Breaking the Rules: Discrete Mathematics Problems You Can Count On

Presented by:

Eric Welch
welche@email.arizona.edu

Jenifer Martin (indisposed)
jgmartin@email.arizona.edu

Anticipatory Set

How many different ways can you make 25 cents?

Put your pencil down when you think you've found them all.

Be ready to explain your reasoning.

If you've done this problem before,
replace US currency with 1-cent, 2-cent,
and 3-cent coins.

What is Discrete Math?

"Three important areas of discrete mathematics are integrated within the Standards:

combinatorics,

iteration and recursion,

and vertex-edge graphs.

… Combinatorics is the mathematics of systematic counting."

NCTM, 2000

Objectives / Overview

Participants will :

- make sense of discrete mathematics problems and explain connections to CCSS-M standards
- model problems to organize information
- use regularities in repeated reasoning to identify opportunities for iteration and recursion
- generalize the structure of solution strategies to construct algorithms
- invent novel discrete problems to provide another avenue to grade-level CCSS-M content

“There are no right answers to wrong questions.”

- Questions students ask:
  - What’s the point of learning discrete mathematics?
  - How do I know when to use what formula?

“There are no right answers to wrong questions.”

- Questions students ask:
  - What’s the point of learning discrete mathematics?
  - How do I know when to use what formula?
- Questions you might be asking:
  - What’s the purpose of teaching discrete mathematics when it’s not in the Common Core?
  - Why can’t students remember the formulas?

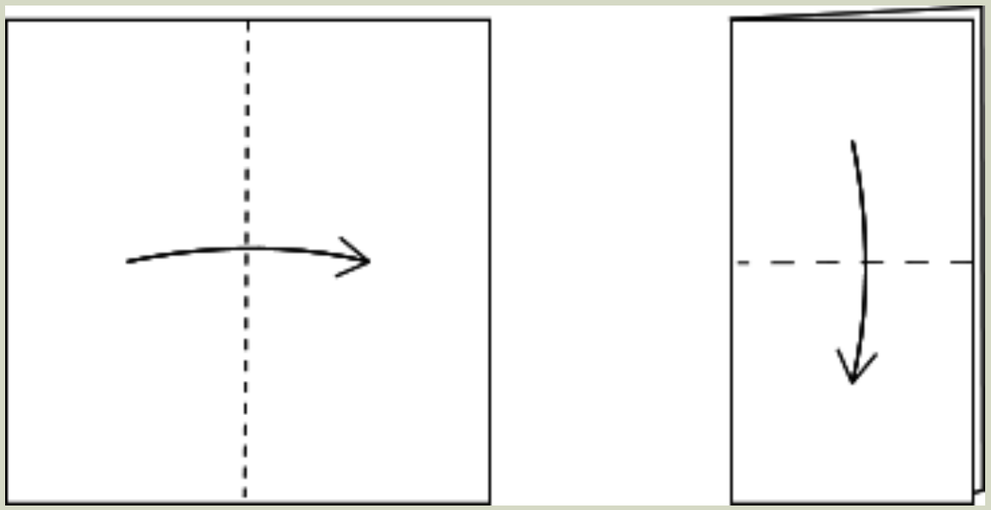
Questions that will drive this session:

- How might existing discrete mathematics curricula help students learn the standards they are required to learn?

- What problem-solving situations can we use to teach discrete mathematics?

A Hands-On Activity for Systematic Listing and Counting

A Paper Folding Activity:



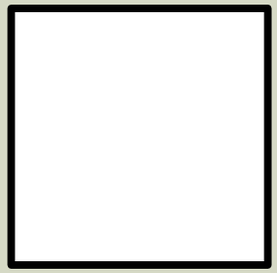
When you unfold this paper, how many squares will there be?

How can you be sure that you've counted them all?

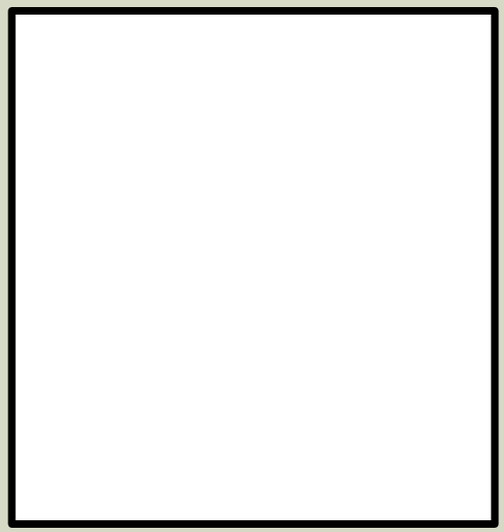
A Hands-On Activity for Systematic Listing and Counting

Organize: What kinds of squares are there?

small squares



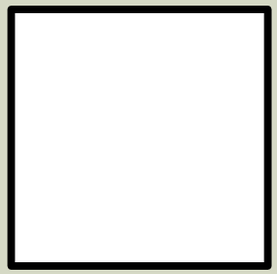
large squares



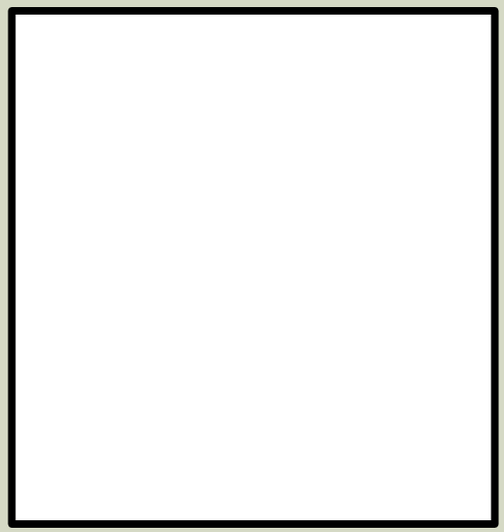
A Hands-On Activity for Systematic Listing and Counting

Iterate: For each kind of square, how many are there?

