

LEVEL

Algebra 2 or Math 3 in a unit on analyzing functions

Algebra 2 or Math 3 in a unit on statistics

MODULE/CONNECTIONS TO AP*

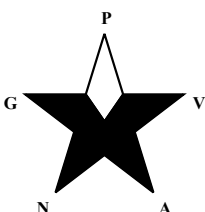
Analysis of Functions

Graphical Displays and Distributions

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MODALITY

NMSI emphasizes using multiple representations to connect various approaches to a situation in order to increase student understanding. The lesson provides multiple strategies and models for using those representations indicated by the darkened points of the star to introduce, explore, and reinforce mathematical concepts and to enhance conceptual understanding.



- P – Physical
- V – Verbal
- A – Analytical
- N – Numerical
- G – Graphical

Analyzing Function Behavior Using Graphical Displays – Part 1

ABOUT THIS LESSON

This lesson incorporates graphical displays with analysis of functions. Students graph linear and absolute value functions over limited domains and then use a calculator’s random number generator to compile a set of function values based on random domain values. Comparing a histogram of the function values to the function’s coordinate graph allows students to analyze the connections between the two types of graphs and to predict the distribution of function values in histograms for additional functions.

OBJECTIVES

Students will

- create histograms based on randomly selected function values.
- interpret the distribution of histograms.
- graph transformations of absolute value functions.
- sketch histograms based on the predicted distribution of function values of absolute value functions.

COMMON CORE STATE STANDARDS FOR MATHEMATICAL CONTENT

This lesson addresses the following Common Core State Standards for Mathematical Content. The lesson requires that students recall and apply each of these standards rather than providing the initial introduction to the specific skill. The star symbol (*) at the end of a specific standard indicates that the high school standard is connected to modeling.

Targeted Standards

- S-IC.1:** Understand statistics as a process for making inferences about population parameters based on a random sample from that population.*
See questions 1e, 2c, 3, 4

Reinforced/Applied Standards

- F-IF.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
*Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**
See questions 2c, 4
- S-ID.3:** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).*
See questions 2c, 2e, 3, 4
- S-ID.1:** Represent data with plots on the real number line (dot plots, histograms, and box plots).*
See questions 1c, 2b, 3, 4

- F-IF.7b:** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*
See questions 1a, 2a, 3, 4

- F-IF.2:** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
See questions 1b, 2b, 3

COMMON CORE STATE STANDARDS FOR MATHEMATICAL PRACTICE

These standards describe a variety of instructional practices based on processes and proficiencies that are critical for mathematics instruction.

NMSI incorporates these important processes and proficiencies to help students develop knowledge and understanding and to assist them in making important connections across grade levels. This lesson allows teachers to address the following Common Core State Standards for Mathematical Practice.

MP.3: Construct viable arguments and critique the reasoning of others.

Students justify their predicted histograms based on their knowledge of the behavior of function values.

MP.5: Use appropriate tools strategically.

Students use the graphing calculator to generate and evaluate random domain values in order to create histograms.

MP.7: Look for and make use of structure.

Students relate the shape of the display of function values to the appearance of the graph of a function on a restricted domain and predict the shape of a histogram of function values.

FOUNDATIONAL SKILLS

The following skills lay the foundation for concepts included in this lesson:

- Create and interpret histograms
- Apply transformations to absolute value functions

ASSESSMENTS

The following formative assessment is embedded in this lesson:

- Students engage in independent practice.

