Check back at the NCTM site for additional notes and tasks next week.

PROOF ENOUGH FOR YOU?

General Interest Session
NCTM Annual Meeting and Exposition
April 19, 2013
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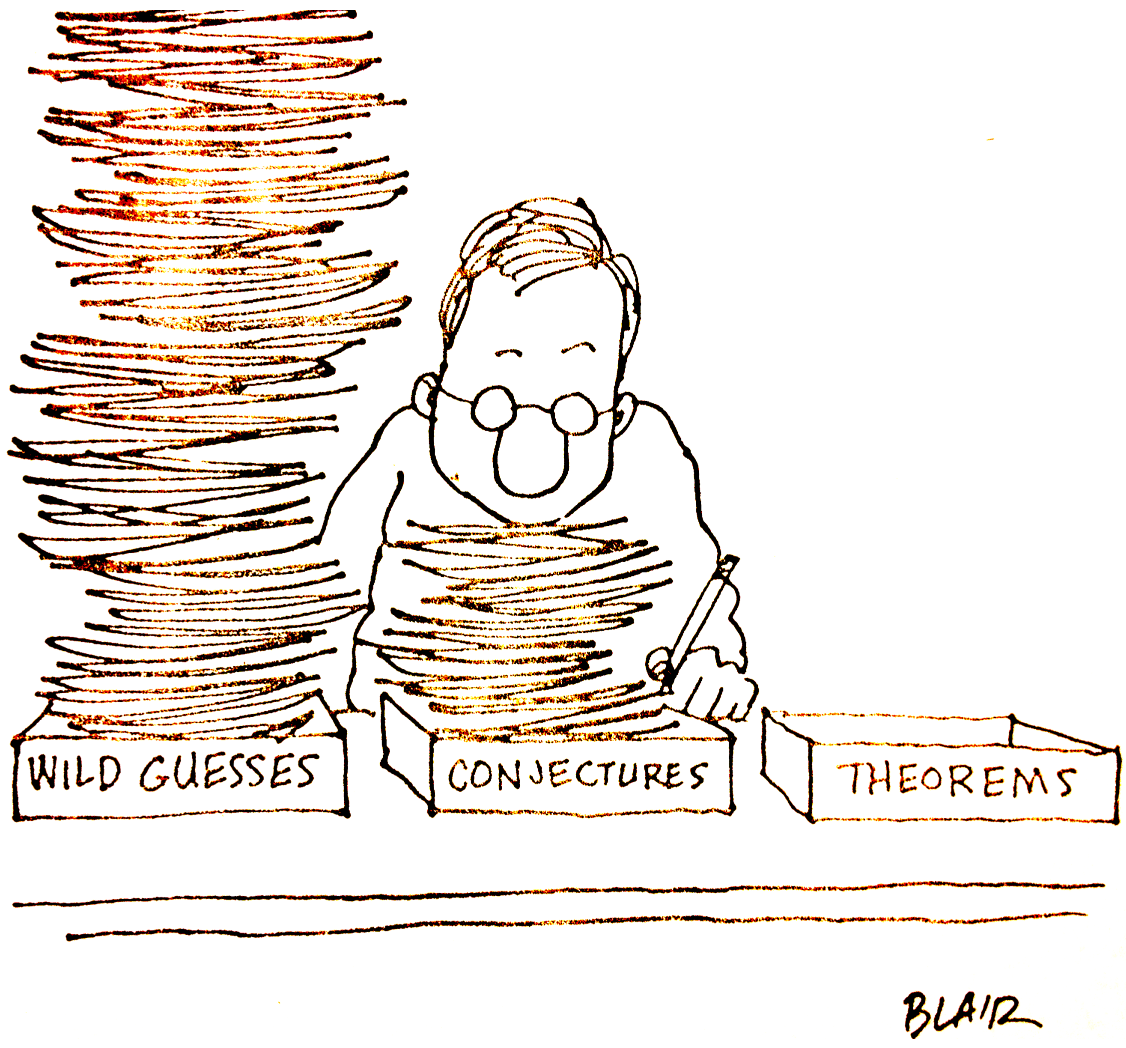
What do mathematicians do?



Suppose n is an odd number. Calculate some values of $n^2 - 1$.

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Strategy: Provide students with contexts where there is opportunity to notice and wonder.

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Strategy: Provide students with contexts where there is opportunity to notice and wonder.

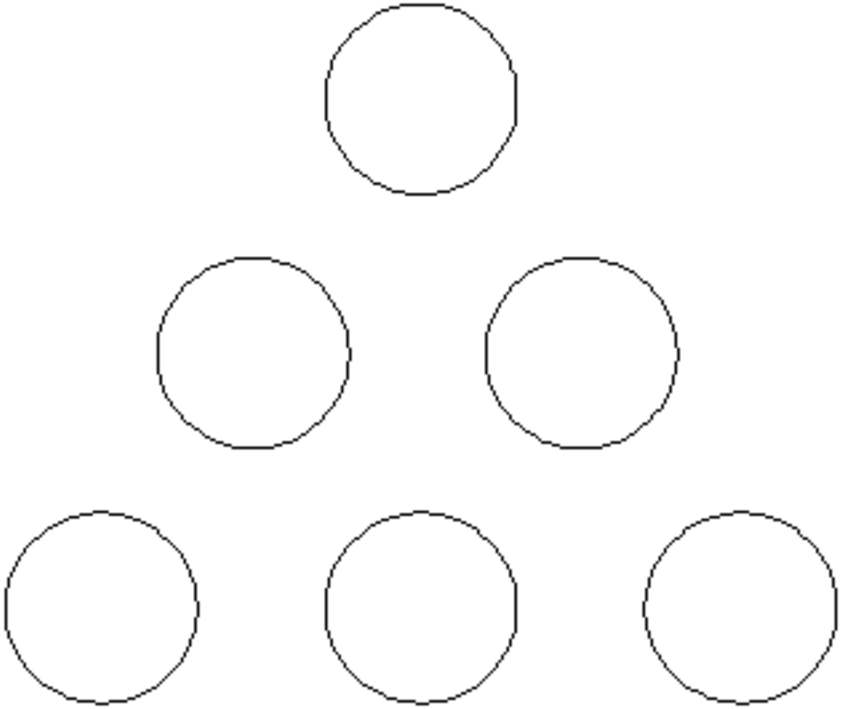
One way is to remove the “question”.

Suppose that you have two numbers, m and n, and that the sum of these two numbers, $m + n$, is an even number.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Think of two whole numbers under 10.
Take one of them and add 1.
Multiply by 5.
Add 1 again.
Double your answer.
Subtract 1.
Add your second number.
Add 2.
Double again.
Subtract 8.
Halve this number and tell me your answer.

Put the numbers 1, 2, 3, 4, 5, 6 in the circles.
Make some observations.



Proofs are constructed for two main purposes: to establish truth and to communicate to others.

Constructing or reading a proof is how we convince ourselves that some statement is true. But I may also have a need to convince someone else.

For both purposes, a proof of a statement must *explain why* that statement is true.

In the first case, convincing myself, it is generally enough that my argument is logically sound and I can follow it later.

Where I have to convince someone else, more is required: the proof must also provide an explanation in a manner that the recipient can understand.

Proofs written to convince others have to succeed communicatively as well as be logically sound.

- Keith Devlin, *Introduction to Mathematical Thinking*

Alone, you can shovel 400 square feet of snow in 60 minutes. With your snow blower, your neighbor can clear the same area in 40 minutes.

Alone, you can shovel 400 square feet of snow in 60 minutes. With your snow blower, your neighbor can clear the same area in 40 minutes.

Strategy: Call it proof.

You can do proof every day.

If $3x + 4y = 12$

and

If $2x + 5y = 14$

What else do you know?

You already do proof every day.