small squares: 4



large squares: 1



A Hands-On Activity for Systematic Listing and Counting

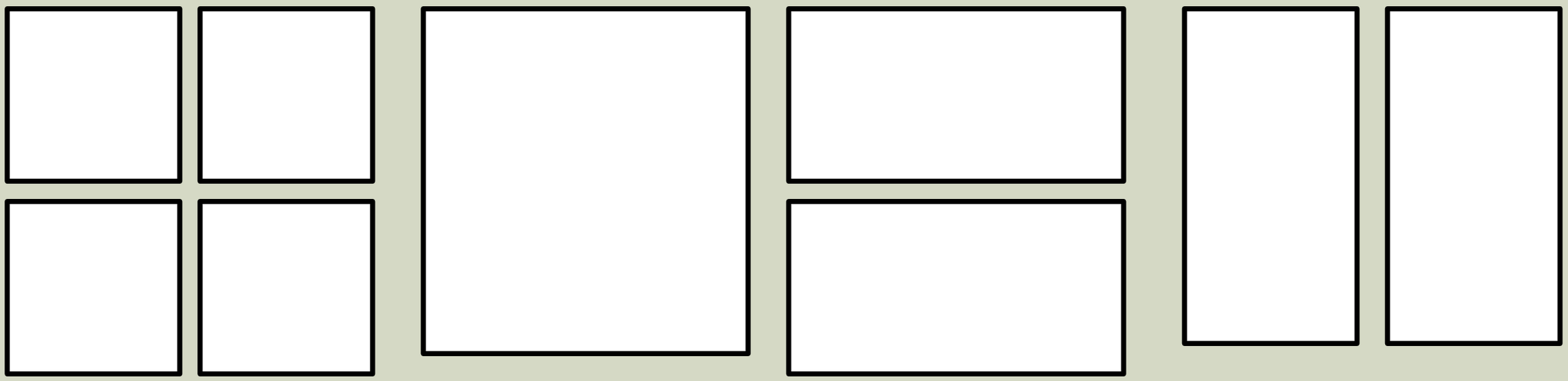
Extension: How many quadrilaterals are there?

How can you be sure that you've counted them all?

A Hands-On Activity for Systematic Listing and Counting

How many quadrilaterals are there?

4 small squares + 1 large square + 4 rectangles = 9 quadrilaterals



But wait!

“Every square is also a rhombus.”

But wait!

“Every square is also a rhombus.”

5 squares

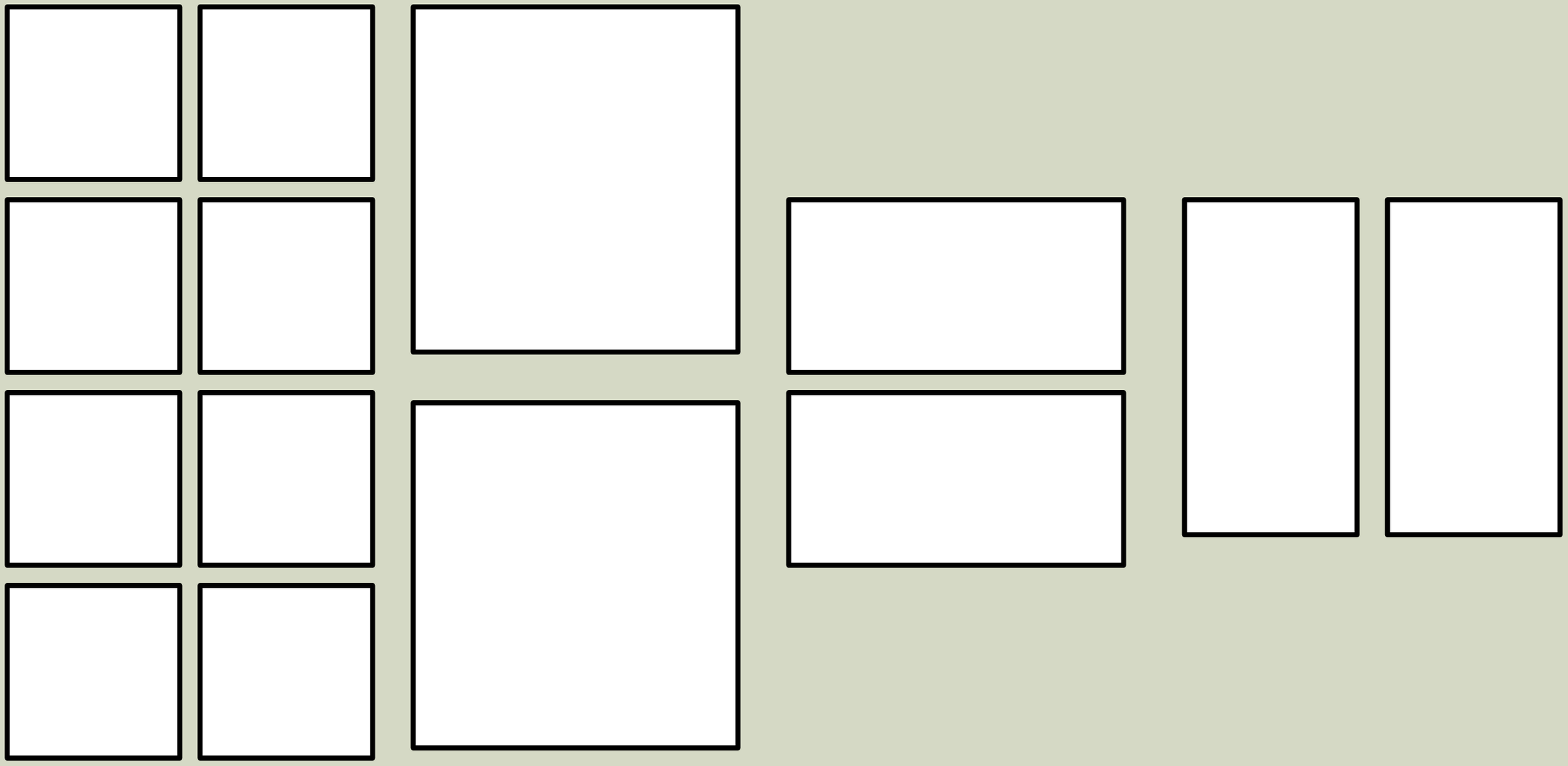
5 rhombuses (rhombi is the Latin plural)

4 rectangles

---

14 quadrilaterals

Watch out for Overcounting...



A General Algorithm:
The Addition Rule of Counting

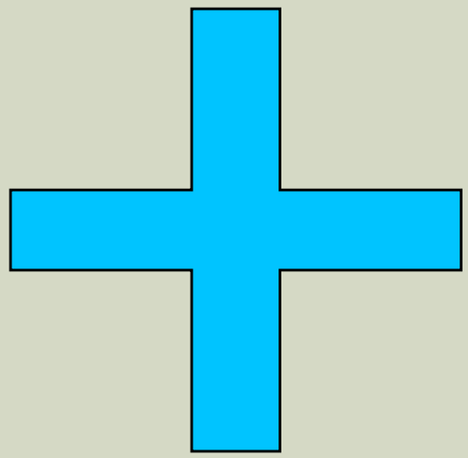
Name all the categories, without overlapping.

Add up how many are in each category.

STEP 1

Small squares
Big squares
Vertical rectangles
Horizontal rectangles

STEP 2



Nonroutine Problems vs Exercises

"Nonroutine problems are problems for which the learner does not immediately know a usable solution method. Nonroutine problems require productive thinking because the learner needs to invent a way to understand the problem."

