Beyond "y=mx+b": Deepening students' understanding of linear relationships

Amy Hillen
Kennesaw State University

Kelly Edenfield
Carnegie Learning

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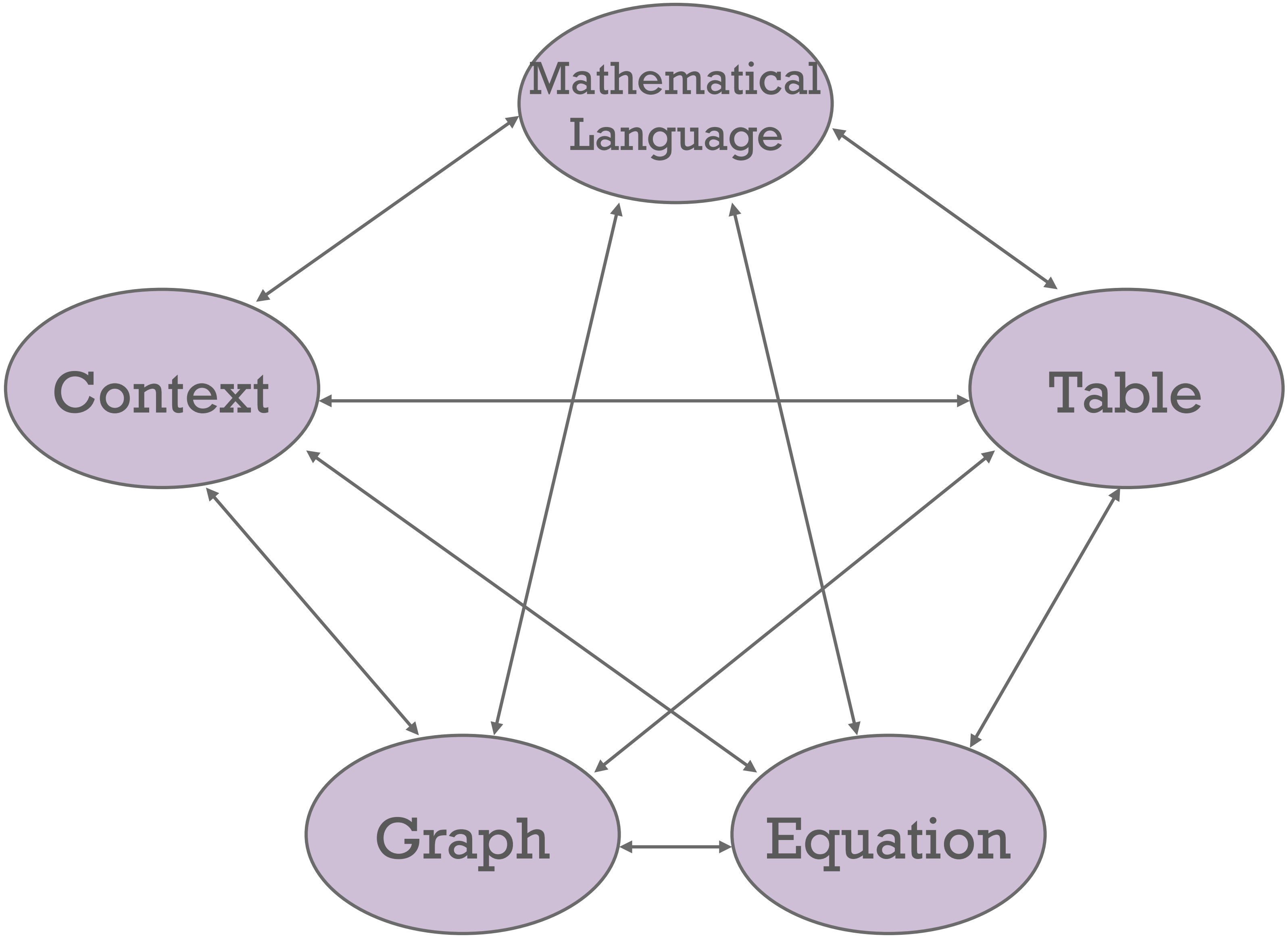
Overview of session

- Brief background on task sequence
- Quick journey through the task sequence
- Looking across the tasks
- Considering how the tasks deepen students' understanding of linear relationships

Why linear relationships? Why multiple representations?