If you believe that $x^4 \bullet x^5 = x^9$,

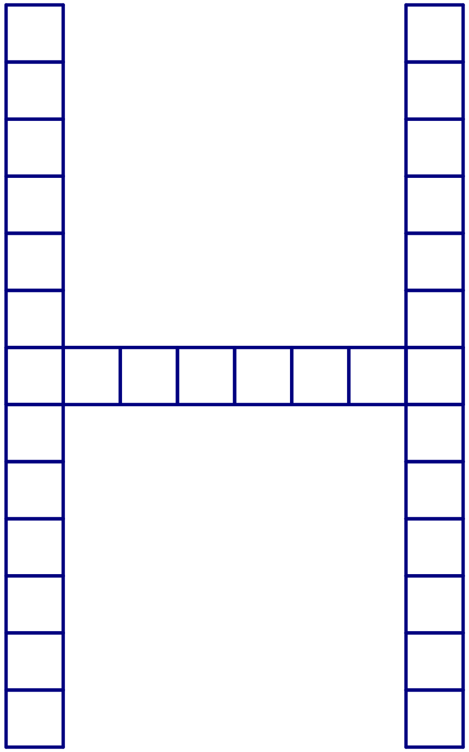
What should we call $x^4 \bullet x^0$?

Or, what should the question mark be in the statement

$$x^4 \bullet x^? = x^4,$$

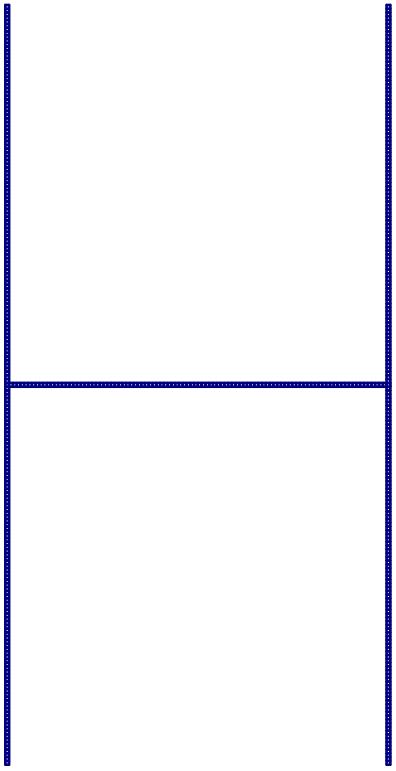
Got something to prove?

S = 6



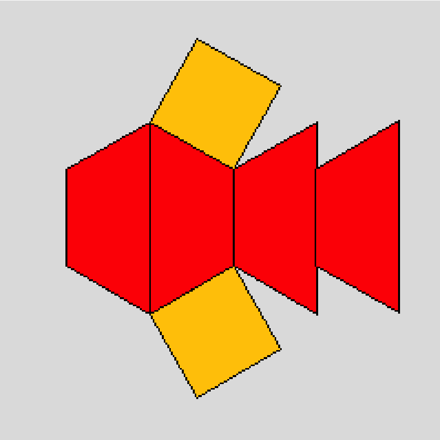
Got something to prove?

S = 100

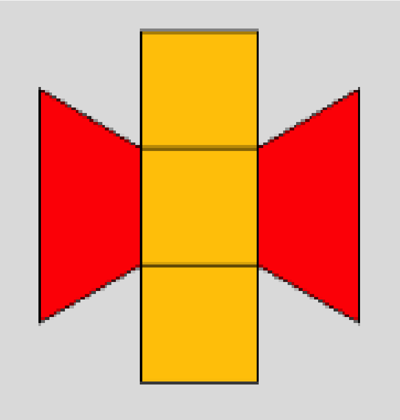


You've got something to prove.

"fish"

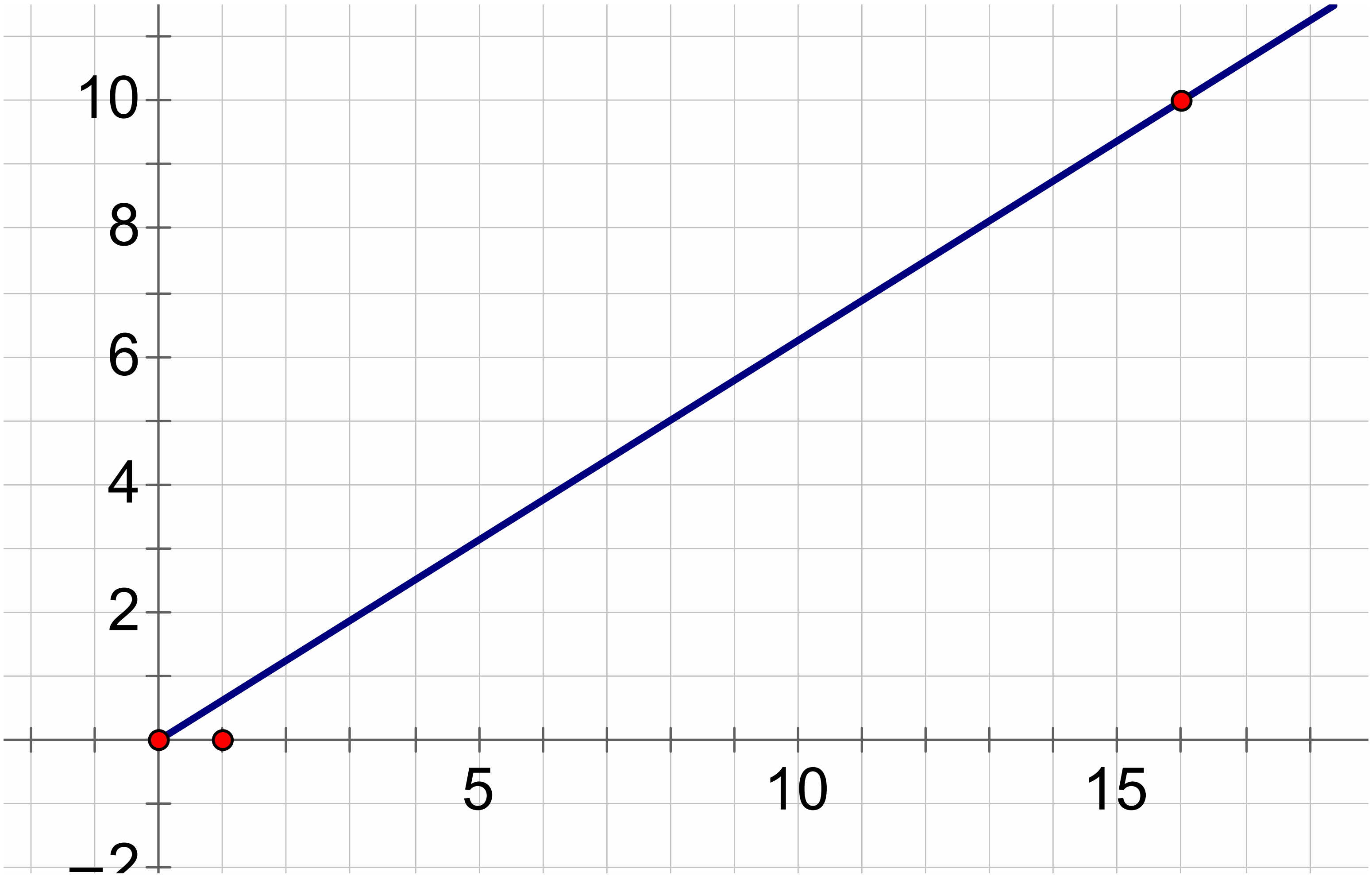


"butterfly"