National Research Council, 2001. *Adding it Up.* (126) National Academy Press. Washington, DC

Reviving Nonroutine Problems

If you quickly come to a solution, **use it to give yourself a new problem**. You might try:

* thinking of how you would explain your solution to a 4th grader

* thinking of how you could extend the problem for an 8th grader

* looking for an algorithm to solve the general case

* ***breaking*** *your algorithm with an added complication*

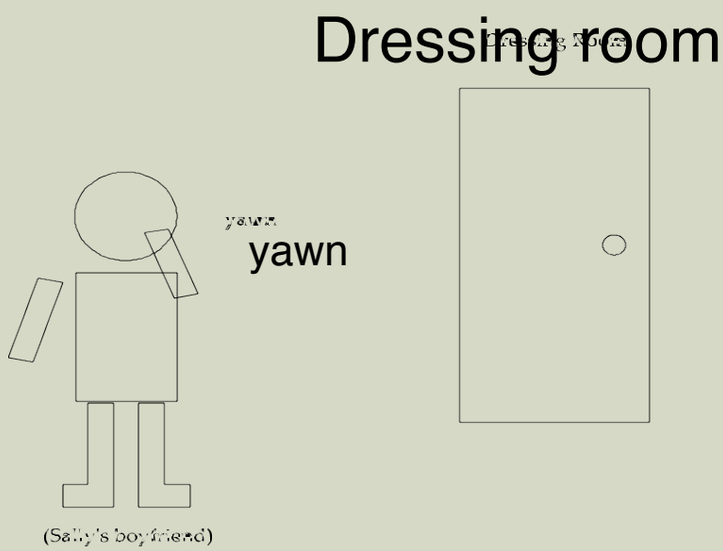
* thinking of how you would justify your work to your college professors

Rodeo Outfits

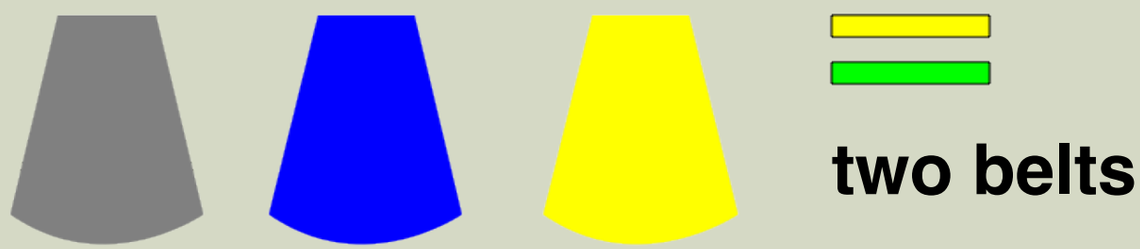
Sally the cowgirl needs to get ready for the rodeo.

In the dressing room, she has 3 dresses (yellow, blue and gray) and 2 belts (green and yellow).

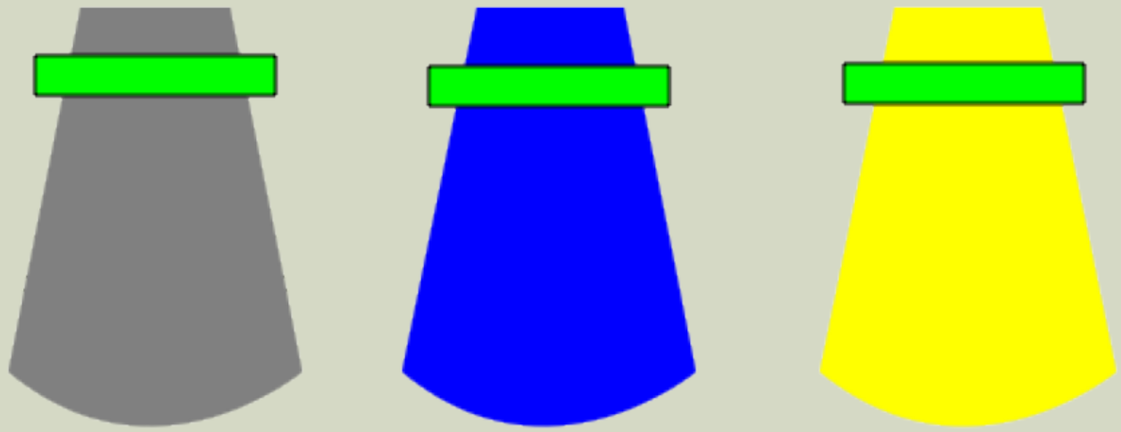
If Sally is to wear one dress and belt, how many different outfits can she create?



