


## Two-Dimensional to Three-Dimensional Transitions in Grades Three Through Five


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## Presentation Overview

- Why Spatial Thinking
- Project Background – Spatial Thinking Skills results: Grades K-2
- Components of Spatial Sense
- Examples of tasks
- Results
  - Ties to the Common Core
  - Tie to Number
  - Ties to SBAC?

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## What is Spatial Sense?

- Spatial sense is an intuitive feel for one's surroundings and the objects in them.

(NCTM, 1989, p. 49)

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## Why Spatial Reasoning?

**“Spatial thinking** is so deeply embedded in the activities of daily life and thought that it is difficult to disentangle and appreciate its role. We may not even realize its role, but it is fundamental to many taken-for-granted activities, underpinning their successful performance and sometimes accounting for their spectacular failure.”

(The National Research Council, 2006, P. 50)


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## Spatial Thinking Supports:

- Mathematics achievement (Battista, 1994)
- Success in numerous occupations (Mitchelmore, 1980; Lord, 1985)
- Computer technology (Pleck, et al, 1990)
- Technical professions (Lord & Rupert, 1995)
- Sciences (Bodner & Guay, 1997; Sorby, 1999)

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## Issues

- Research has been reported that estimates half or more of the adults in the United States struggle to manipulate or control iconic images (Maccoby & Jacklin, 1974; McGee, 1979 ).
- The institution of school is a large contributor to the deficiencies currently seen in spatial thinking by generating a disinterest in visualization. (Sommer, 1978)

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**Issues**

- Topics that support spatial thinking such as Geometry have been deemphasized to create more time for symbolic, procedural thinking. (Sorby, 1999)

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**PROJECT HISTORY – SPATIAL THINKING IN GRADES K-2**

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**Four Predictive Constructs**

```

    graph LR
      PMA[PMA] --- Number[Number]
      PMA --- Space[Space]
      Number --- FF[Fluency and Flexibility]
      Number --- IR[Interpreting Relationships]
      Space --- Measurement[Measurement]
      Space --- SR[Spatial Reasoning]
  
```

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**Sample Items**

Grade 2: Number-Reasoning About Quantity

What number should be in the missing space?

$$15 + 8 = \square + 9$$

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**Sample Items**

Grade 1: Measurement

Which line is longer?

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**Sample Items**

Grade K: Number-Reasoning about Quantity

Here are some bugs. Some you can see and some are hiding in the box. If there are 5 bugs altogether, how many are hiding in the box?

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Imagine you could move these two objects. Which object would they make if you put them together?

A + B

A B C

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is rotated to

as is rotated to

A B C D E

(Bottista, Wheatley & Talsma, 1982)

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### PMA Findings

- There is a strong relationship between PMA results and a state standardized achievement test;
  - 0.51 to 0.72 based on different constructs.
- Students need more opportunities to:
  - Solve contextual problems and compare their strategies
  - Use facts they know to solve facts they don't know
  - Compose, decompose, and draw shapes instead of simply 'labeling' shapes
  - Investigate relationships between quantities and develop relational thinking

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## DESCRIPTION AND STRUCTURE OF SPATIAL THINKING

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### Spatial Constructs

Visualization

Orientation

Relations

(McGee, 1979; van Nes & de Lange, 2007; & Lee, 2005)


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### Construct 1

- Spatial Visualization:
  - the ability to make object-based transformations
  - only the positions of the objects are moved with respect to the environmental frame of reference
  - frame of reference of the observer remains constant.

(van Nes & de Lange, 2007)

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## Construct 2

- Spatial Orientation:
  - comprehension of the arrangement of elements within a visual stimulus pattern
  - remaining unconfused by the changing orientations in which a configuration may be presented
  - the body orientation of the observer is an essential part of the problem.”

(Lee, 2005)

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## Construct 3

- Spatial Relations:
  - Recognizing spatial distributions and patterns
  - Connecting locations
  - Associating and correlating spatially distributed phenomenon
  - Comprehending and using spatial hierarchies
  - Orienting to real-world frames of reference


(Lee, 2005)

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## DESCRIPTION AND EXAMPLES


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## Participants

- Random sample of students from 3<sup>rd</sup>, 4<sup>th</sup>, and 5<sup>th</sup> grade classrooms.
  - 3<sup>rd</sup> Grade N=60
  - 4<sup>th</sup> Grade N=59
  - 5<sup>th</sup> Grade N=51


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## Instrument

- Presentation to class – 51 questions
- 1-10: 2D Composition – Multiple Selection
- 11-20: 3D Composition – Multiple selection (same figures as 1-10 with depth added)
- 21-30: Nets of cubes and tetrahedrons – Yes or No selection
- 31-36: Silhouettes of 3D objects – Multiple Choice

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## Instrument

- 37-41: 2D Rotation – Multiple Choice Selection
- 42-51: 3D Rotation – Multiple Choice Selection

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**Ask yourself the following:**


- How did I know which solutions to choose?
- How did I visualize the object in my mind?
  - Did I take it apart?
  - Did I move it in some manner?
  - What attributes caught my attention?
- Be prepared to discuss the answers to these questions with a partner.