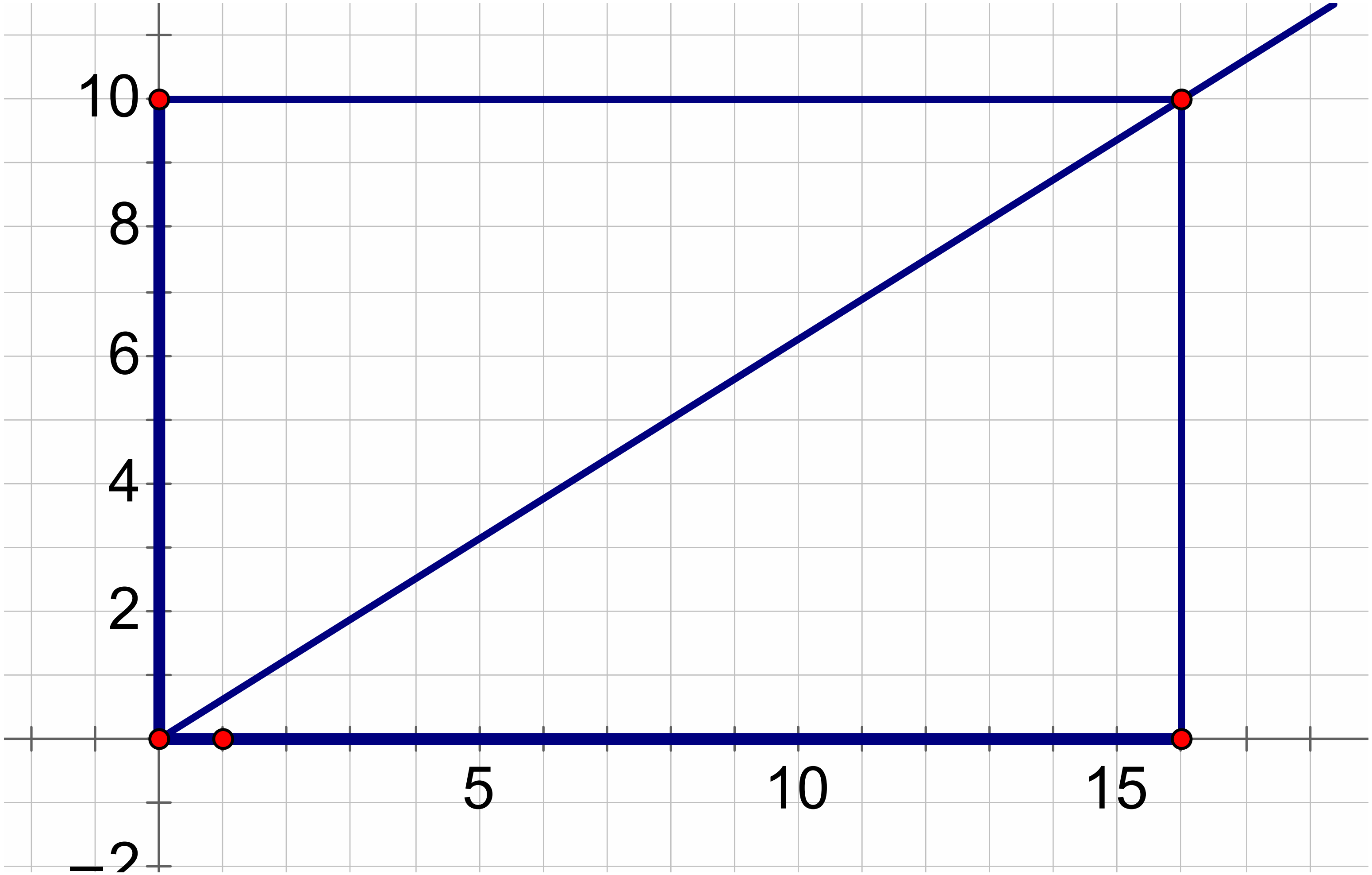


If you have 32 trapezoids available, and 24 squares available, how many of each animal can you make? Is it possible to use all the pieces?

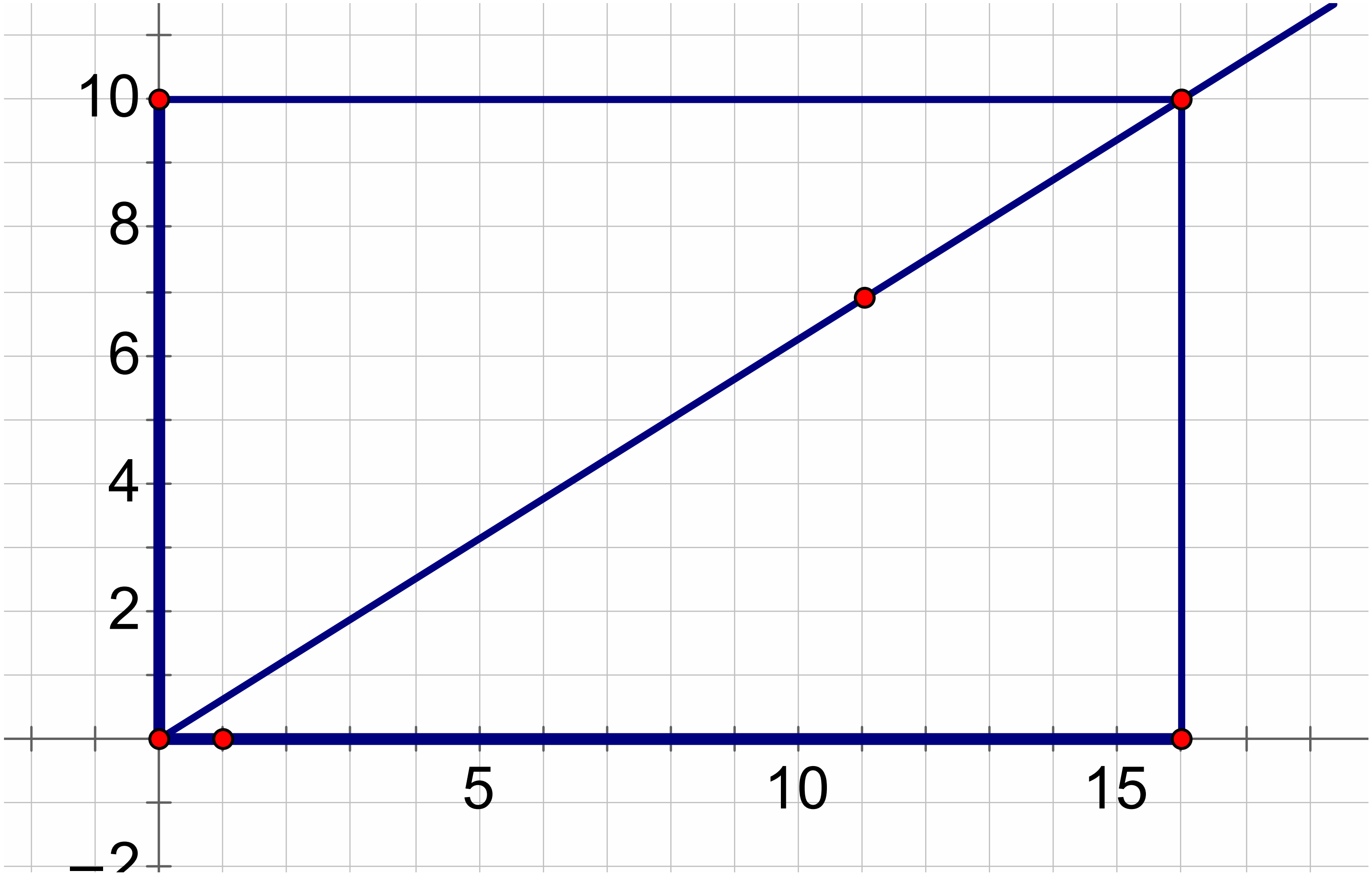
Draw a ray and pick a point on it.