- Key concept in the middle grades (CCSS, 2010)



Five Representations of a Function

Van de Walle, 2004

+ Why linear relationships? Why multiple representations?

- Key concept in the middle grades (CCSS, 2010)
- "A fluency in linking and translating among multiple representations seems to be critical in the development of algebraic thinking. The learner who can…move fluidly among different mathematical representations has access to a perspective on the mathematics in the problem that is greater than the perspective any one representation can provide." (Driscoll, 1999, p. 141)
- "Research has shown that children who have difficulty translating a concept from one representation to another are the same children who have difficulty solving problems and understanding computations. Strengthening the ability to move between and among these representations improves the growth of children's concepts." (Lesh, Post, & Behr, 1987)

Our approach

- Allow “y = mx + b” structure to emerge through the exploration of problems situated in variety of real-world contexts

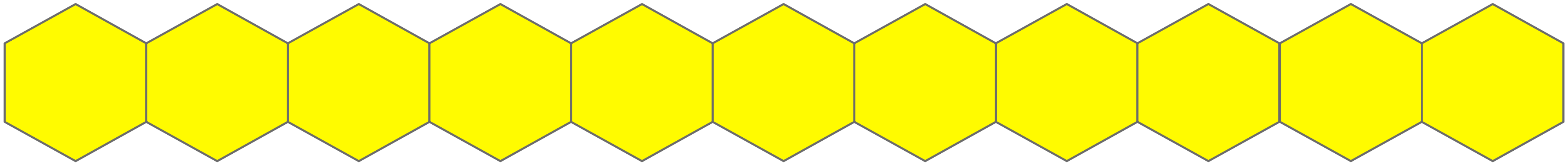
- Promote understanding of **why** this structure works by making connections between the real-world contexts and other representations

Exploring the three tasks

- Consider the three tasks on the handout
- In what ways would the learners with whom you work approach each task?
  - Consider multiple ways if you have time

- Consider: Is the "$y = mx + b$" structure the most useful structure for each task? Why or why not?