The following additional assessments are located on the our website:

- Graphical Displays and Distributions – Algebra 2 Free Response Questions
- Graphical Displays and Distributions – Algebra 2 Multiple Choice Questions

MATERIALS AND RESOURCES

- Student Activity pages
- Graphing calculators
- Mathematica demonstration “Analyzing Functions Using Graphical Displays” explores the relationship between the distribution of randomly selected function values and the behavior of the function and is available on the NMSI website.
- NMSI video clip on random domain generation on the TI-84
- NMSI video clip on creating histograms on the TI-84

TEACHING SUGGESTIONS

Before beginning the lesson, a review of histograms, shapes of distributions, and measures of center may be helpful. “Histograms,” a middle grades lesson, can be used to introduce or review these concepts. Since students will describe the shapes of distributions, discuss the definitions of skewed, mound-shaped, symmetric, and uniform distributions using the sample graphs displayed in the teaching suggestions. Students also need to be able to recognize the relationship between the measures of central tendency (mean and median).

In order to ensure that all groups will generate different random numbers, seed the graphing calculators by storing the last four digits of the student’s phone number in rand on the graphing calculator.

The key sequence is as follows:

Step 1: Students enter the last four digits of their phone numbers

Step 2: Press **STO▶**

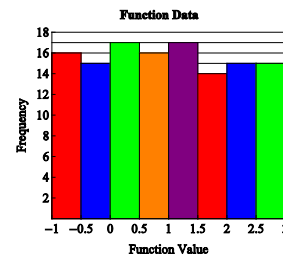
Step 3: Press **MATH**, arrow over to PRB, press **1**

Step 4: Press **ENTER** and the new number is the seed for the random number generator

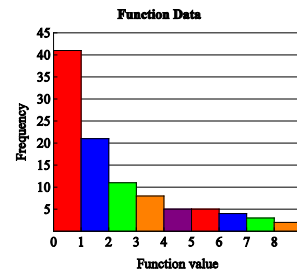
```
1234→rand
```

Discuss the meaning of the syntax of the calculator key strokes when generating the random numbers over the limited domains. The calculator instructions for generating the domain values at random are provided on the last page of the student activity.

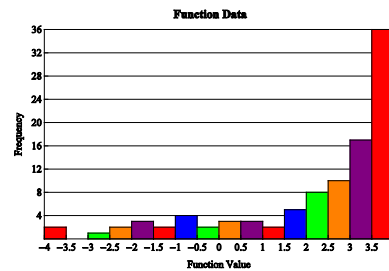
Memory errors may occasionally occur when generating large lists of random numbers since these lists take large amounts of RAM. If memory errors occur, instruct students to clear the lists prior to generating random numbers.



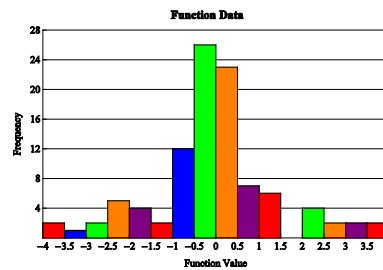
This histogram would be described as uniform since the bins have the same or similar frequencies throughout the distribution. Since this distribution is fairly symmetric, we expect the mean and median function values to be approximately the same.



This histogram would be described as skewed right or skewed to the larger numbers since the tail of the distribution is on the right hand side. Since this distribution is skewed right, the mean function value will typically be larger than the median.



This histogram would be described as skewed left or skewed to the smaller numbers since the tail of the distribution is on the left hand side. Since this distribution is skewed left, the mean function value will typically be less than the median.



This histogram would be described as mound shaped. Since the distribution is approximately symmetric, we expect the mean and median function values to be approximately the same.

TEACHER PAGES

Suggested modifications for additional scaffolding include the following:

- 4 Allow the use of the calculator to draw the histograms. Provide a fill-in the blank format for the justification. For example,
“The histogram shows that the function values from ____ to ____ occur twice as often as those from ____ to ____.”

NMSI CONTENT PROGRESSION CHART

In the spirit of NMSI’s goal to connect mathematics across grade levels, a Content Progression Chart for each module demonstrates how specific skills build and develop from sixth grade through pre-calculus in an accelerated program that enables students to take college-level courses in high school, using a faster pace to compress content. In this sequence, Grades 6, 7, 8, and Algebra 1 are compacted into three courses. Grade 6 includes all of the Grade 6 content and some of the content from Grade 7, Grade 7 contains the remainder of the Grade 7 content and some of the content from Grade 8, and Algebra 1 includes the remainder of the content from Grade 8 and all of the Algebra 1 content.