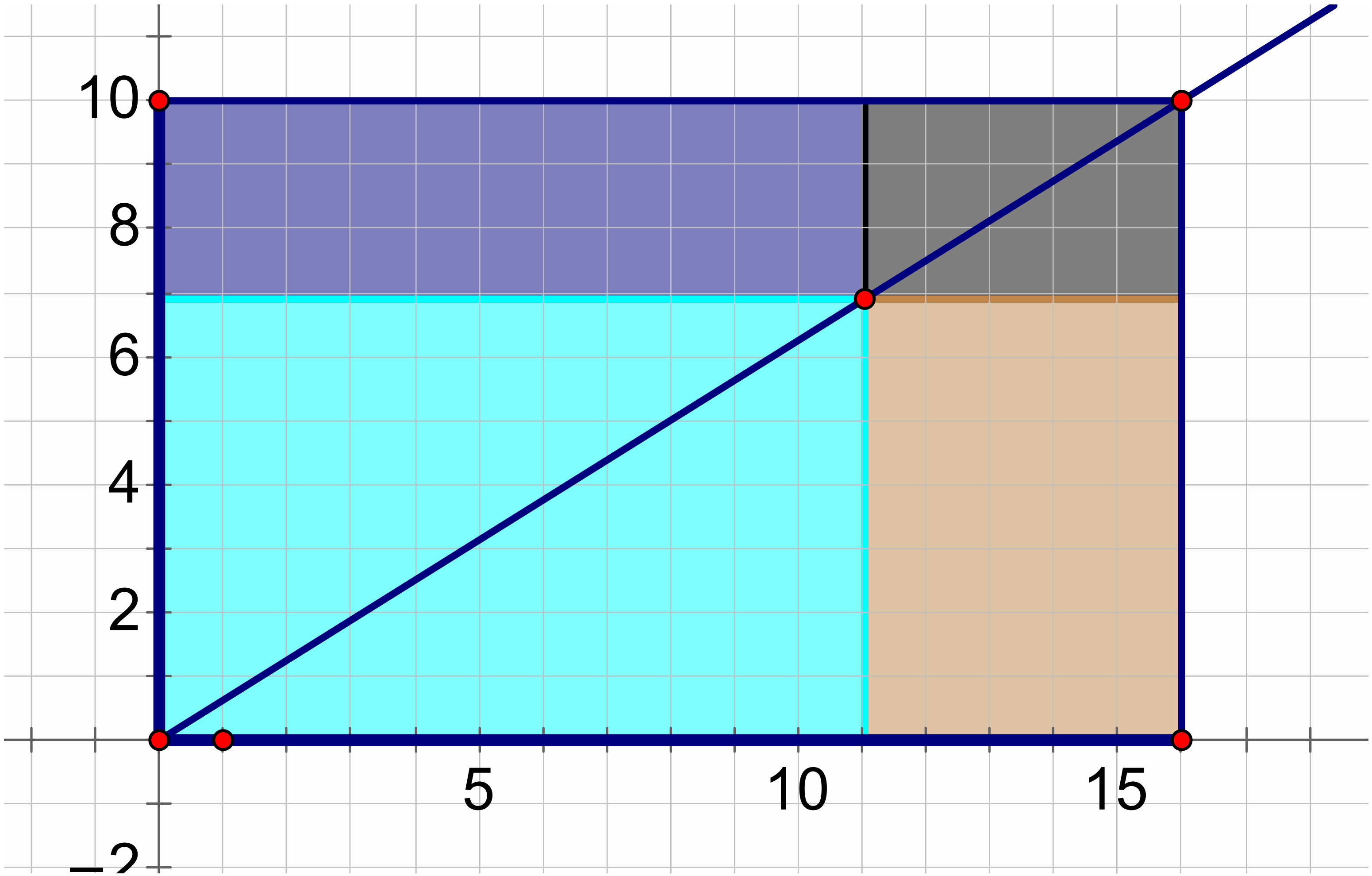
Make a rectangle.



Pick a point on the ray inside the rectangle.



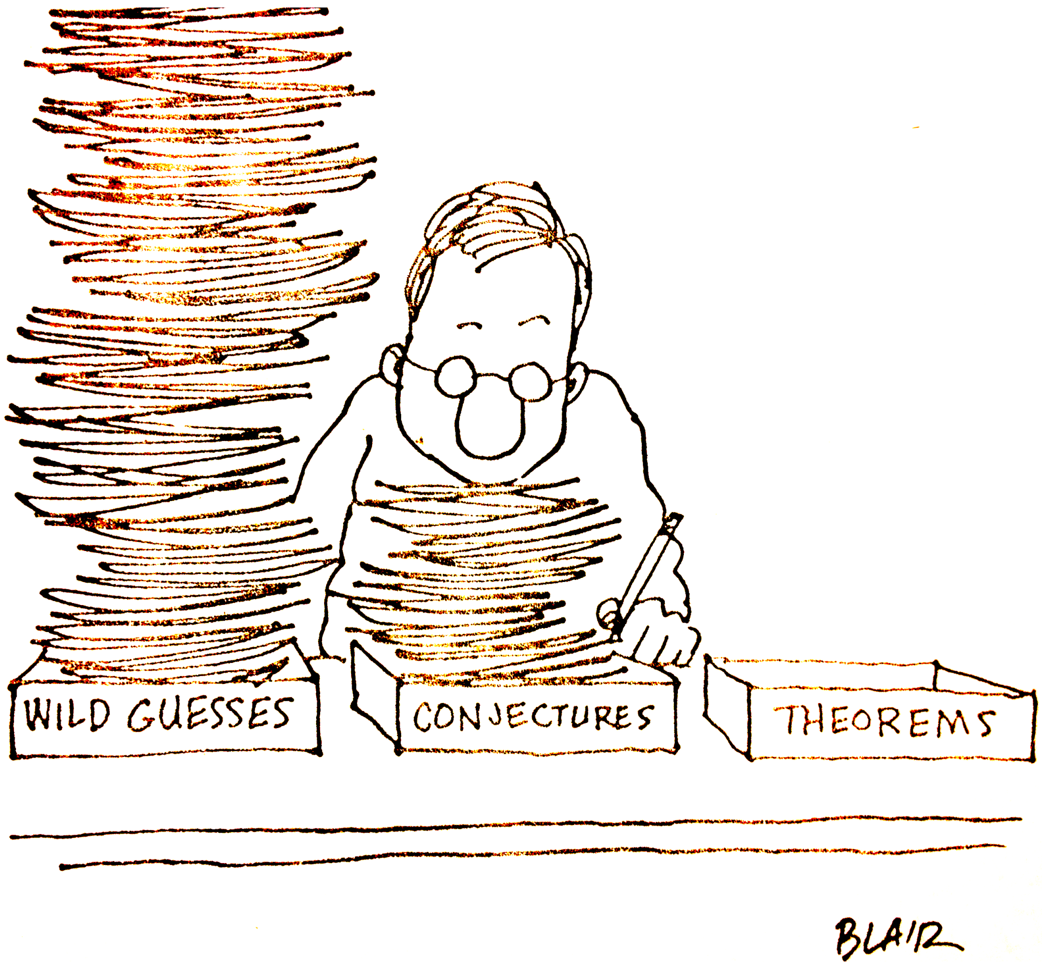
Use it to form 4 rectangles.



Students Want to do Proofs…

- “How did all these algorithms for many different types of math come about? Who discovered or figured them out?”
- “How do different types of math develop?”
- “How were numbers created?”
- “Who decided how math would work?”
- “Why do the angles of a triangle sum up to 180 degrees, and not some other number?”
- “How did people come up with our number system?”

What do we mathematicians do?



http://nrich.maths.org/9078

Be a Mathematician

Mathematicians often know lots of mathematical facts, but, more importantly, they think about maths in different ways. These activities are grouped to help you to practise thinking like a mathematician.

http://nrich.maths.org/9078

Working Systematically

Mathematicians try to work systematically so they can see how they worked something out, and see patterns which messy work might not reveal. Here's a selection of tasks where having good ways to sort and organise can be very helpful.

http://nrich.maths.org/9078

[Saying What You See.](http://nrich.maths.org/9078)

You and your friends are probably quite good at imagining things and seeing things in lots of different ways. Here you'll put that to use in doing some maths challenges.

http://nrich.maths.org/9078

[What Can You Find Out?](http://nrich.maths.org/9078)

Have a go at exploring as you look at these challenges, maybe with others. Talk about how it is going and if a slip-up occurs, then find a way out!

http://nrich.maths.org/9078

[**What If. . . ?**](http://nrich.maths.org/9078)

Here are some exciting activities for you - have a go at them and then see what happens if you change one of the little questions. You may be able to change it more than just once!

http://nrich.maths.org/9078

Convince Me!

Here are some challenges that you can work on and then see if you can convince someone that your solutions are right! Have a go!

http://nrich.maths.org/9078

What's Your Plan?

Sometimes it's not easy to know how to start a problem. Try talking to friend about how to start, and what sort of plan you'll have to carry on.