The Hexagon Task



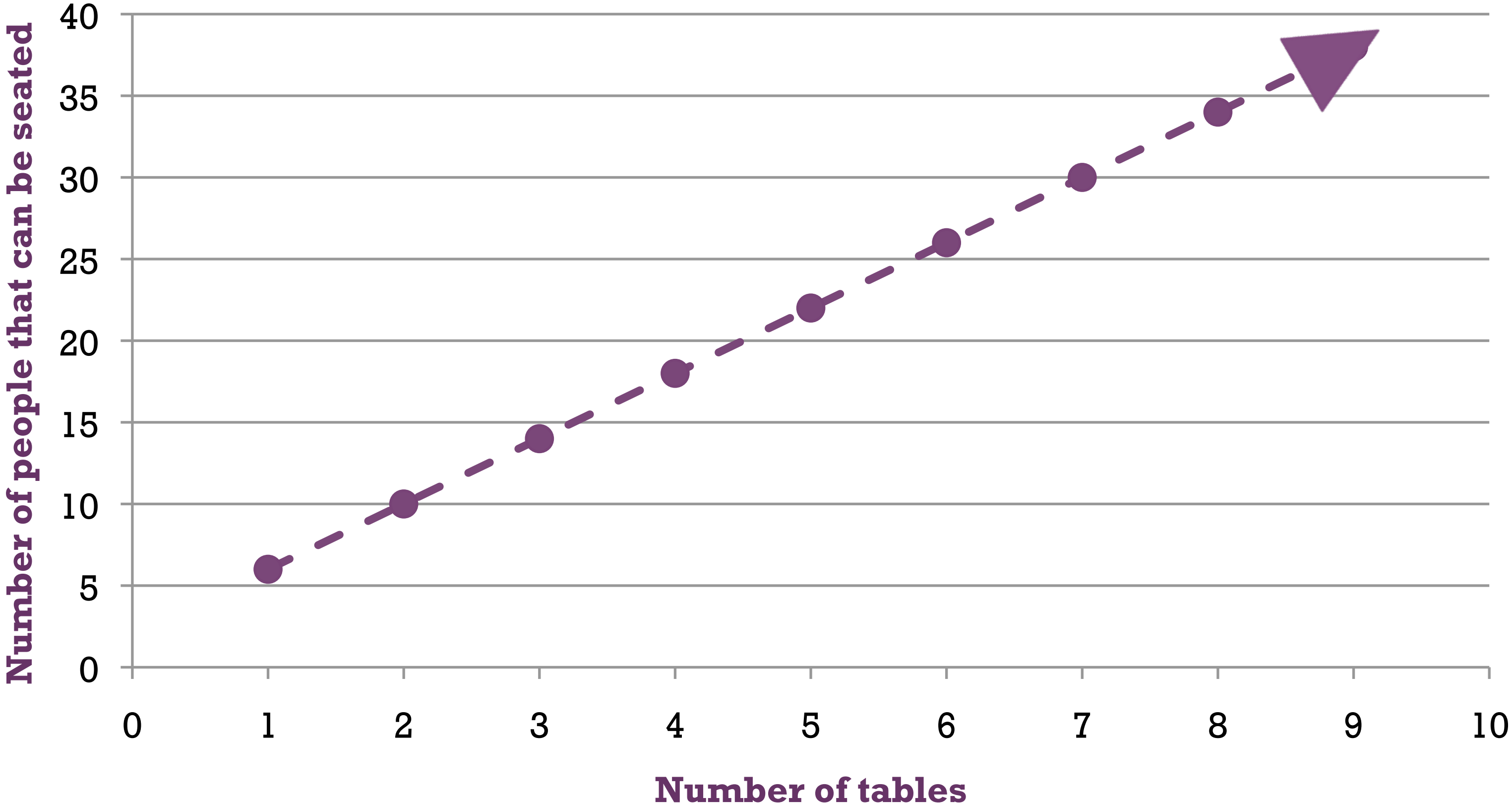
11 tables

How could you ***efficiently*** *determine the number of people that can be seated at 11 tables? n tables?*

The Hexagon Task Generalizations

1. $P = 4(x - 2) + 10$
2. $P = 6x - 2(x - 1)$
3. $P = 4x + 2$
4. $P = 6 + 4(x - 1)$
5. $P = 2(2x + 1)$

