(s) incorporate reasoning-and-proving?
- How did the modification(s) support other mathematical practices, including the primary practice?

CONNECTIONS AMONG THE PRACTICES

All of the practices are a part of “doing mathematics.”

Students can engage in multiple practices within a single task, and their work doing one practice can support their abilities to do other practices.

PRACTICE 3 IN TERMS OF PRACTICE 1

- Practice 3: “Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments.”

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- Practice 3: "Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments."
- In terms of Practice 1: "Mathematically proficient students analyze givens, constraints, relationships and goals to plan a 'pathway' for justifying their reasoning."

PRACTICE 2 IN TERMS OF PRACTICE 4

- Practice 2: “Mathematically proficient students make sense of quantities and their relationships in problem situations.”

PRACTICE 2 IN TERMS OF PRACTICE 4

- Practice 2: "Mathematically proficient students make sense of quantities and their relationships in problem situations."
- In terms of Practice 4: "Mathematically proficient students identify important quantities in a situation and analyze relationships to draw conclusions."

THANK YOU!

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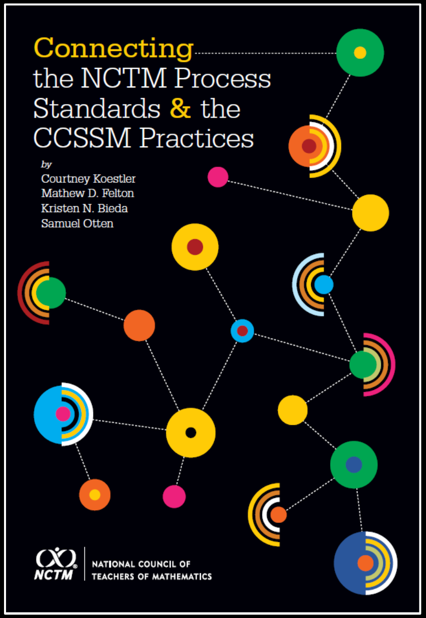

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