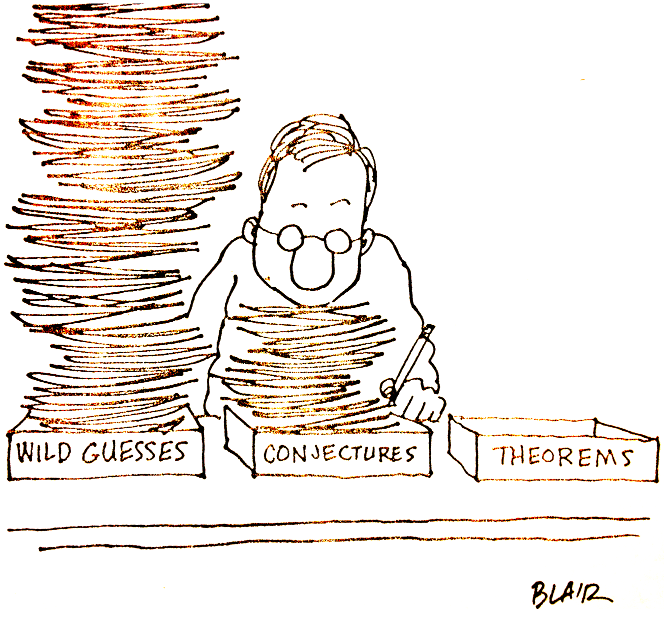
http://nrich.maths.org/9078

Practice Makes Perfect.

These activities make use of things you probably already know and help you to understand them even better for solving problems!

What do we mathematicians do?

Strategy: Talk about math like we talk about other subjects. Name what we do.



Proof: a sequence of statements, each of which either follows from previous statements in the sequence or from agreed axioms.

Proof: a sequence of statements, each of which either follows from previous statements in the sequence or from agreed axioms.

This about as informative as describing a **novel** as a sequence of sentences, each of which either sets up an agreed context or follows credibly from previous sentences.

Both definitions miss the essential point: that both a proof and a novel must tell an interesting story.

Both definitions miss the essential point: that both a proof and a novel must tell an interesting story.

They do capture a secondary point, that the story must be convincing, and they also describe the overall format to be used; but a good story line is the most important feature of all. A mathematical proof is a story about mathematics that works.

-Ian Stewart, *Nature's Numbers*

A Story

Theorem 5-6:
Any two right angles are congruent.

Given:	**Angle (A) is a right angle**
	Angle (B) is a right angle
Prove:	**Angle (A) = Angle (B)**

A Story?

Proof:

1. Angle (A) is a right angle	1. Given
2. Angle (B) is a right angle	2. Given
3. Measure of angle (A) = 90°	3. Definition of a right angle
4. Measure of angle (B) = 90°	4. Definition of a right angle
5. Angle (A) is congruent to Angle (B)	5. Congruent angles are angles that have the same measure

More to this story?

But wait! There's more:

We have forgotten here to include,

3a. For all numbers x), x = x equality;

3b. 90 = 90

3a. Reflexive property of

3b. Substitution rule for universally quantified statements.

Proof is what we do…all the time.

The purpose of proof is understanding. The choice of whether to present a proof as is, to elaborate, or to abbreviate, depends on which is likeliest to increase the students' understanding of concepts, methods, and applications – Rueben Hersh

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Strategy: Tell stories.

Tell me a story…

$$1+2=3$$

$$4+5+6=7+8$$

Tell me a story…

$$1+2=3$$

$$4+5+6=7+8$$

$$9+10+11+12=13+14+15$$

The story never ends…

There is a difficulty in this policy. It depends on the notion of "understanding," which is neither precise nor likely to be made precise. Do we really understand what it means "to understand"? – Rueben Hersh

Keep students in the conversation.

Proof comes from inquiry, conversation, discussion, debate, revision.

In the end, a proof is just a convincing argument, as judged by competent judges – the mathematical community in which we live.

The judge!

Just kidding.

- Relax. There is no committee that is coming to judge if your proofs are “good enough.”

Not kidding.

- However, students will face measures of their understanding.

Try to relax.

- Understanding is always under construction. Aim for it and your students will go farther than you might think.

Get messy.

- Engage students in the definition-making process.
- Polished proofs are the result of lots of collaboration and debate!

Poincaré says you can relax.

Without (intuition), the geometrician would be like a writer well up in grammar but destitute of ideas.

Keep students in the conversation.

What a proof should do for the student is to provide insight into why the theorem is true.

- Hersh

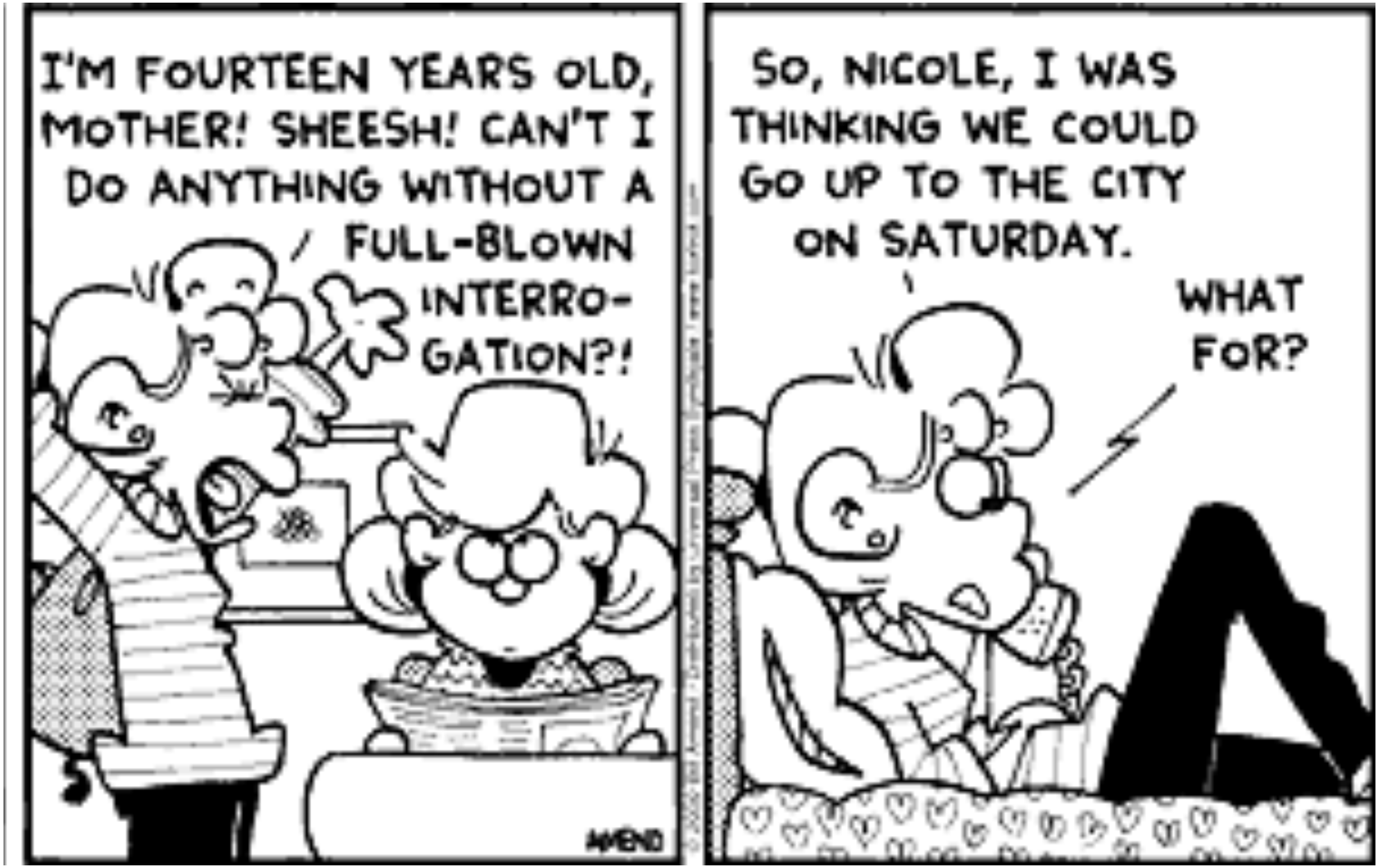
My Theorems

- Let students notice and wonder.
- Don’t be reluctant to use the word proof.
- Use proof for different reasons.
- Use many forms of proof – in a variety of math contexts.
- Discuss the purposes of proof.