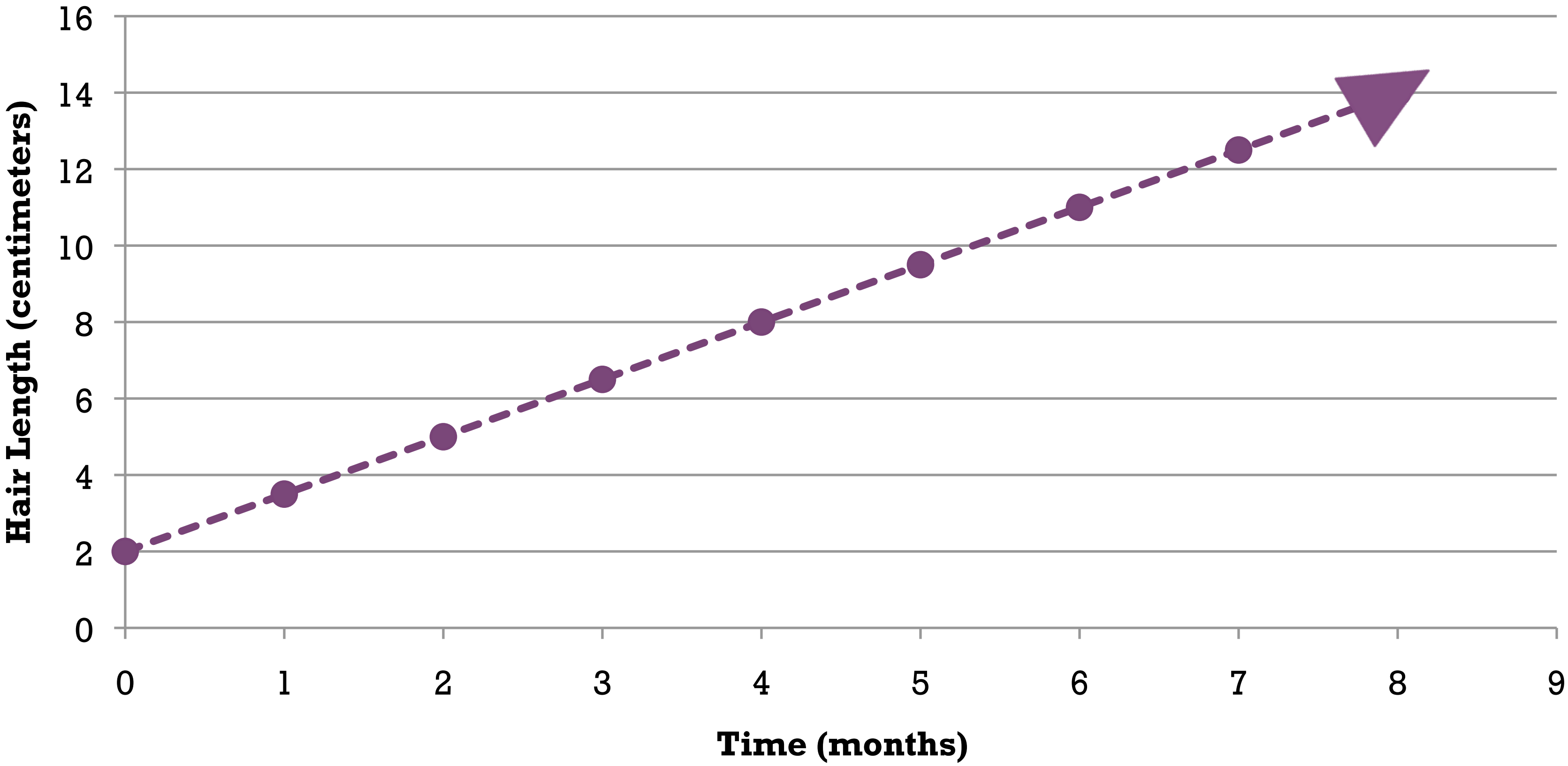
The Hexagon Task



Paul's Hair Growth

Month	Length of Paul's hair
1	3.5
2	5
3	6.5

Paul's Hair Growth



+ Paul's Hair Growth Generalizations

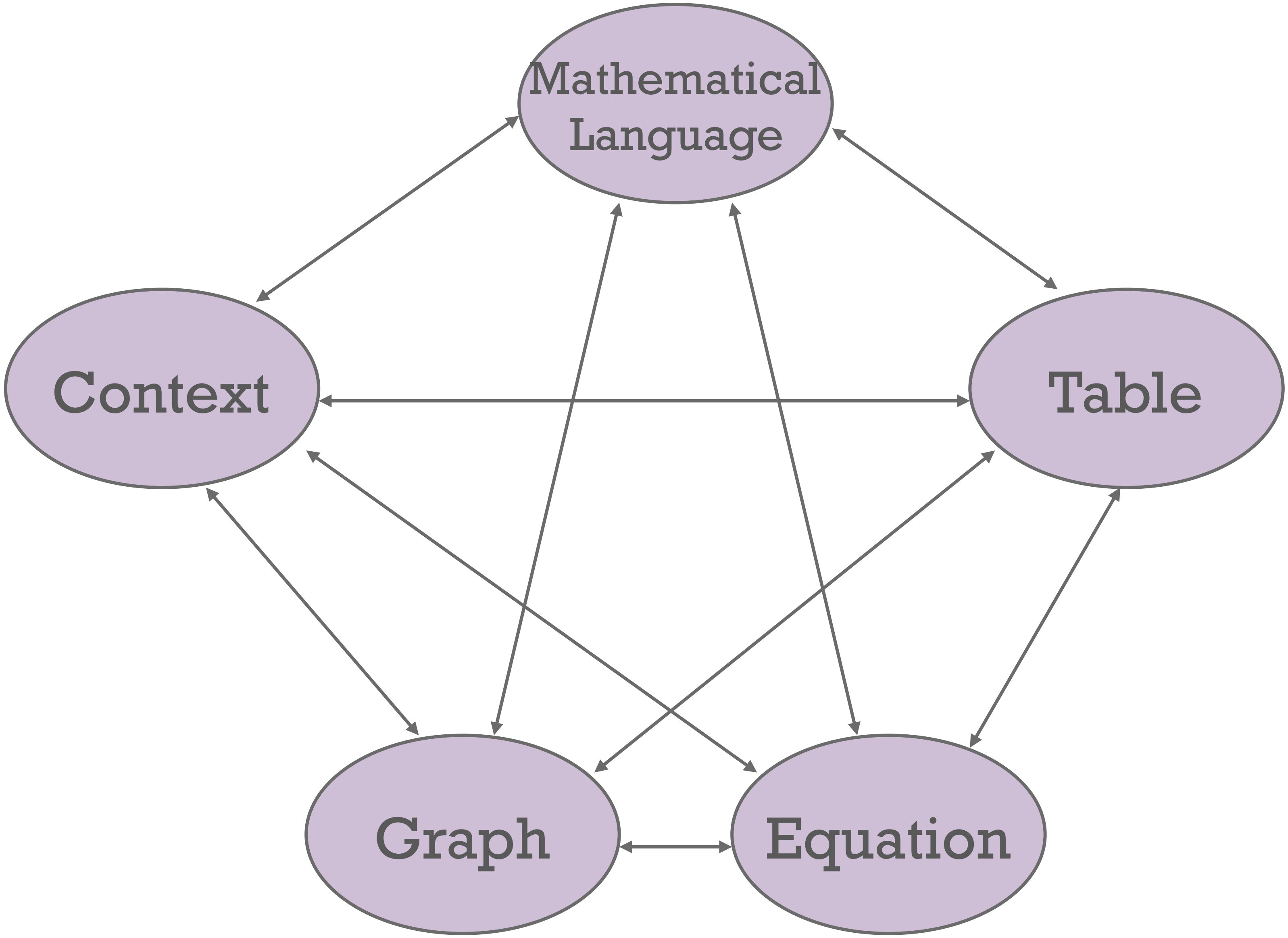
1. $y = 1.5x + 2$

- Teachers we've worked with often comment that writing Equation #1 as "$y = 2 + 1.5x$" is "better."
  - How might they be thinking? How might writing Equation #1 as "$y = 2 + 1.5x$" support students' understanding?

2. $y = 3.5 + 1.5(x - 1)$

"So, does this mean that we could make another equation that started at the 2nd row of the table – like $5 + 1.5(x - 2)$? And that we could keep doing this for the other rows of the table?"

- *Teacher from Amy's class*



Five Representations of a Function

Van de Walle, 2004

Paul's Hair Growth
Inserting some mathematical language/ terminology

$$y = 2 + 1.5x$$

We call the 1.5 the *slope*, or the *rate of change for the two quantities* (Sowder, Sowder, & Nickerson, 2010, p. 287)

We call the 2 the *y-intercept* – the point where the graph of a line crosses the y-axis. The y-intercept has the value of the dependent variable when the value of the independent variable is zero (Sowder, Sowder, & Nickerson, 2010, p. 288)

+ The Supermarkets Carts Task Generalizations *(in scale drawing cm)*

1. Thinking about the length of the first cart, plus the additional length that each additional cart adds (i.e., the part that's 'sticking out'): $S = 3.8 + 1.1(x - 1)$
