

Goal: The students will work in cooperative groups to explain why a fractal will perform better as an antenna than a flat, solid piece of metal by building successive stages of the Sierpinski tetrahedron, its complement, and find patterns in each.

Materials:

- 4 poster boards to create a 16 inch tetrahedron (stage 0)
- Sierpinski Tetrahedron (per group)
 - 16 -- 8 –inch equilateral *triangles*
 - 16 -- 4 – inch tetrahedron
 - 64 -- 2 –inch tetrahedron
- Complement (in different color, per group)
 - 8 -- 8 – inch equilateral triangles
 - 8 -- 4 – inch tetrahedron
 - 32 -- 2 – inch tetrahedrons
- Scissors
- Transparent Tape
- Finding Patterns in Sierpinski's Tetrahedron Handout (one per student)
- Comparing Volume in Sierpinski's Tetrahedron and Its Complement Handout (one per student)

Templates for triangles and tetrahedrons can be found at:

http://math.fau.edu/Teacher/CATEs_PDF/3%20sierpinski/sierpinski%20unit%204.pdf

Set Up: Create a 16 inch stage 0 tetrahedron for the front of the room

Directions:

1. Watch “The Evolution of Cell Phone” (<http://www.youtube.com/watch?v=JcnXOhrmDB8>)
 - a. Discuss how cell phones have changed. “Where have the antennas gone?”
 - b. Show picture of internal fractal antenna. Explain what a fractal is.
 - c. Pose the question why would a fractal work better than a flat piece of metal?
2. Provide the “old antenna”, a 16 inch tetrahedron. Have students calculate volume, surface area, and Edge length of this tetrahedron and record it as stage 0 on the Finding Patterns Table.
3. Have each group construct stage 1 using 4 – 8 inch triangles to make a tetrahedron. Repeat this step to make 3 additional tetrahedrons. After making all 4 tetrahedrons, use them to construct the shape of the 16 inch tetrahedron. Calculate and record measurements for Stage 1 on Finding Patterns Table.
4. REPEAT for Stage 2 (using the 4 inch tetrahedrons) and Stage 3 (using the 2 inch tetrahedrons).
5. Generalize what would happen to volume, surface area, and total edge length as the Stage number increases.
6. Discuss why this makes fractals more beneficial than a flat piece of metal. (volume approaches 0, surface area remains the same, edge length increases to infinity)

Complement: How can something maintain its size but have no volume?

1. Help students construct an octahedron to physically represent the empty space of Stage 1. Calculate and record volume on Comparing Volume sheet.
2. Have students construct physical models of the additional empty space gained during Stage 2 and 3. Have students place these octahedrons in corresponding locations to begin building the complement. (Note: each stage of the complement will build onto the previous one)
3. Calculate the total volume of the complement for stage 2 and 3 and record on Comparing Volume Sheet.
4. Discuss observations – i.e. the sum of the volume of each Stage and the volume of the resulting complement equal 0 .

For more discussion of the complement: (<http://www.fractalnature.com/sierpinitetrahedron.html>)

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