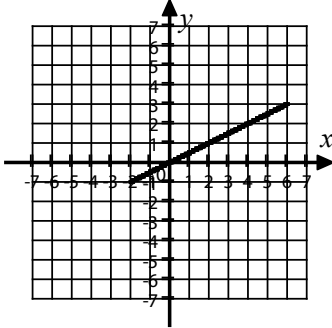
The complete Content Progression Chart for this module is provided on our website and at the beginning of the training manual. This portion of the chart illustrates how the skills included in this particular lesson develop as students advance through this accelerated course sequence.

6th Grade Skills/Objectives	7th Grade Skills/Objectives	Algebra 1 Skills/Objectives	Geometry Skills/Objectives	Algebra 2 Skills/Objectives	Pre-Calculus Skills/Objectives
Create, interpret, and compare dotplots (line plots), stemplots, and bar graphs. (200_06.GD_D.01)	Create, interpret, and compare dotplots (line plots), stemplots, bar graphs, histograms, and boxplots. (200_07.GD_D.01)	Create, interpret, and compare dotplots (line plots), stemplots, bar graphs, histograms, and boxplots. (200_A1.GD_D.01)	Create, interpret, and compare dotplots (line plots), stemplots, bar graphs, histograms, and boxplots. (200_GE.GD_D.01)	Create, interpret, and compare dotplots (line plots), stemplots, bar graphs, histograms, and boxplots. (200_A2.GD_D.01)	Create, interpret, and compare dotplots (line plots), stemplots, bar graphs, histograms, and boxplots. (200_PC.GD_D.01)
Calculate the mean, median, mode, range, and mean absolute deviation from tabular or graphical data or data presented in paragraph form. (200_06.DI_D.01)	Calculate the mean, median, mode, range, and mean absolute deviation from tabular or graphical data or data presented in paragraph form. (200_07.DI_D.01)	Calculate the mean, median, mode, range, and standard deviation from tabular or graphical data or data presented in paragraph form. (200_A1.DI_D.01)	Calculate the mean, median, mode, range, and standard deviation from tabular or graphical data or data presented in paragraph form. (200_GE.DI_D.01)	Calculate the mean, median, mode, range, and standard deviation from tabular or graphical data or data presented in paragraph form. (200_A2.DI_D.01)	Calculate the mean, median, mode, range, and standard deviation from tabular or graphical data or data presented in paragraph form. (200_PC.DI_D.01)

Analyzing Function Behavior Using Graphical Displays - Part 1

Answers

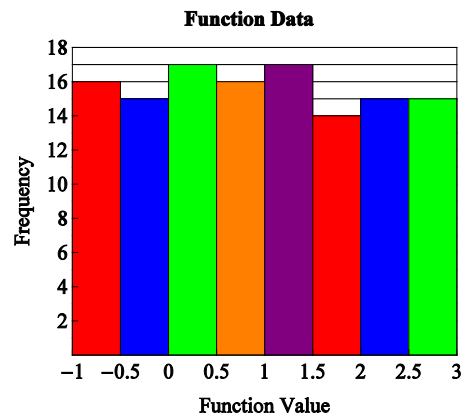
1. a.



b. Students generate 5 random domain values on their calculators.

c. A sample completed table and a histogram are shown. The class histogram contains 125 randomly generated values.

x	$f(x)$
1.842	0.921
2.313	1.157
1.117	0.559
0.067	0.033
4.078	2.039



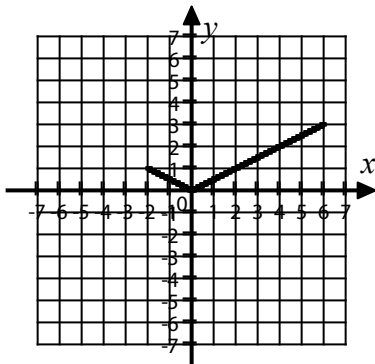
d. The distribution has a fairly uniform shape. All the bins seem to have approximately the same height or frequency.

e. The function is linear, and all the values have the same chance of being selected.