2. Thinking about the length of all the carts if they weren't nested, and then subtracting the length that gets lost when you nest them: $S = 3.8x - 2.7(x - 1)$
3. Thinking about every cart as having the 'sticking out' part identified in #1, plus the length of the front of the first cart: $S = 2.7 + 1.1x$

Looking across the three tasks

Mathematical Practice 7: Look for and make use of structure.

"Mathematically proficient students look closely to discern a pattern or structure. …. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5–3(x–y)^2$ as 5 minus a positive number times a square and use that to realize that its value can not be more than 5 for any real numbers x and y." (CCSS, p. 8)

Teachers we've worked with often note that "our approaches to Supermarket Carts are similar to approaches we've used to solve some of our other problems."

+ Looking across the three tasks

- Structure 1:

  $y = 3.8 + 1.1(x - 1)$ (Supermarket Carts)

  $y = 3.5 + 1.5(x - 1)$ (Paul's Hair Growth)

  $y = 6 + 4(x - 1)$ (Hexagon Task)

- Structure 2:

  $y = 3.8x - 2.7(x - 1)$ (Supermarket Carts)

  $y = 6x - 2(x - 1)$ (Hexagon Task)

- Structure 3:

  $y = 2.7 + 1.1x$ (Supermarket Carts)

  $y = 2 + 1.5x$ (Paul's Hair Growth)

  $y = 2 + 4x$ (Hexagon Task)

"So does this mean that the number by itself is always the y-intercept? And that the number by the n or x is always the slope?"
- *Teacher from Amy's class*

How would you want the learners with whom you work to respond to this wondering?