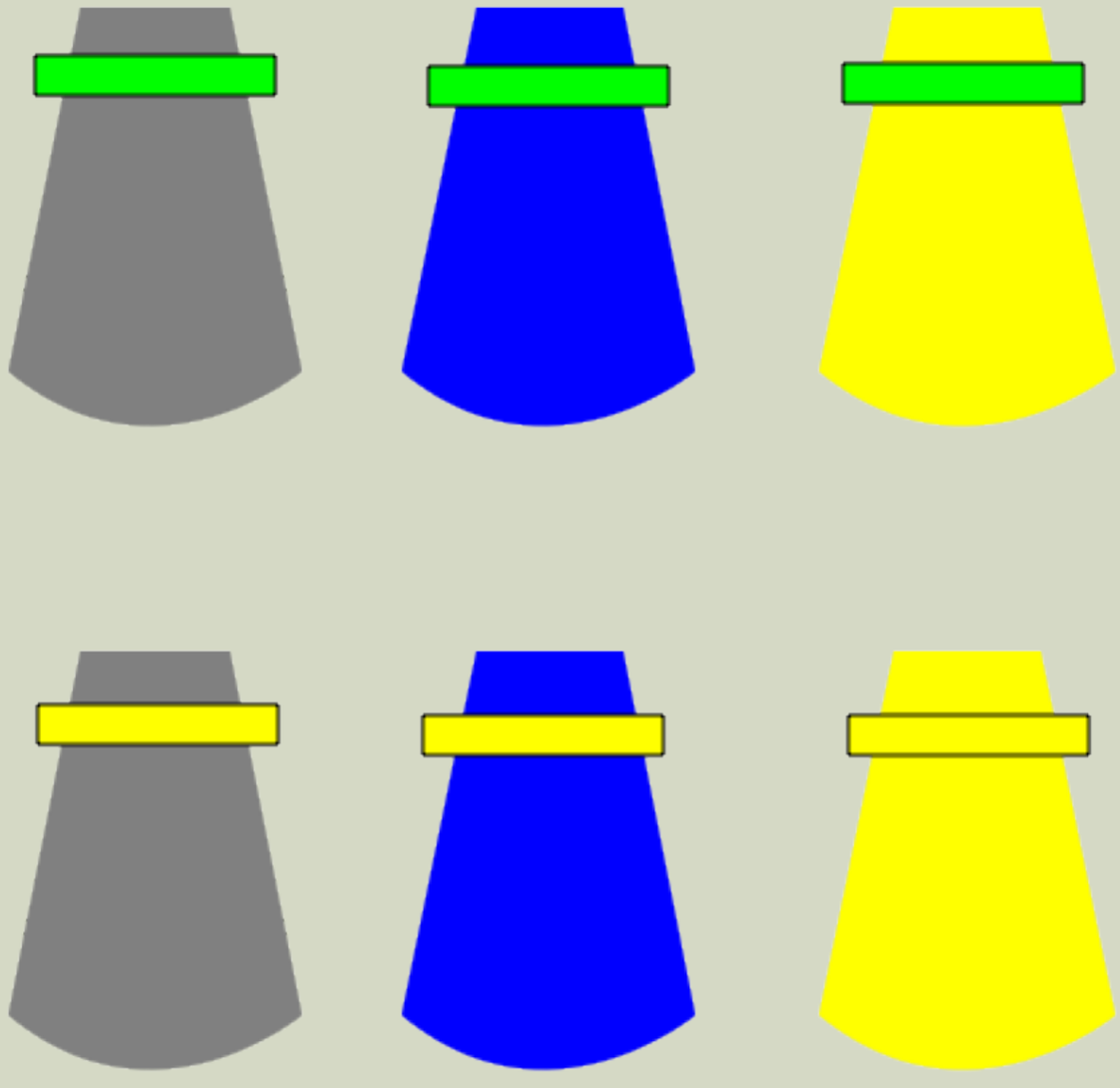
Sally’s boyfriend



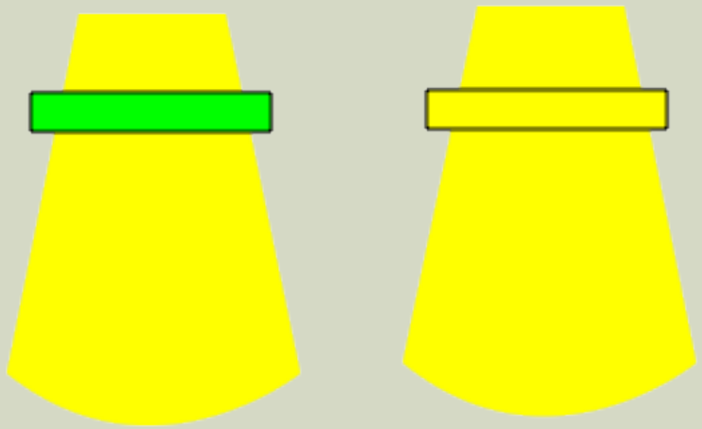
One Way to Organize and Iterate



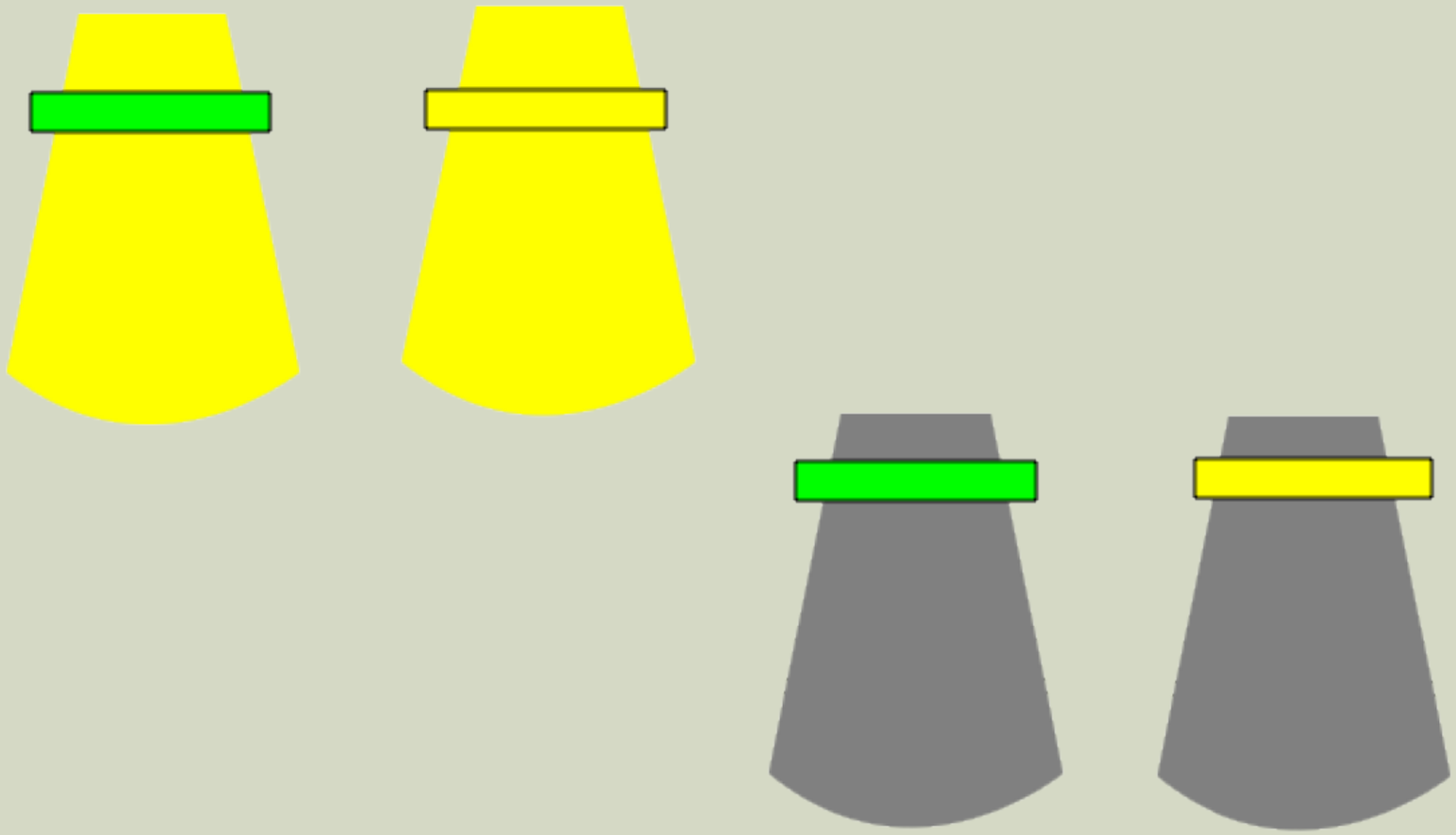
One Way to Organize and Iterate



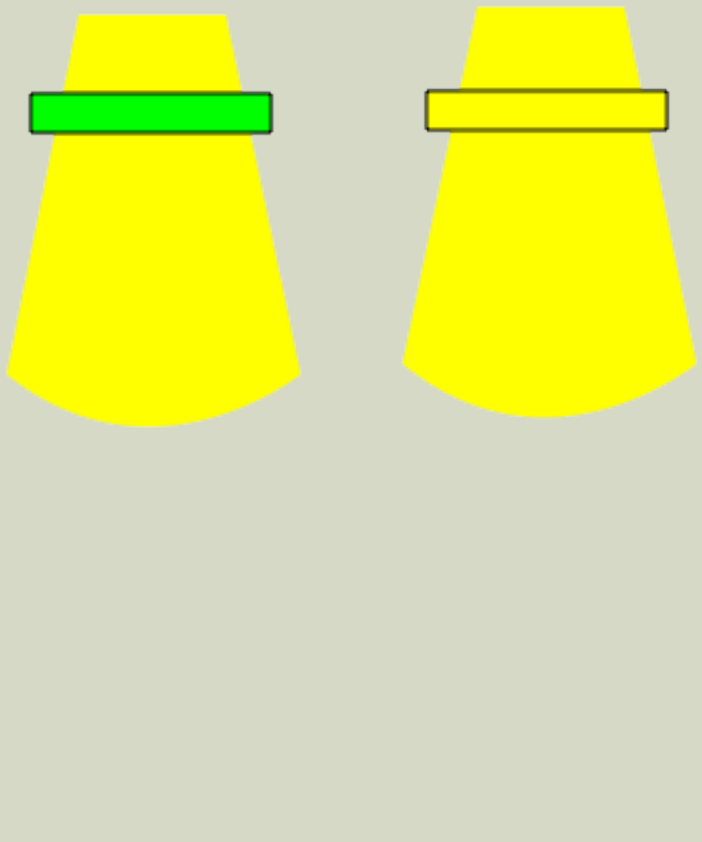
Another Way to Organize and Iterate

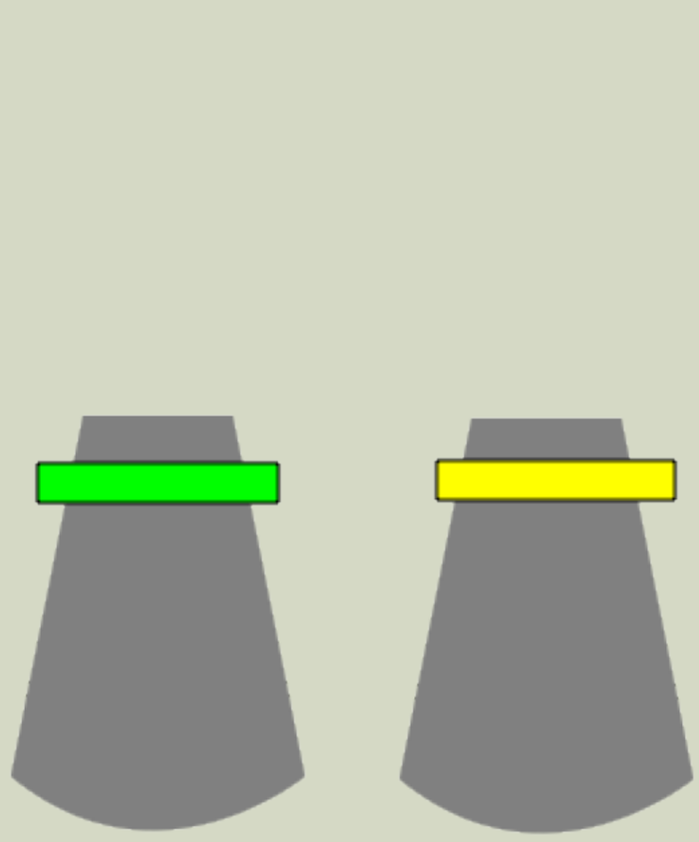


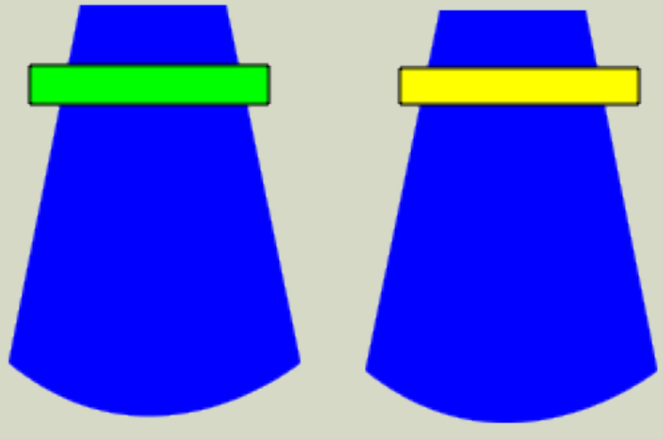
Another Way to Organize and Iterate



Another Way to Organize and Iterate

