Discrete Mathematics Problem Set

(Problems taken from *Mathematics Teaching in the Middle School* unless otherwise noted.)

Problem A

- Adam has won a radio contest that includes a chance to win money. A box is filled with \$100, \$50, \$20, \$10, and \$5 bills. Adam will be blindfolded and allowed to draw bills, one at a time, until he has drawn five bills of the same denomination. What is the least amount of money that Adam can win? What is the most that he can win?

- The least amount that Adam could win is \$25; the most is \$840. If Adam immediately picks five \$5 bills, he wins only \$25. Although very unlikely, he could pick four of each bill, \$5, \$10, \$20, \$50, and \$100, before picking out a fifth \$100 bill to win \$840.

Problem B

- In 1980, a typical telephone number in the United States contained seven digits. Several areas of the country now must use ten-digit telephone numbers. If the entire country follows, exactly how many different ten-digit telephone numbers are available such that the first digit cannot be a 0 or 1 and the fourth digit cannot be a 0?
- 7,200,000,000 different numbers. Since the first digit cannot be a 0 or 1, there are only 8 possible choices. Since the fourth digit cannot be 0, there are only 9 possible choices. There are 10 possible choices for each of the remaining digits. Therefore, $8 \times 10 \times 10 \times 9 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 =$ 7,200,000,000.

Problem C

- Sarah is taking 6 classes at Odyssey Middle School for next year. How many different ways could her classes be assigned? She asked to have her PE class at the end of the day so that she would be ready for after-school sports. She also must have Student Council during 3rd period, since she is the vice president. How many fewer possible schedules does she have, given these conditions?

- 720 different ways; 696 fewer ways. Use the counting principle to find the number of possible assignments of six classes in a six-period day. There are $6 \times 5 \times 4 \times 3 \times 2 \times 1$, or 720, different possibilities (and this does not count different teachers!). However, with restrictions set on the assignment, the number decreases to 24 ($4 \times 3 \times 1 \times 2 \times 1 \times 1 = 24$). The bold 1's in periods 3 and 6 represent the single choice for 3rd-period Student Council and 6th-period PE, neither of which can be changed. So there are 720 – 24, or 696, fewer possible schedules, given these restrictions.

Problem D

- How many different eight-bead necklaces can be made using combinations of only black and white beads? Be careful not to count rotations or flips; for example, wwwbwbbb and wwbwbbbw represent identical necklaces.

- Thirty different necklaces. This problem requires an organized approach. One approach is to consider the numbers of ways to make necklaces with no white beads, with one white bead, and so on.

Original problem

Problem E

- At Jane's ice-cream shop, five different flavors are offered daily. Today's flavors include vanilla, chocolate, strawberry, cookies and creme, and raspberry. Homemade waffle, pretzel, and sugar cones are also available. If Jane orders a double-dip cone, from how many different ice-cream cones can she choose? The order of the dips does not matter.

- A total of 45 double-dip cones. Make an organized list of the fifteen ice-cream combinations (v-v, v-c, v-s, v-cc, v-r, c-c, c-s, c-cc, c-r, s-s, s-cc, s-r, cc-cc, cc-r, r-r). For each ice-cream combination, 3 different cone options exist, making a total of 45 double-dip cone options.

Problem F

- The medals for the singles luge at the Salt Lake City Olympics went to Italy, Austria, and Germany. In how many ways could the gold, silver, and bronze medals be awarded to these three countries?

- Six. This number represents three choices for gold, two for silver, and one for bronze.

Problem G

- Suppose that there are 3 empty seats in a classroom. Two new students join the class. How many different ways can they be assigned to the empty seats? How many ways can they be assigned if there were 4 empty seats? Five empty seats? Try to find a rule that will work for any number of seats and 2 new students.

- With 3 empty seats, there are 6 ways; 4 empty seats, 12 ways; 5 empty seats, 20 ways. To solve, use the rule $n(n - 1)$, where n is the number of empty seats. With 3 empty seats, there are three possible seats for the first student, and then two possible seats left for the second student. For 4 students, we have 4 possible seats for the first student and 3 possible seats for the second student, and so on. Each time, we multiply the number of possible seats times 1 less than the number of seats.

Problem H

Earth. On Alpha, spaceship license plates only use three letters (no number or other characters). Two license plates are considered identical if and only if they contain the same three letters in the same order. How many planet Alpha license plates are possible if the letter A must be followed directly by the letter Z?