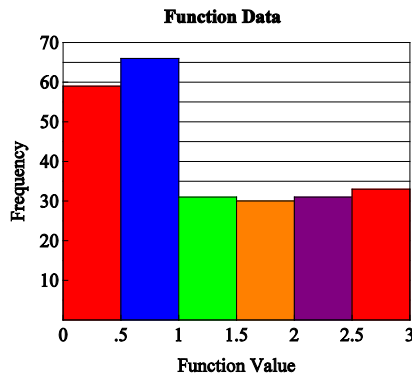
f. Answers will vary. The median is the 63rd of the 125 function values. For the sample data, the bin $0.5 \leq f(x) < 1$ contains the median.

g. Since the distribution is fairly symmetric, the mean and median function values should be approximately the same.

2. a.



b. A sample histogram is provided.

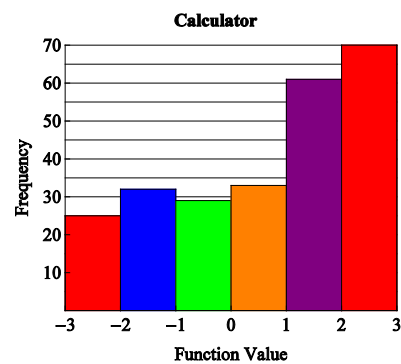
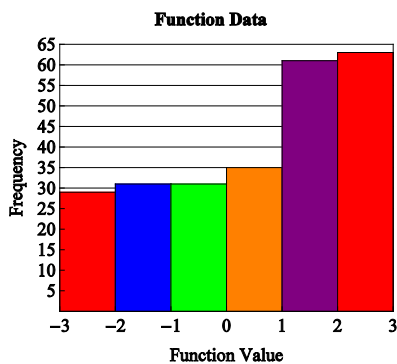


c. If enough sample data is collected, the histogram will be skewed right. The function values between 0 and 1 occur twice while the values between 1 and 3 only occur once so the frequencies for the bins between 0 and 1 should be twice as high.

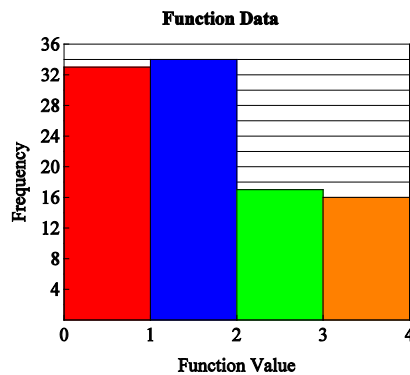
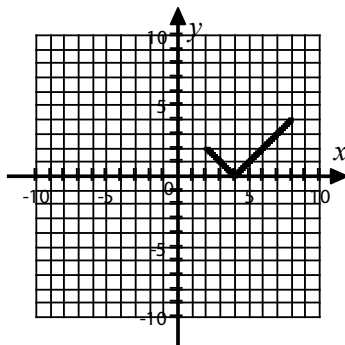
d. Answers may vary. The median occurs between the 125th and 126th of the 250 function values. For the sample data, the bin $0.5 \leq f(x) < 1$ contains the median value.

e. If enough sample data is collected, the distribution will be skewed right, so the mean should be larger than the median.

3. Answers will vary. If enough sample data is collected, the student’s histogram may not be exactly like the one in the key, but the pattern should be apparent that the bins between 1 and 3 will be about twice as tall as the bins between -3 and 1.



