

Teaching Content through Problem Analysis:

Driven by CCSS Mathematical Practices

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A Problem to Start

Solve this problem in as many ways as you can.

The first three figures in a tile pattern are shown. Find the total number of tiles in the n th figure.

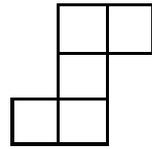


Figure 1

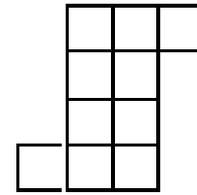


Figure 2

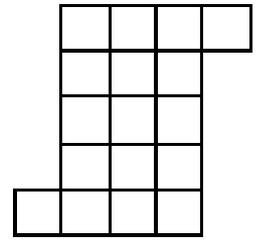


Figure 3

A Snapshot: Teaching Through Problem Analysis

“Instructional programs from prekindergarten through grade 12 should enable all students to –

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

A Snapshot: Teaching Through Problem Analysis

Research shows that:

- Students must solve problems on a regular basis in order to improve their problem-solving ability.
- Asking students to generate solutions that are generalizable is important.
- Instruction aimed at engaging students in problem solving results in deeper understanding of mathematics.

A Snapshot: Teaching Through Problem Analysis

The Standards for Mathematical Practice state that mathematically proficient students should:

- Make sense of problems
- Analyze givens, constraints, relationships
- Try special cases
- Solve simpler problems
- Explain relationships between representations

A Snapshot: Teaching Through Problem Analysis

- Introduce a problem that will lead us into a particular topic (or topics) and analyze:
 - **Approaches** to solutions
 - **Generalizations** of solutions
 - **Concepts** involved in solving the problem
 - **Connections** between the concepts involved and other areas of mathematics or topics in other fields of study
- 

Sharing Solutions

Solve this problem in as many ways as you can.

The first three figures in a tile pattern are shown. Find the total number of tiles in the n th figure.

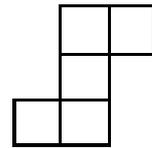


Figure 1

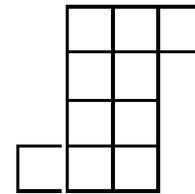


Figure 2

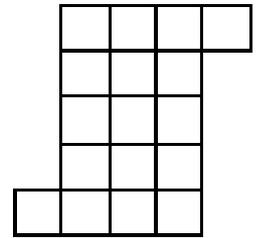


Figure 3

Generalizing our Solution

Generalize the solution you presented.

The first three figures in a tile pattern are shown. Find the total number of tiles in the n th figure.

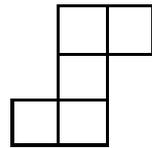


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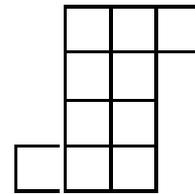


Figure 2

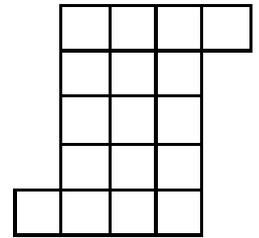


Figure 3

Generalizing our Solution

Possible generalizations:

1) $f(n) = n^2 + 2n + 2$

Generalizing our Solution

Possible generalizations:

2) Replace the terms of the sequence 5, 10, 17, 26, ... with a variable sequence such as $y_1, y_2, y_3, y_4, \dots$ that has a constant second difference. Then,

$$f(n) = an^2 + bn + c \text{ where}$$

- $a = .5y_1 - y_2 + .5y_3$
- $b = -2.5y_1 + 4y_2 + -1.5y_3$
- $c = 3y_1 + -3y_2 + y_3.$

Generalizing our Solution

Possible generalizations:

3) In recursive form:

$$y_{n+1} = y_n + (y_3 - 2y_2 + y_1)(n-1) + (y_2 - y_1).$$

Analysis of our Generalization

$$f(n) = n^2 + 2n + 2$$

$$f(n) = an^2 + bn + c \quad \text{where}$$

$$a = .5y_1 - y_2 + .5y_3$$

$$b = -2.5y_1 + 4y_2 + -1.5y_3$$

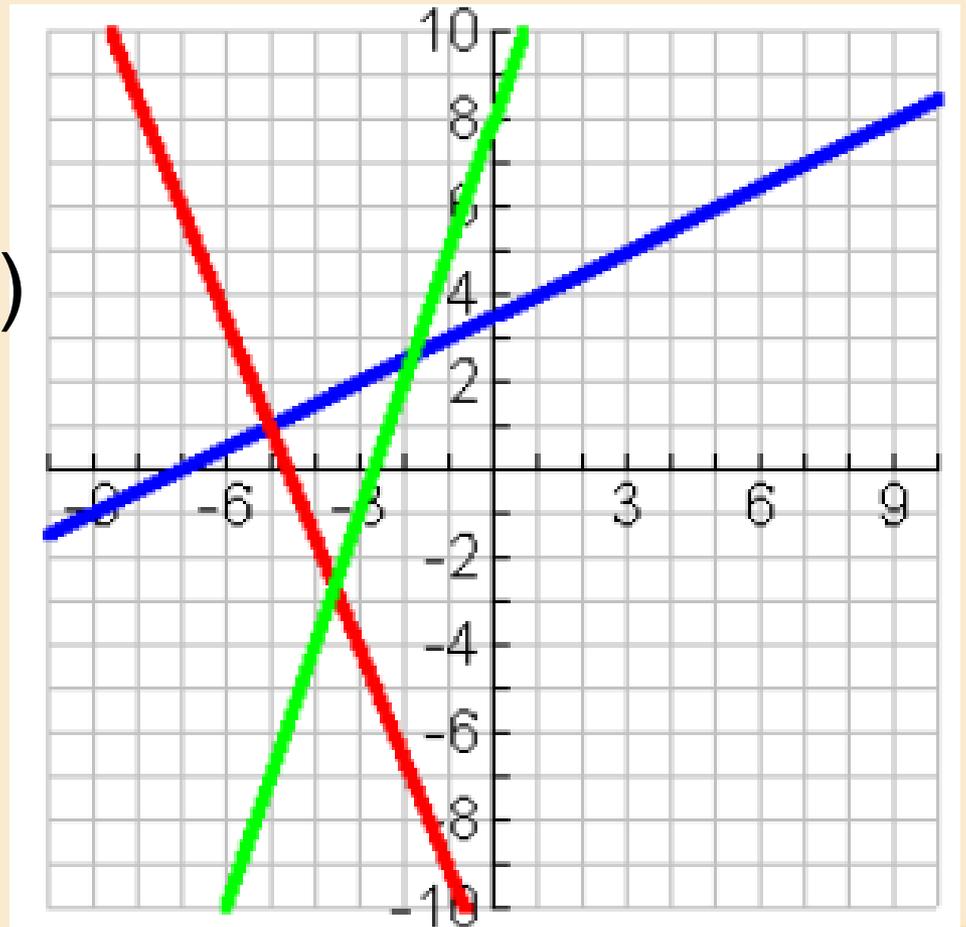
$$c = 3y_1 + -3y_2 + y_3$$

Analysis of our Generalization

$$a = 3.5 + .5y_1 \text{ (blue)}$$

$$b = -11.5 - 2.5y_1 \text{ (red)}$$

$$c = 8 + 3y_1 \text{ (green)}$$



Concepts

What mathematical concepts were involved in the solution you presented?

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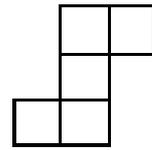


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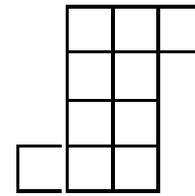


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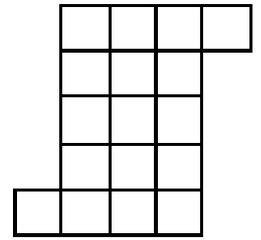


Figure 3

Concepts

Possible Concepts:

- Pattern Recognition
 - Quadratic Functions
 - Features of Parabolas
 - Recursive Form of a Function
 - Explicit Forms of a Function
 - Method of Finite Differences
- 

Connections

What connections to other mathematical topics or other fields of study can be made?

The first three figures in a tile pattern are shown. Find the total number of tiles in the n th figure.

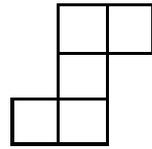


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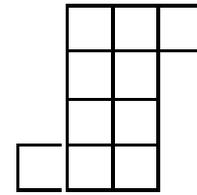


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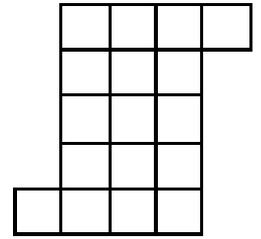


Figure 3

Connections

Possible Connections:

- *Linear and quadratic functions*
 - *Transformations of functions*
 - *Parabolas and complex numbers*
 - *Triangular Numbers*
 - *Pascal's Triangle and Combinatorics*
- 

Supporting the Standards

- Where is this content in the Common Core?
 - What Standards for Mathematical Practice were addressed?
- 

Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them.**
 - 2. Reason abstractly and quantitatively.**
 - 3. Construct viable arguments and critique the reasoning of others.**
 - 4. Model with mathematics.**
 - 5. Use appropriate tools strategically.**
 - 6. Attend to precision.**
 - 7. Look for and make use of structure.**
 - 8. Look for and express regularity in repeated reasoning.**
- 

Questions or Comments?

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