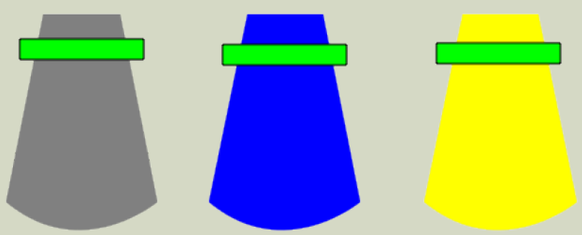




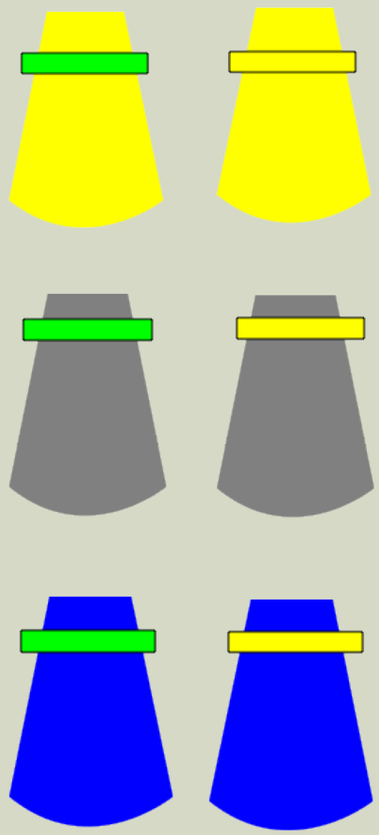


The Multiplication Rule of Counting Reinforces the Commutative Property

3.OA.5 Apply properties of operations as strategies to multiply and divide



Hold the belt constant,
Iterate through the dresses

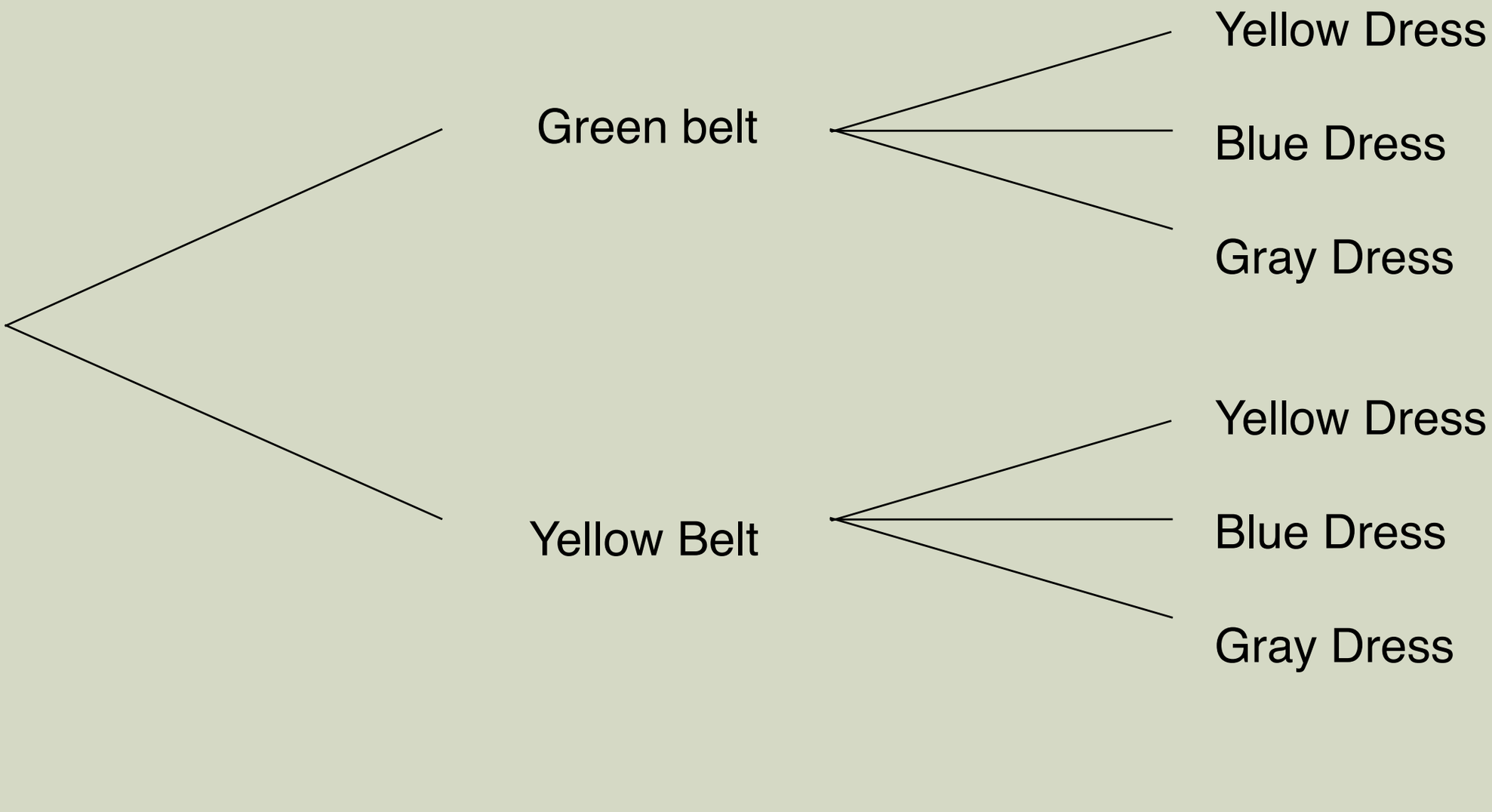


Hold the dress constant,
iterate through the belts

Cartesian Product

	Green Belt	Yellow Belt
Yellow Dress	*Yellow Dress with Green Belt*	*Yellow Dress with Yellow Belt*
