## Median - Statistical and Geometric: How Are They Related?

**Statistical mean**—is the fulcrum (balance point) of the data
**Statistical median**—is the midpoint of the data

**Geometric median**—the line dividing a planar shape in half—providing a line of balance
**Centroid**—the intersection of the medians; the fulcrum of the planar shape

**Applications** to centers of balance—include architecture, centers of population (using weighted mean) and toys.

Resources:

Kranendonk, Henry and Witmer, Jeffrey, *Exploring Centers*, New York, Dale Seymour Publications, 1998, Print.

"What is so remarkable about the idea of "center" to mean an extensive range of investigations that connect geometry to population studies? After all center is simply the midpoint of the diameter of a circle—or the mean of a data set—or a median value of a sorted list—or the location that minimizes the sum of distances—or…? Exactly! The concept of "center" is not as simple as we thought. Its meaning depends on the context of the application. When it is used to summarize and analyze an entire set of data, it can be a powerful tool for investigative student projects." Henry Kranendonk, Rufus King High School, Milwaukee, WI.

Berlinghoff, William, et.al., Math Connections, New York, It's About Time, 2000, Print.

Sources for toys:

Balancing birds:
http://www.officeplayground.com/Balancing-Eagle-Mini-P2070.aspx?utm_source=amazonpa%20&utm_medium=comparisonshopping&utm_term=3567&utm_campaign=productfeed $0.49/each ordered 50

Paper bird
http://www.ellenjmchenry.com/documents/BalancingBirdToyPatternPage.pdf

Ladybug Crawlers
https://www.smilemakers.com

Summary:

Centers! Students need an understanding of the significance of the statistical mean—that it is the fulcrum (balance point) of the data. Their appreciation of mathematics is enhanced by seeing the connections between the content domains. In statistics, the median divides the data in half; in geometry, the median divides the area in half, providing a line of balance. The point of intersection of the geometric medians is the centroid (balance point) of the shape. The centroid of a graphed region can be determined by calculating the mean of the vertices of the region. Applications include the center of population which can be determined similarly, but with a weighting of the regions of the territory being studied by their populations.

Centroid of a triangle:

1. estimated by using a pencil as the fulcrum to find the balance point
2. determining the mean of the x- and y-coordinates of the vertices
3. locating the intersection of the medians
4. 2/3 of the distance from a vertex to the midpoint of the opposite side (along the median from that vertex)
5. Kranendonk's method of "collapsing the raisins"

Centroid of a quadrilateral:

1. estimated by using a pencil as the fulcrum to find the balance point (except in some concave examples)
2. determining the mean of the x- and y-coordinates of the vertices
3. midpoint of the segment between the midpoints of 2 non-adjacent sides
4. the midpoint of the medians (lines that divide the area in half)
5. Kranendonk's method of "collapsing the raisins"
6. Parallelogram: locating the intersection of the diagonals

   (aside: the figure formed by connecting the midpoints of each side of a quadrilateral is a parallelogram.)

Activities:

Balancing Pennies

Move the Weights

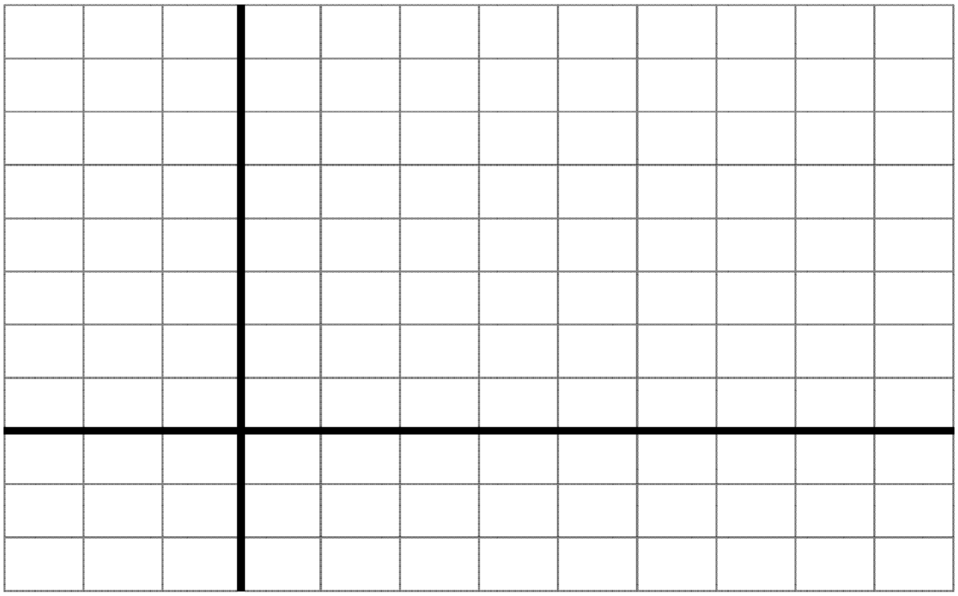
Move Fulcrum

Planar shapes

Graphing

Applications: Balancing toys (birds), wall crawlers, architecture;
Centers of population

## Center of Population



Graph the cities using the coordinates on the table below. Find the weighted coordinates of each point and the mean of the populations and the weighted coordinates. Plot the center of population (the mean of the weighted coordinates).

| Community | Population (p) | | x | y | | Weighted x p*x | Weighted y p*y |
|---|---|---|---|---|---|---|---|
| Begium | 892 | | 5.25 | 6.2 | | | |
| Port Wash | 8612 | | 4.8 | 4.6 | | | |
| Milwaukee | 636236 | | 5 | 1.7 | | | |
| Waukesha | 50365 | | 2.2 | 1.7 | | | |
| Racine | 85725 | | 6.25 | − 1.3 | | | |
| Beloit | 35207 | | − 1.3 | − 2.1 | | | |
| | | | | | | | |
| Totals | | | | | | | |
| Mean | | | | | | | |
| | | | | | | | |

# Geometric Centers Quiz

Name________________________

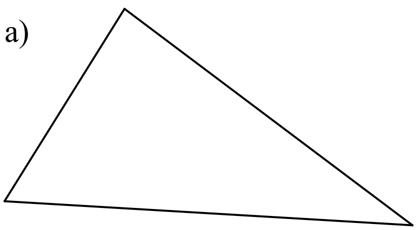
1. Define centroid.

2. Explain 3 ways of finding the centroid of a triangle.

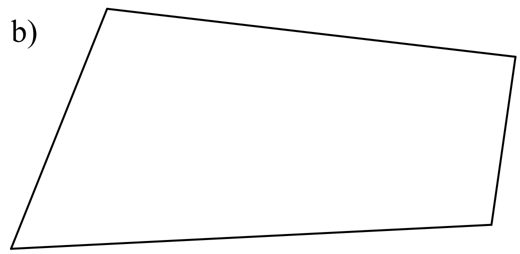
3. Explain 2 ways of finding the centroid of a quadrilateral.

4. Find the geometric centers of the following shapes to the nearest millimeter.

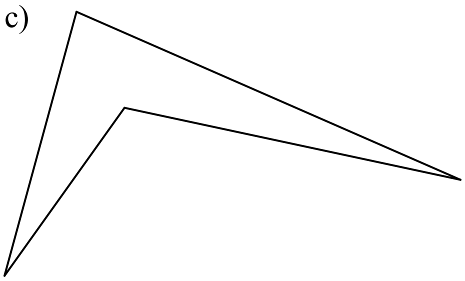
a)



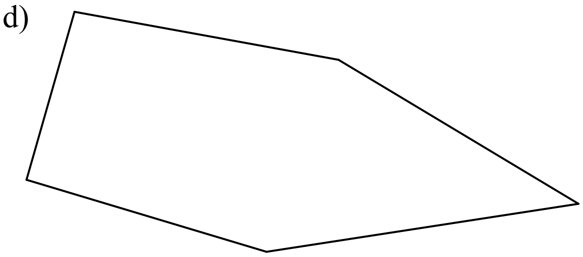
b)



c)



d)



5. Plot and label the following sets of points and
   - Estimate the location of the centroid geometrically
   - Verify the location of the centroid arithmetically (Show work)

a) A (4, 6) B (6, 9) C (5, 15)

b) A (– 3, – 5) B (4, – 3) C (10, 5) D (1, 9)

c) A (– 7, – 2) B (– 1, 15) C (3, 10) D (12, 4) E (5, – 4)

6. On the back, describe some situations where finding the centroid might be useful.