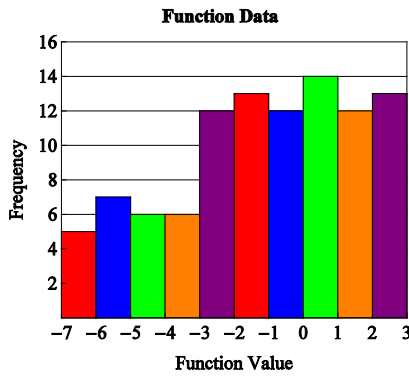
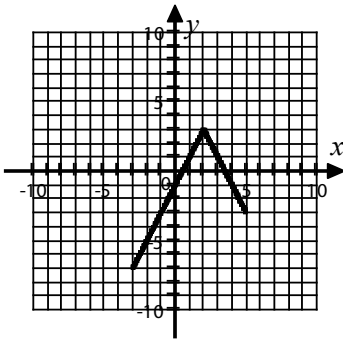
4. a.



Justification:

The histogram shows that the function values from 0 to 2 occur twice as often as those from 2 to 4. The graph shows that there are 2 domain values for each of the function values from 0 to 2 and only 1 domain value for the function values from 2 to 4.

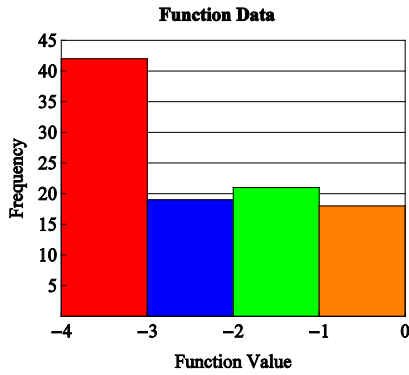
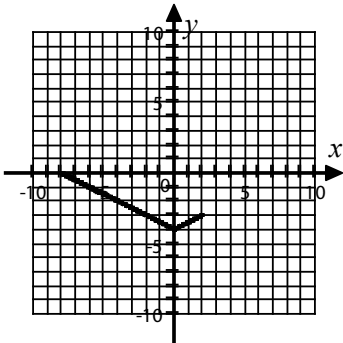
b.



Justification:

The histogram shows that the function values from -3 to 3 occur twice as often as the function values from -7 to -3 . The graph shows that there are 2 domain values for each function value from -3 to 3 and only 1 domain value from -7 to -3 .

c.

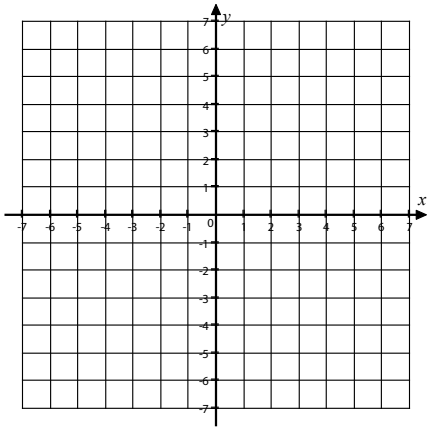


Justification:

The histogram shows that the function values from -4 to -3 occur twice as often as the function values from -3 to 0 . The graph shows that there are 2 domain values for each function value from -4 to -3 and only 1 domain value from -3 to 0 .

Analyzing Function Behavior Using Graphical Displays - Part 1

1. Use $f(x) = \frac{1}{2}x$ on the interval $[-2, 6]$ to answer the following.
- a. Sketch a graph of $f(x)$.



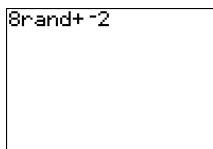
- b. Randomly select 5 domain values from the interval $[-2, 6]$, evaluate $f(x)$ at each of those domain values, and record those values in the table in part (c). In order to generate random numbers over a limited domain, one at a time, follow these calculator steps:

Step 1: Enter the width of the domain

Step 2: Press $\boxed{\times}$, $\boxed{\text{MATH}}$, arrow over to PRB, press $\boxed{1}$

Step 3: Press $\boxed{+}$, enter the lowest value in the domain and press $\boxed{\text{ENTER}}$