Blue Dress	*Blue Dress with Green Belt*	*Blue Dress with Yellow Belt*
Gray Dress	*Gray Dress with Green Belt*	*Gray Dress with Yellow Belt*

Modeling with a Tree Diagram



Choosing a Representation

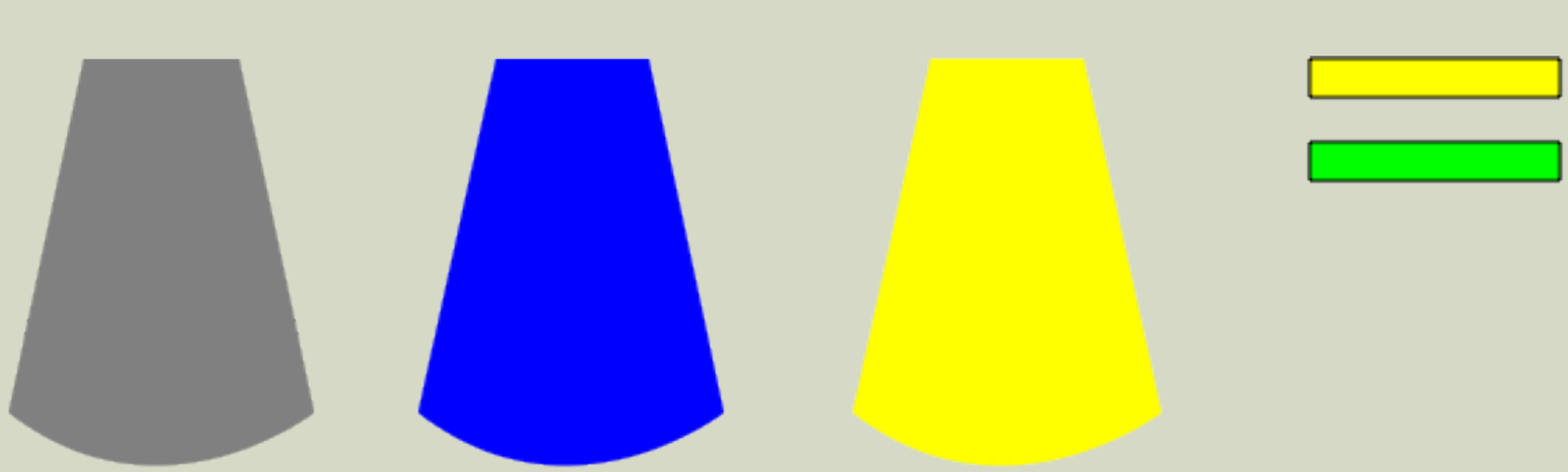
Suppose we extend the problem by giving Sally a choice of two different hats: lavender and pink. How many outfits are possible?

How would you represent your solution to this problem? Why?