My Theorems

- Tell stories – consider every proof to be an attempt to communicate an idea, not simply confirm truth.
- Make the stories hard to put down.
- Proof should flow from students' needs to understand.
- Let students skip to the end of the story sometimes.
- Provide more detail when it serves a need.

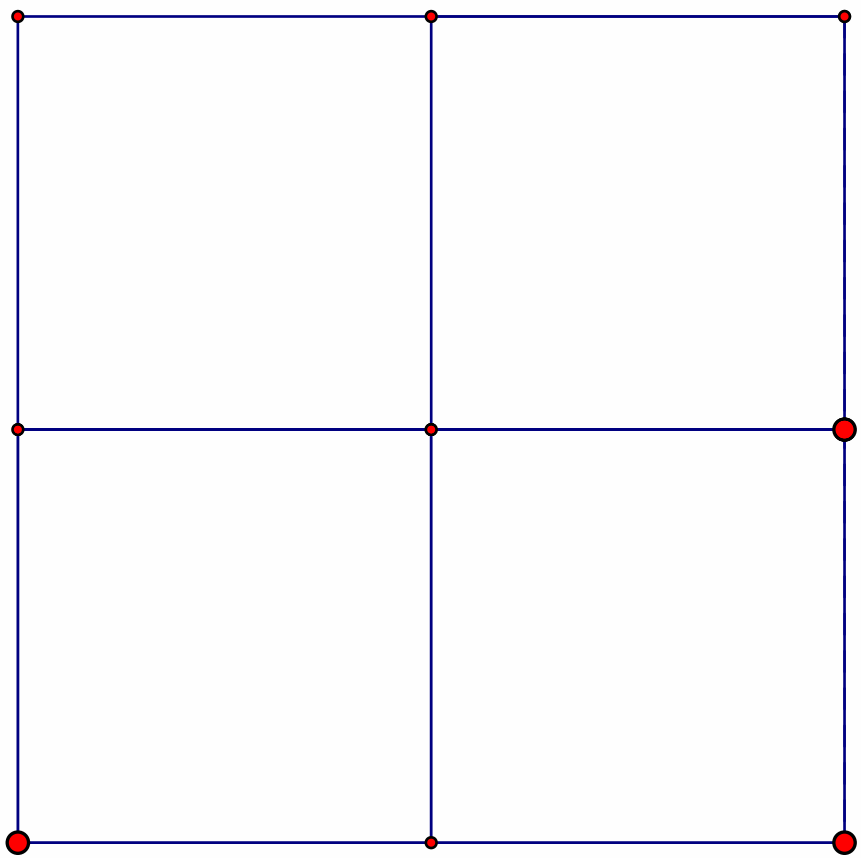

CAN NICOLE AND I GO UP TO THE CITY ON SATURDAY?
WHAT FOR?
WHAT DO YOU MEAN, WHAT FOR?! DO I HAVE TO GIVE YOU A REASON FOR EVERYTHING?!


I'M FOURTEEN YEARS OLD, MOTHER! SHEESH! CAN'T I DO ANYTHING WITHOUT A FULL-BLOWN INTERRO-GATION?!
AMEND
SO, NICOLE, I WAS THINKING WE COULD GO UP TO THE CITY ON SATURDAY.
WHAT FOR?

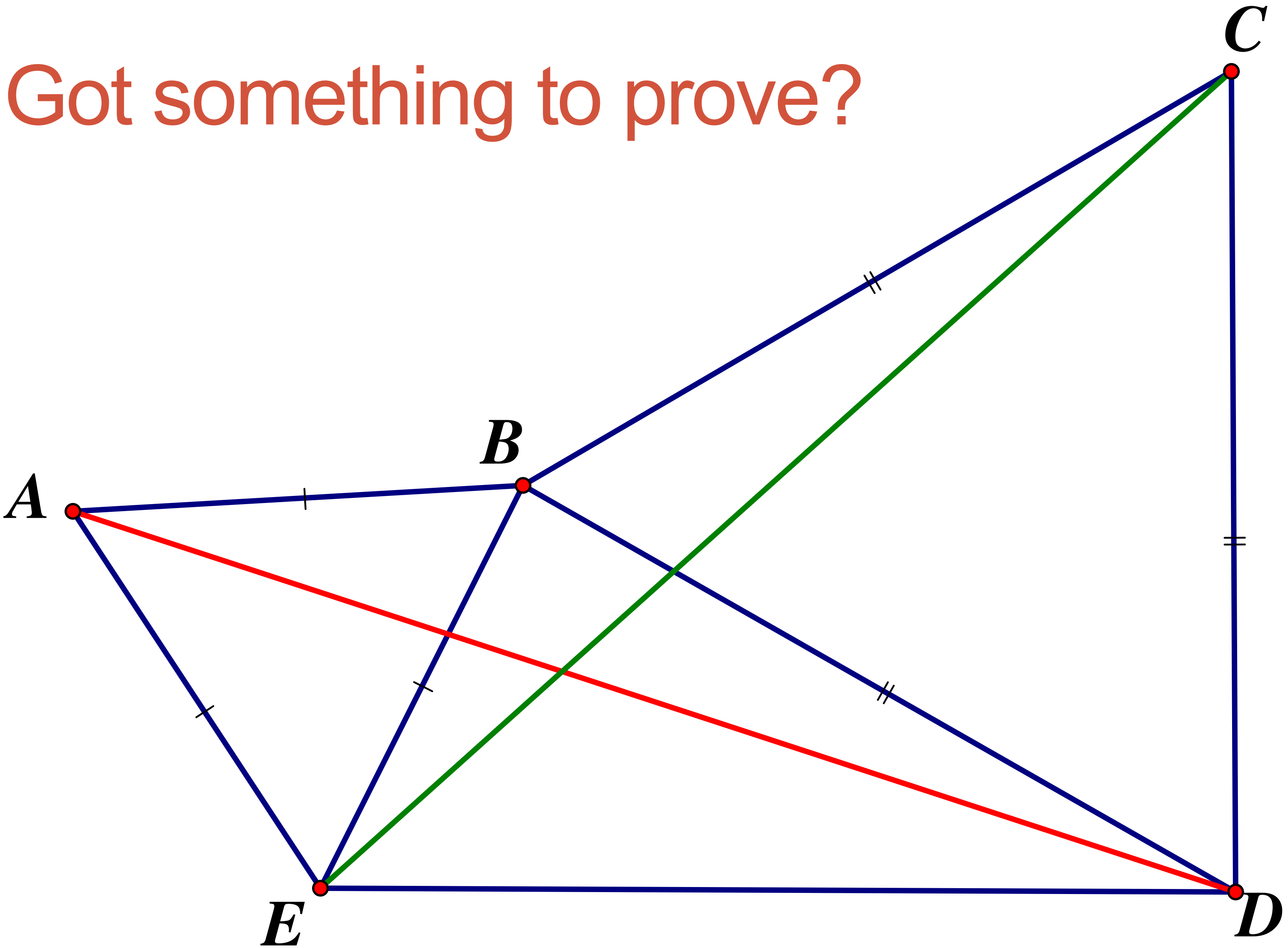
Corollaries

- Formal proof is only truly appreciated when a student appreciates a need for it.
- Appreciation takes time – time spent having experiences with questions that seem, to the student, to require explanation.
- It is not really a proof if you do not understand it, no matter how well established the result!

Got something to prove?