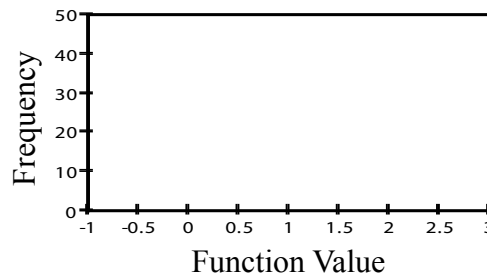
For example to generate random values on the domain $[-2, 6]$ on the calculator, the display would be:



$W \cdot \text{rand} + A$, where W is the width of the domain, $W = B - A$, A is the smallest value in the domain, and B is the largest value in the domain.

- c. Combine your data with the class data and create a histogram.

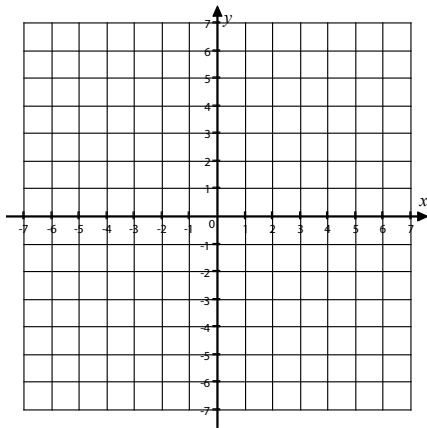
x	$f(x)$



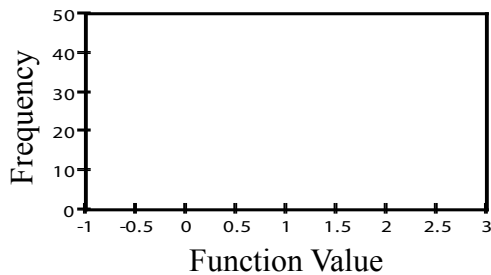
- d. Describe the shape of the distribution.
- e. Why do you think the distribution is shaped this way?
- f. Which bin contains the median function value?
- g. Based on the shape, will the mean or median be larger? Explain.

2. Use $g(x) = \frac{1}{2}|x|$ on the interval $[-2, 6]$.

a. Sketch a graph of $g(x)$.

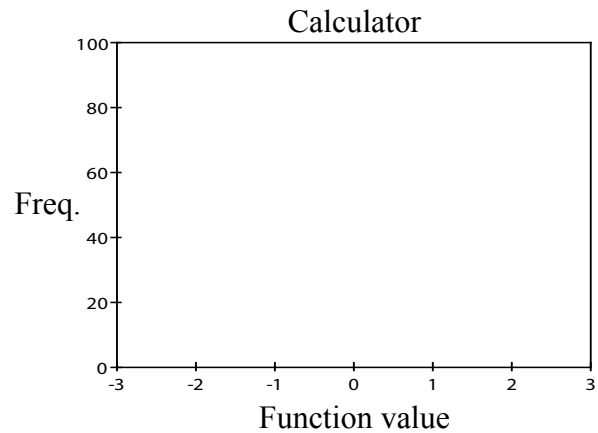
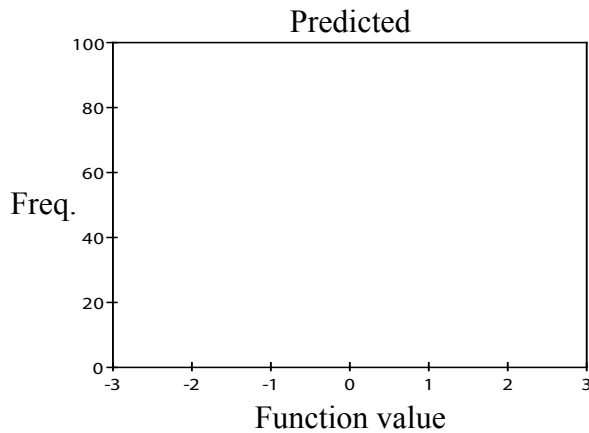
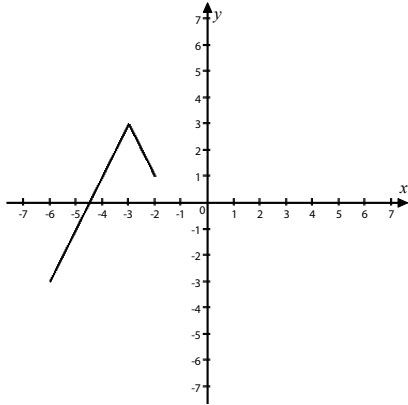