Choosing a Representation

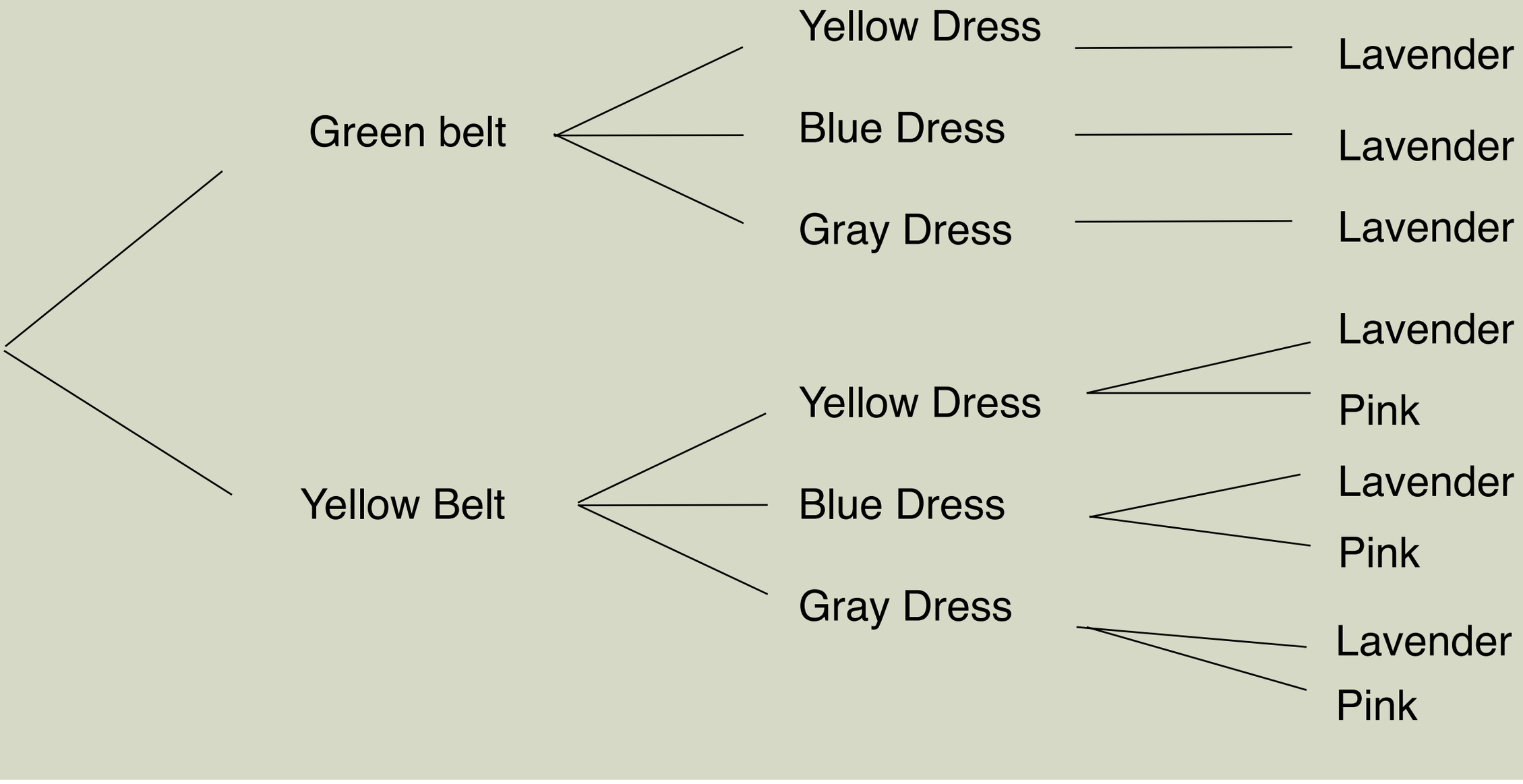
Suppose we extend the problem by giving Sally a choice of two different hats: lavender and pink. How many outfits are possible?

How would you represent your solution to this problem? Why?



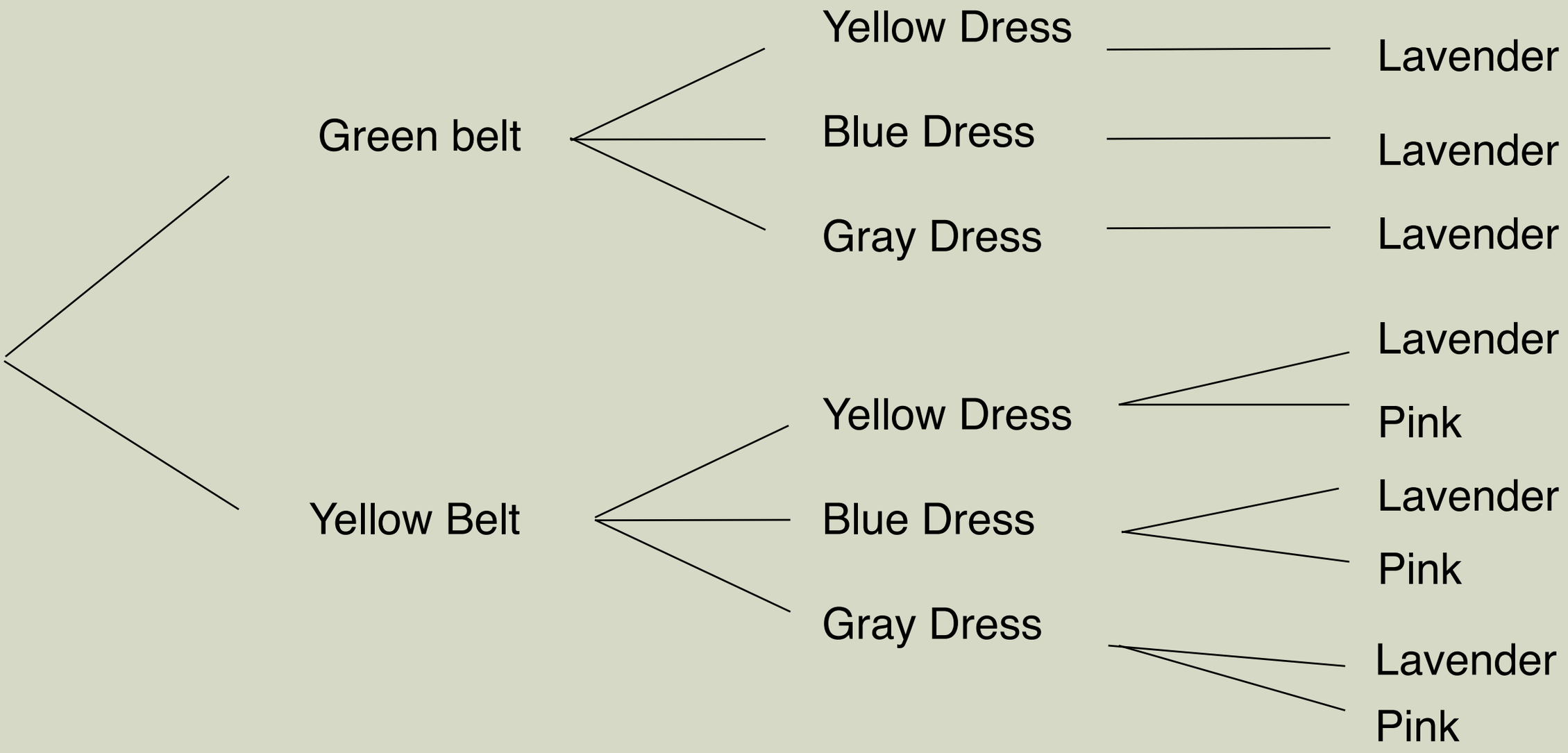
Extension: Obviously, the pink hat just doesn't go with the green belt.
Now how many outfits are possible? Justify your solution!