- 15,675. If the letter A were not in the alphabet, the number of possible license plates would be $25 \times 25 \times 25$, or 15,625. We must add to this amount the number of plates that include the letter A. If A is the first letter, then the second letter is Z, and any letter other than A can be the third. If A is the second letter, than the third letter must be Z, and the first letter

Problem I

- Suppose 8 large crates of books are to be shipped to a new library. The crates have the following weights in hundreds of pounds: 32, 60, 56, 40, 48, 20, 60, and 64. If the moving trucks have a capacity of 12,000 pounds each, what is the minimum number of trucks needed so that all the books can be sent at the same time? (NCTM Yearbook 1991)

Problem J

- You are in charge of a math team, and have to break students into pairs for a certain competition. How many ways can you split the students into pairs if there are 4 students? What if there are 6 students? Make a conjecture for how many ways there would be to split 2n students into pairs, and see if it works for n = 4.

- How does the problem and the solution change if there are an odd number of students, and one student must be left out?

Problem K

- You are in charge of a sports team and have the students break up into groups of 3 for a practice activity. How many ways can you split 6 students into groups of 3? How many ways can you split 21 students into groups of 3?

Original problem

Problem L

- You want to create a set of dominoes using cardstock (also known as oak tag). Every domino has 0, 1, 2, 3, 4, 5, or 6 pips on each side. A set of dominoes has only one domino of each type. How many dominoes will you need to make?

- Practical context: Accounting for a 1/2” border, a sheet of paper has 7.5” x 10” of usable surface. If you want to fit all the dominoes onto one piece of paper, what's the largest size that each domino be? (Conventionally, a domino is a rectangle with a side length ratio of 1:2)

Problem M

- A group of high school students plays in a punk rock band. The band has 5 songs, and plays them all at every show, but doesn't like to play them in the same order. How many different shows can the band play before they play the set in the same order?

- How does the problem change if they only play 4 songs at each show?

Original problem

Problem N

- The number 2,938 has the following properties:
- it is a four-digit number
- there is an even digit in the thousands place
- there is an odd digit in the hundreds place
- none of the digits repeats
- How many other numbers have this property?

Problem O

- In hopscotch, children go across a path on either one foot or two feet. As you can see, there are only three possible hopscotch paths using 3 squares. How many hopscotch paths can be made using 4 squares? 6 squares? How about n squares? Form a theory, then test to see if it works with 5 squares.

Problem P

- Every telephone in the United States has a 10-digit telephone number, consisting of a 3-digit area code and a 7-digit local number. The first digit of the area code cannot be a 1 or a 0, and the first digit of the local number cannot be 0. How many possible phone numbers are there? How many are eliminated by the convention of disallowing local numbers starting with 555, since such numbers are used for fake phone numbers in film? How many usable numbers are left?

Problem Q

- In Arizona, license plates originally had 3 letters followed by 3 digits. Then, when all those possibilities were used up, new license plates were made, each one consisting of 3 digits followed by 3 letters. Now that those are used up, license plates start with 3 letters and ending with 4 digits.

- Two years ago, the first car I saw with one of these new license plates was AAB4386, and last week, I saw a car with the license plate ALX5349. Assuming that license plates are printed systematically, ie, starting with AAA0000 and incrementing the rightmost place first, how many license plates have been printed in Arizona, ever?

Original problem

Problem R

- How many factors does a prime number have?
- What about the square of a prime?
- The cube of a prime?
- How many factors does a product of two distinct primes have?
- How many factors does 4*3 have?
- How many factors does 17*19 have?
- How many factors does 4*3*4*3*17*19*17*19 have?

Original problem

Problem S

- If you go to France and eat lunch at a fancy restaurant, the waiter will be upset and act extremely condescending if you don't order wine, an appetizer, two courses, and a dessert. The menu at Le Petit Andouillette offers 4 wines, 2 appetizers, 4 main courses, and 3 desserts.
- Wines: Syrah, Zinfandel, Merlot, Chardonnay
- Appetizers: Baguette avec brie, Fougasse avec tomme
- Main Courses: Steak frites, Coq au vin, Boudin blanc, Foie gras
- Dessert: Creme brulee, Eclair, Tartes aux fruits
- How many different dining experiences could you have at this restaurant?

Original problem

Problem T

- It's been said that a pair of deuces ain't much, but you can still win with it. How many ways can you get a pair of deuces, when playing 5-card draw? How many hands will this beat? How many hands are possible in poker?

Original problem