b. Using the steps at the end of the lesson and a graphing calculator, randomly select 250 domain values on the interval $[-2, 6]$. Evaluate $g(x)$ at each of those domain values and draw a histogram of the function values. Sketch the histogram on the grid provided.



- c. Describe the shape of the distribution. What characteristics of the function affect the shape?
- d. What bin contains the median value?
- e. Based on the shape, what is the relationship between the mean and the median function values?

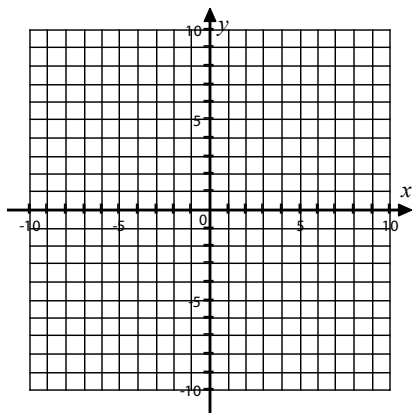
3. Given the graph of $f(x) = -2|x + 3| + 3$ on the interval $[-6, -2]$, predict a histogram of 250 function values if those function values were generated by evaluating 250 randomly selected domain values. Then using your calculator, generate 250 domain values and evaluate $f(x)$ at each function value. Use bin widths of 1.



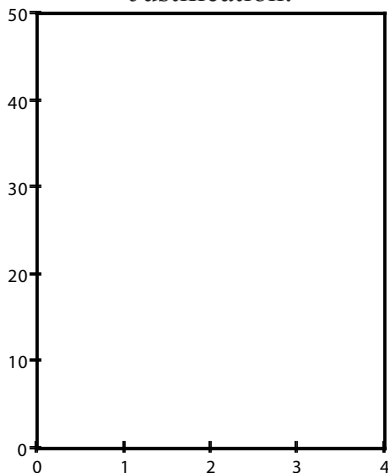
Compare and contrast the predicted histogram with the one generated on the calculator.

4. Without the aid of a calculator, sketch each function on the given interval and draw a possible histogram of 100 function values if those function values were generated by evaluating 100 randomly selected domain values, using bin widths of 1. Justify your predictions.

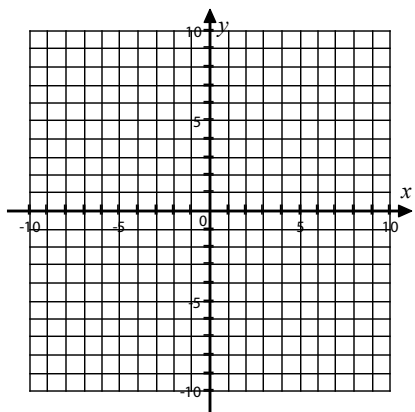
a. $f(x) = |x - 4|, [2, 8]$



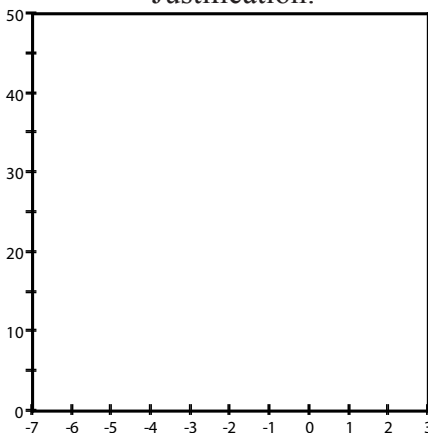
Justification:



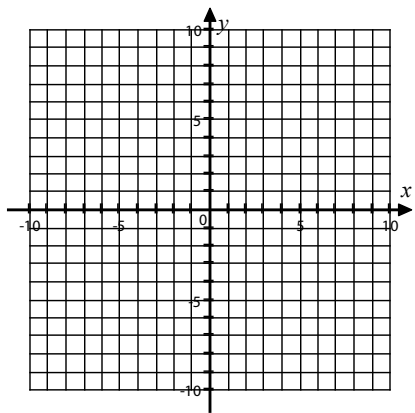
b. $f(x) = -2|x - 2| + 3, [-3, 5]$



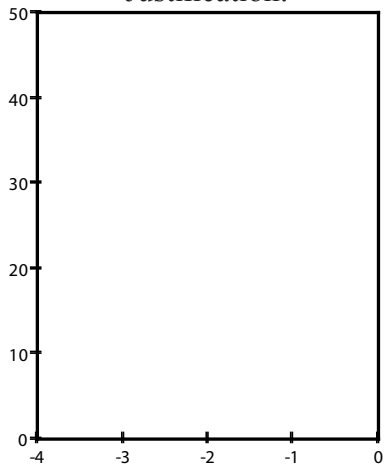
Justification:



c. $g(x) = \frac{1}{2}|x| - 4, [-8, 2]$



Justification:



To generate a large quantity of random numbers at one time over a limited domain and store these values in List 1 on the calculator, use the following steps:

Step 1: Enter the width of the domain

Step 2: Press **[MATH]**, arrow over to PRB, press **[1]**

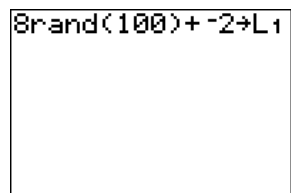
Step 3: Press **[\square]**, enter the number of random values required, press **[\square]**

Step 4: Press **[+]**, enter the smallest value in the domain

Step 5: Press **[STO \blacktriangleright]**, **[2nd]**, **[1]**

The syntax is $W \cdot \text{rand}(n) + A \rightarrow L1$ where W is the width of the domain, A is the smallest value in the domain, and n is the number of values you want to generate.

For example to generate 100 random values on the domain $[-2, 6]$ on the calculator, the display would be:



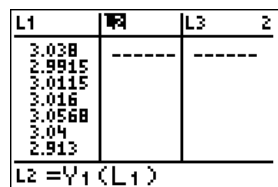
To evaluate a function that is in Y1 at the values in a list, use the following steps:

Step 1: Press **[STAT]**, press **[1]**, arrow to the header of L2 and press **[CLEAR]**, press **[ENTER]**

Step 2: Arrow to the header of L2, press **[VARS]**, arrow to [Y-VARS], press **[1]**, press **[1]**

Step 3: Press **[\square]**, **[2nd]**, **[1]**, **[\square]** then press **[ENTER]**

The calculator display would be:



Once all your data is in the lists, create a histogram of the data, using the following steps:

Step 1: Press **[2nd]**, **[Y=]**, **[ENTER]**

Step 2: Make sure that On is highlighted by pressing **[ENTER]** when the cursor is on On

Step 3: Arrow down and over to the picture of a histogram  and press **[ENTER]**

Step 4: Arrow down and press **[2nd]** and the number of the list where the function values reside

Step 5: Set the window so the Xmin is slightly less than or equal to the minimum function value

Step 6: Set the Xmax to be slightly greater than or equal to the maximum function value

Step 7: Set the Xscl to the chosen bin width and the Ymax to at least half the number of values selected

Step 8: Press **[GRAPH]**