The Versatility of Tree Diagrams



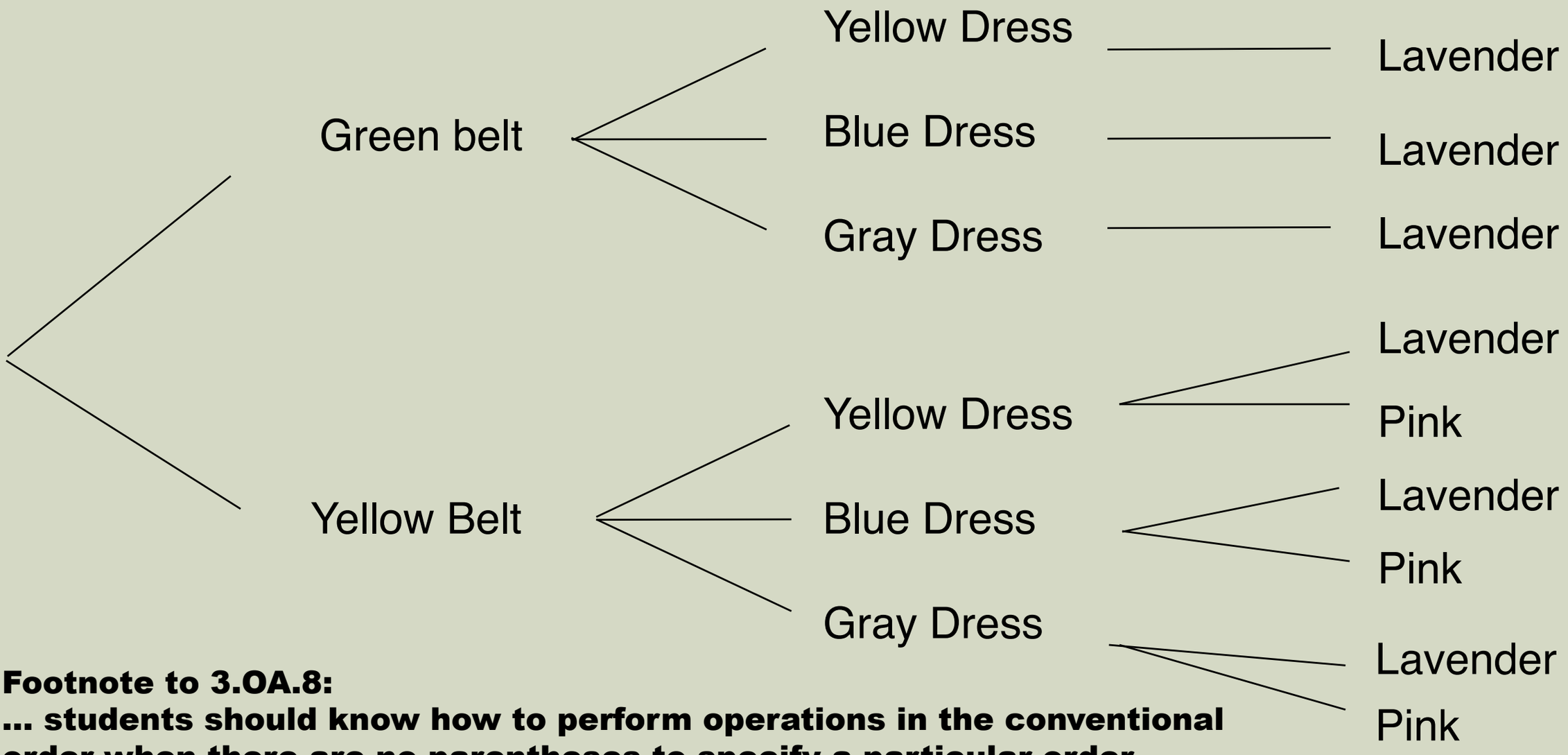
Subtraction Rule of Counting?

(2 belts x 3 dresses x 2 hats) – 3 outfits that clash



Order of Operations
Multiplication is "Stickier"

2 belts x 3 dresses x 2 hats – 3 outfits that clash



Footnote to 3.OA.8:
… students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order

The Very Particular Boss

You have been hired by a real estate firm to design a logo for their new website. Draw a building on each of the four different colored post-it's.

The Very Particular Boss

You have been hired by a real estate firm to design a logo for their new website.
Draw a building on each of the four different colored post-it's.

Your boss likes your buildings but is unsure about the order they are in.
How many ways can you arrange your buildings from left to right?

The Very Particular Boss

You have been hired by a real estate firm to design a logo for their new website.
Draw a building on each of the four different colored post-it's.

Your boss likes your buildings but is unsure about the order they are in.
How many ways can you arrange your buildings from left to right?

Extensions:

1) Your boss wants to simplify. How many ways of choosing just two of your colored buildings are there?

2) How many ways of choosing and arranging two buildings are there?

3) Can you find a general solution to the problem of choosing *n* items out of a group of *r* items, based on your work? Test your solution with a few easy examples.

4) What relationships exist between the problem of choosing objects and the problems of arranging distinct objects? How are these relationship reflected in formulas?

Structures for Counting, Expressions

Recursion

Factorials

4 x 3 x 2 x 1

4 x 3

Dividing out swaths of possibilities
The Division Rule of Counting?

(4 x 3 x 2 x 1) / (2 x 1)

(4 x 3) / 2

6.EE.2b: view one or more parts of an expression as a single entity

Your Late, Eccentric (and Wealthy) Uncle

$ -> cats?

Your Late, Eccentric (and Wealthy) Uncle

How many ways can you rearrange the letters of the following names:

BOB ROB

LISA LILY LULU

Your Late, Eccentric (and Wealthy) Uncle

How many ways can you rearrange the letters of the following names:

BOB ROB

LISA LILY LULU

STACY BOBBY

DEBBIE LILIANA

PERSEVERANCE

Open Space

Try solving some problems! We'll regroup in **n minutes** to discuss them.

Questions to ask:

1. What possibilities are there?
2. How can I organize them?
3. What can I hold constant?
4. How am I iterating through the possibilities?
5. Is there a way to solve any problem of this type?
6. Is there a formula I can construct? Why does it work?
7. How can I connect a challenge like this to grade-level work?

Closure

Think of a standard you are required to teach.

Describe a discrete mathematics problem that would provide an interesting challenge to your students, and help them gain another perspective on that standard.

DISCRETE MATH GAMES

- Everyday games that incorporate discrete mathematical thinking:
  - Yahtzee or Triple Yahtzee
  - Poker

- Online Games
  - http://www.cyberbee.com/games/mastermind.html
  - http://illuminations.nctm.org/ActivityDetail.aspx?ID=3

- Play a “tricky” dice game…
  - Roll two dice.
  - If the product is odd you win
  - If the product is even I win.
  - The odds are 50/50…or are they?