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Which of these pieces can be put together to make the shape below:


  
A

  
B

  
C

  
D


---





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**B**  
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Which of these shapes can be put together to make the shape below:

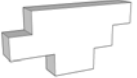
  
A

  
B

  
C

  
D


---





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**B**  
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Which of these pieces can be put together to make the shape below:

  
A


  
B

  
C

  
D

  
E


---





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**B**  
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Which of these shapes can be put together to make the shape below:

  
A


  
B

  
C

  
D

  
E


---



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Can I fold this shape to make a cube?



Yes No

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Can I fold this shape to make a cube?

Yes      No

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Which one of the black shapes shows what the shape would look like from the side?

A

B

C

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Which gray shape is the same size and shape as the white shape in the black rectangle?

A      B      C      D

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Is this figure →

the same as this figure after it has been turned? →

Yes      No

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**RESULTS**

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**Items Identified**

- 2D Composition: 5 items
- 3D Composition: 4 items
- Nets: 4 items
- 2D Rotation: 5 items
- 3D Rotation: 5 items
- Orientation: 6 items
  - These items were among the most reliable


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## Experimental Factor Analysis

- Spatial Visualization
  - 2D Composition
    - Items 1, 3, 4, 6, and 8
  - 3D Composition
    - Items 13, 18, 19, and 20
  - Visualize 3D from 2D (Nets)
    - Items 21, 22, 23, and 24 (2 Factors)


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## Experimental Factor Analysis

- Spatial Visualization Continued:
  - 2D Rotation
    - Items 37, 38, 39, 40 and 41
  - 3D Rotation
    - Items 43, 44, 46, 49, and 51
- Spatial Orientation (silhouettes)
  - Items 31, 32, 33, 34, 35, and 36 (all items)


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## Internal Reliability

- Spatial Visualization by subscale and order
  - 3D Rotation: .63
  - 2D Composition: .55
  - 2D Rotation: .41
  - 3D Composition: .40
  - Visualize 3D from 2D (Nets): .36
  - Overall: .67
- Spatial Orientation: .61


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## Gender Differences

- Differences seen between grade levels, means and reliabilities
- Differences between males and females, mostly seen at 5<sup>th</sup> grade
- 5<sup>th</sup> Grade female mean of 17.39 was less than the male mean at 4<sup>th</sup> grade of 17.56.


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## Relationships

- Spatial Orientation was most closely correlated to the total score
  - Also strongest relationship with 2D and 3D Composition and 2D Rotation
  - Weakest relationship with 3D Rotation
- Was a small relationship between the 2D and corresponding 3D shapes

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## Recommendation

- Spatial thinking skills should be an integral part of all areas of mathematics – not a separate topic that is visited once or twice in isolation.

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**STRUCTURAL TIES TO NUMBER AND OPERATIONS AND THE CCSSM**

### Structural Components

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### Initial Ideas of Geometry

- Geometry is about understanding the **space** around us and **shapes**.
- **Spatial reasoning** involves the mental manipulation of an object in order to complete a task.
- Why focus on spatial reasoning?
  - shown to have a very high predictive value for mathematics achievement
  - helps develop fluency with flexible operations in arithmetic
  - strengthens and supports students' ability in measurement
  - builds proportional reasoning
  - allows students to make sense formulas
    - E.g. The formula for the area of a rectangle ( $A = bh$ ) and a right triangle ( $A = 1/2bh$ )

### Initial Ideas of Geometry

- E.g. The formula for the area of a right triangle,  $A = 1/2bh$ .

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Initial Ideas of Geometry


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Initial Ideas of Geometry


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Initial Ideas of Geometry


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Initial Ideas of Geometry


- E.g. The formula for the area of a right triangle,  $A=1/2bh$ .



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Initial Ideas of Geometry


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Initial Ideas of Geometry

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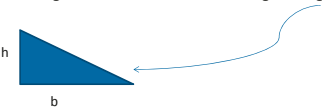


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## Initial Ideas of Geometry

- E.g. The formula for the area of a right triangle,  $A = \frac{1}{2}bh$ .




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## Common Core Examples: K-1

- K.G.3: Identify shapes as two-dimensional or three dimensional.
- K.G.6: **Compose** simple shapes to form larger shapes.
- 1.G.2: **Compose** two-dimensional shapes or three-dimensional shapes to create a composite shape, and compose new shapes from the composite shape.


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## Common Core Examples: 3-4

- 3.G.2: **Partition** shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.
- 4.OA.5: Generate a number or a **shape pattern** that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.


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## Common Core Examples: 4-5

- 4.G.3: Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be **folded** along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.


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Grade 3 Mathematics Sample ER Item Claim 3

## CONNECTIONS TO ASSESSMENT (SBAC)


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## Assessment Item from SBAC

- Focus on SBAC – Idaho selection
- Where do you see spatial skills as a component of this task?


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Ms. Flinn has a garden that is 12 feet long and 3 feet wide.



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- What is the area of Ms. Flinn's garden?  
\_\_\_\_\_ square feet
- She wants to make another garden that has the same area, but a different perimeter. Use the grid to show a rectangle that has the same area, but a different perimeter than Ms. Flinn's garden.


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### A Call for Future Research


- A need for understanding the progression from 2-Dimensional to 3-Dimensional thinking.
- A need for research in spatial relations.
- A need for research in assessment items that are not paper/pencil, but more real life and 3-Dimensional.

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## QUESTIONS?

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