Got something to prove?



http://www2.edc.org/makingmath/handbook/teacher/proof/proof.asp

n people are standing in the plane and the distances between all pairs of individuals are distinct (no two alike). Each person is armed with a cream pie that they hurl at their nearest neighbor. Everyone's throw is accurate. If the number of pie throwers is odd,…

Got something to prove?

$$27 - 19 = ?$$

$$28 - 20 = ?$$

Got something to prove?

$$6-\left({}^{-}5\right)=?$$

$$7-\left({}^{-}4\right)=?$$

$$8-\left({}^{-}3\right)=?$$

…

$$11-(0)=?$$

Got something to prove?

$$6-\left({}^{-}5\right)=?$$

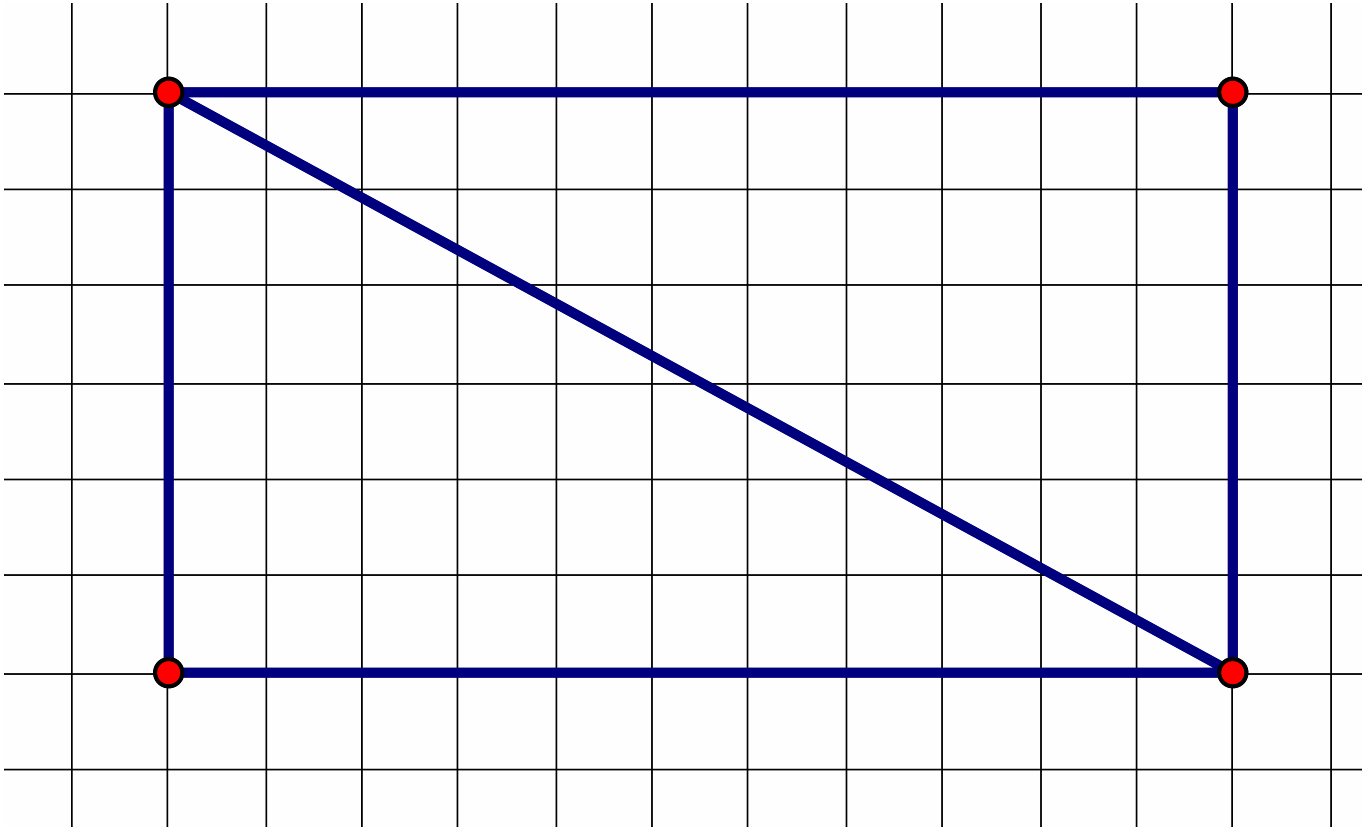
$$6+0-\left({}^{-}5\right)=?$$

$$6+5+\left({}^{-}5\right)-\left({}^{-}5\right)=?$$

$$6+5+\left({}^{-}5\right)-\left({}^{-}5\right)=?$$

$$6+5+0=?$$

Got something to prove?



Got something to prove?

$^{-}5+4+8+7$ $4+9$

Got something to prove?

P points are to be joined in pairs by straight line segments but they may not cross each other. What is the maximum number of line segments there can be?

Got something to prove?

Is there a number with exactly 12 divisors?

Got something to prove?

How long must a mirror be for you to see all of your body? Do you see more of your body as you move back from the mirror?

If you know that 77×77 is 5929, what is 77*78? What is 76*77? How about 78*78?

Got something to prove?

Why is the formula for the area of a trapezoid

$$\frac{1}{2}h(b_1 + b_2)$$

I've seen 6 different proofs…

Got something to prove?

A frog climbs up the side of a well and slides back while resting. Every minute the frog leaps upward 5 feet (and it leaps forward precisely at the end of the minute). Then it rests for a minute. During the rest, the frog slips back 3 feet. At the end of the minute it leaps upward (5 feet), then it slides back (3 feet), and so on.

Got something to prove?

Are there any fractions that get larger when one is subtracted from the numerator and two is subtracted from the denominator?

I claim that if the average of four integers is 94, then at least one of the integers must be greater than or equal to 97.

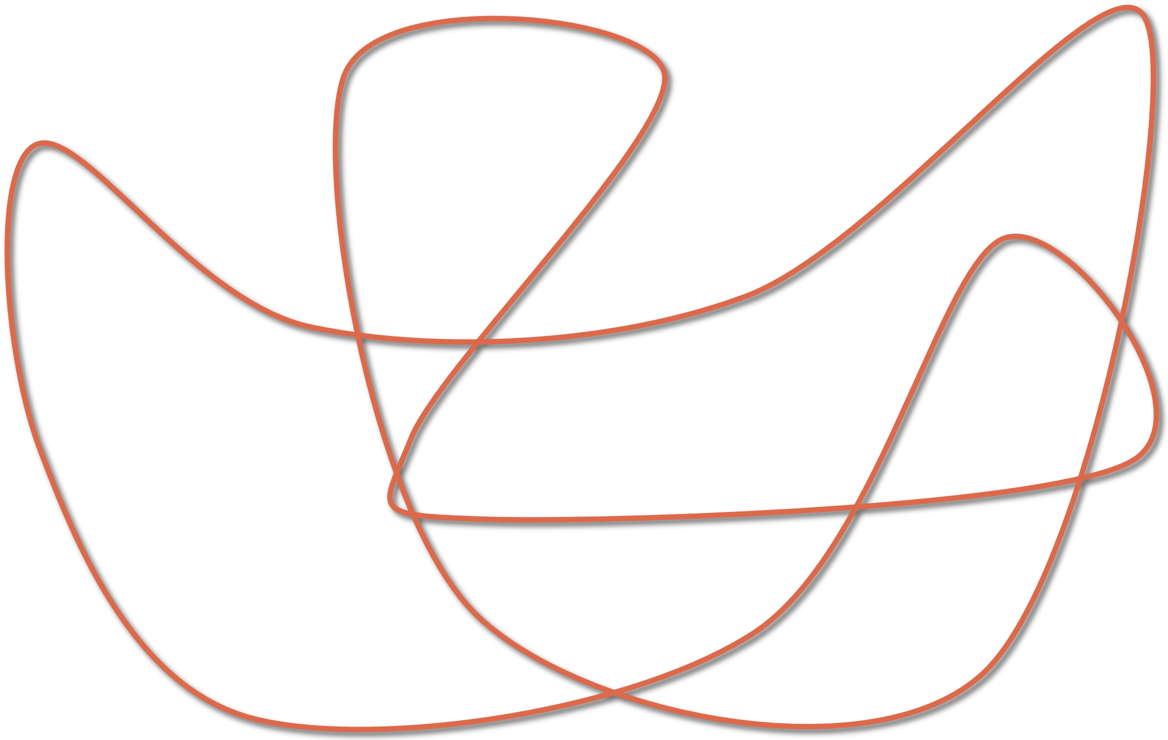
Got something to prove?

If you know what 4 * 2 is, what else do you know?

If you know that 37 times 42 is 1554, what do you think 3.7 * 4.2 is? Why? Prove it.

If 50 is 40% of some larger number, what might that number be?

Got something to prove?



Got something to prove?

Potatoes have skin, I have skin, therefore for I am a potato.

- If a figure is equilateral, then it is equiangular.
- If a figure is equiangular, then it is equilateral.

Got something to prove?

Call two students to the front of the classroom and show them and the class one $10 and two $1 bills. Ask the pair to close their eyes and to raise a hand over and behind their head where it cannot be seen (by the student herself). Place a $1 bill in each raised hand and put the $10 bill away. Before instructing them to open their eyes, explain to them the situation (not the part about which bill they were given) and ask them to tell the class when they know whether they are holding a $1 or $10 bill. When they open their eyes, each is able to see the other's bill but not their own.

Got something to prove?

When the logician's little son refused again to eat his vegetables for dinner, the father threatened him: "If you don't eat your veggies, you won't get any ice-cream!" The son, frightened at the prospect of not having his favorite dessert, quickly finished his vegetables.

What happened next?

Got something to prove?

When the logician's little son refused again to eat his vegetables for dinner, the father threatened him: "If you don't eat your veggies, you won't get any ice cream!" The son, frightened at the prospect of not having his favorite dessert, quickly finished his vegetables.

What happened next?

After dinner, impressed that his son had eaten all his vegetables, the father sent his son to bed without any ice cream...

Got something to prove?

A mad veterinarian (that is, a mad scientist who studies animals) has invented an animal transmogrifying machine. If you put in two cats or two dogs, then one dog comes out of the machine. If you put in one cat and one dog, then one cat comes out.

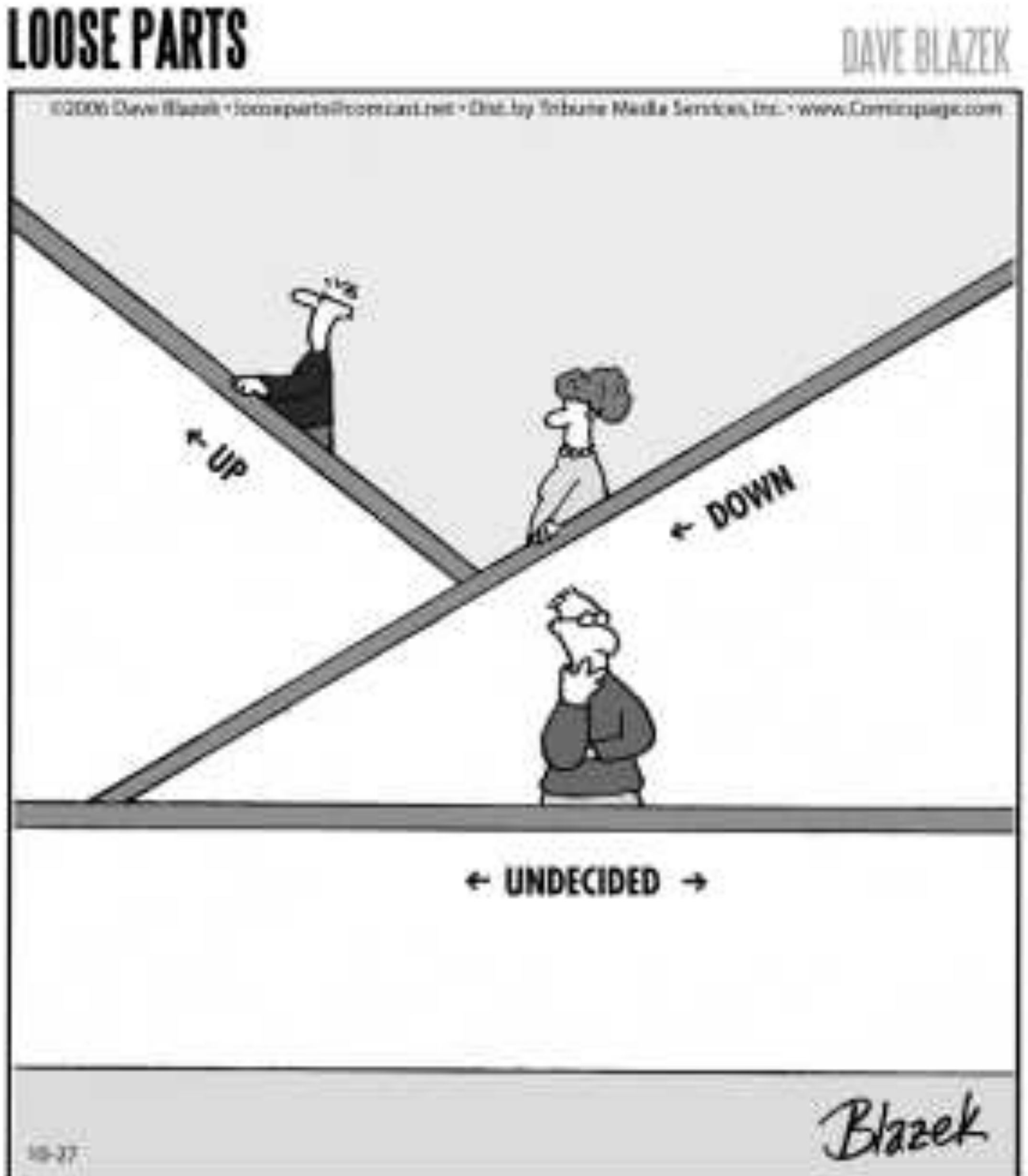
Try this and see what happens.

Got something to prove?

The dragon of ignorance has three heads and three tails. However, you can slay it with the sword of knowledge by cutting off all its heads and tails. With one swipe of the sword you can cut off one head, two heads, one tail, or two tails.

But . . .

- When you cut off one head, a new one grows in its place.
- When you cut off one tail, two new tails replace it.
- When you cut off two tails, one new head grows.
- When you chop off two heads, nothing grows.
- Help the world by slaying the dragon of ignorance.


LOOSE PARTS
DAVE BLAZEK
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← UP
← DOWN
← UNDECIDED →
Blazek

Got something to prove?

- Julia Hall Bowman Robinson, (1919 - 1985) the first female mathematician to be elected to the National Academy of Sciences, and the first female president of the American Mathematical Society in was required to submit a description of what she did each day to Berkeley's personnel office.

Got something to prove?

As she related to a close friend, she wrote to the office:

"Monday--tried to prove theorem, Tuesday--tried to prove theorem, Wednesday--tried to prove theorem, Thursday--tried to prove theorem; Friday--theorem false."

•

Got something to prove?

Robinson’s description was likely delivered tongue-in-cheek, while at the same time being an apt description of the life of a mathematician. Elizabeth Meckes, a 2006 Ph.D. in mathematics from Stanford, and now a professor at Case Western Reserve University, noted in a 2010 interview for girlsangle.org, “So for one thing, I, like most mathematicians, am wrong a lot.”

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Corollaries

“Organized deductive methods in mathematics should be introduced as soon as they can legitimately contribute to the student’s ability to understand and simplify the problems before him. The approach must be axiomatic in the sense I have described above because the overriding consideration is relevance. There is no necessity for completely organized axiom systems as long as we are honest about where we are using deduction and when we are being empirical.”

– Andrew M. Gleason

Why prove?

- **Verification**: validating correctness
- **Explanation**: providing insight, illumination, satisfying curiosity.
- **Communication**: transmitting mathematical knowledge and ideas.
- **Enjoyment**: Meeting an intellectual challenge elegantly.
- **Discovery**: Inventing new results.

Why prove?

- **Exploration**: Of the meaning of a definition or the consequences of an assumption.
- **Connection**: Linking ideas into a framework.
- **Convincing**: I really believe this! Removing doubt.
- **Understanding**: I get it!
- **Extension**: Inspiring new explorations.