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Take a few minutes to consider…

- In what situations is using the structure $y = mx + b$ *especially* useful? In what situations is using an alternate structure *more* useful than the $y = mx + b$ structure?
- How might this sequence of tasks support students' understanding of linear relationships, and in particular, why $y = mx + b$ "works"?
- How might you might adapt this sequence of tasks to meet the needs of the learners with whom you work?
  - What challenges might you face in implementing this sequence of tasks?
  - What misconceptions might students demonstrate? How might you address these misconceptions?
  - What task(s) might you use to follow this sequence of tasks?

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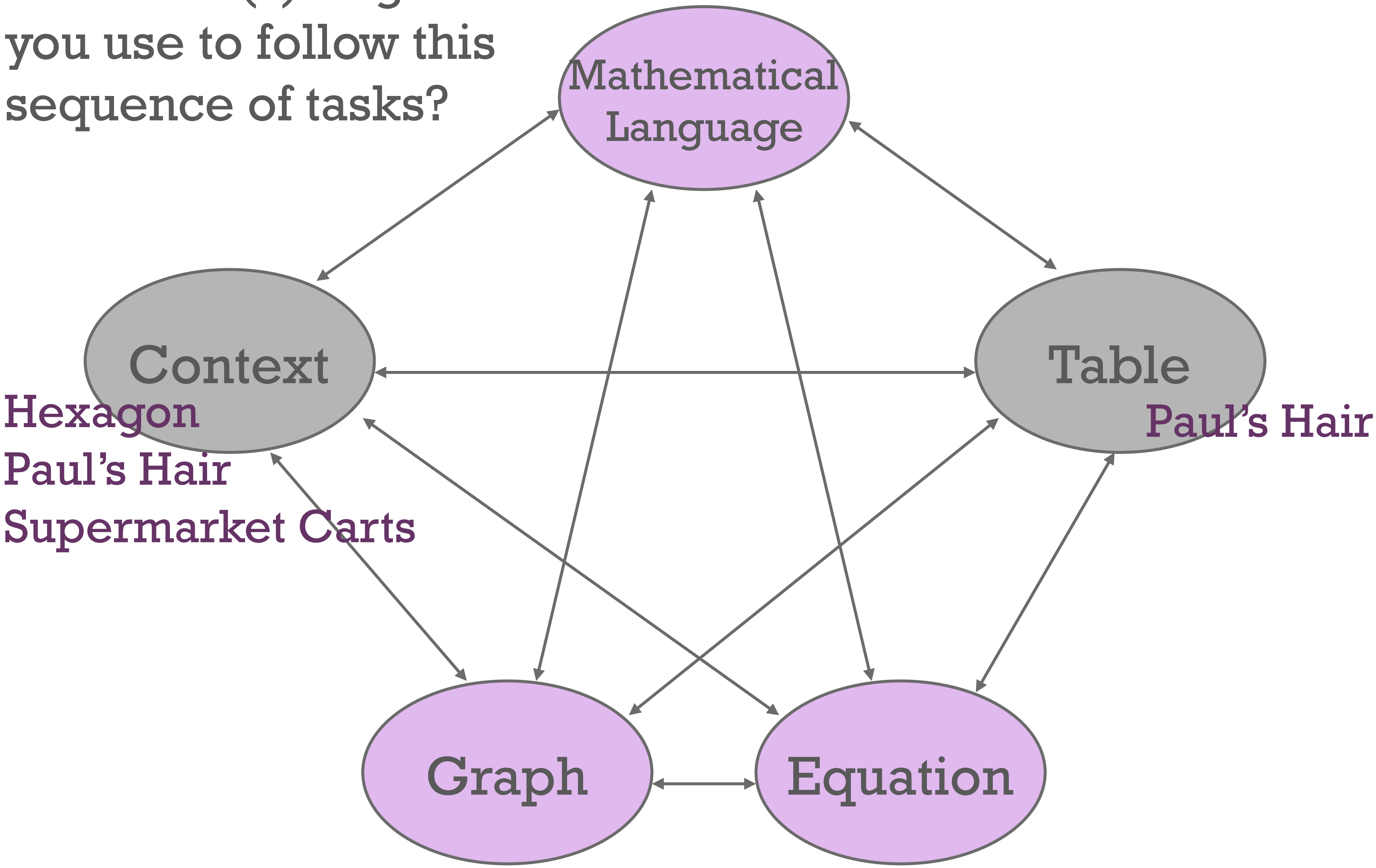

Five Representations of a Function

Van de Walle, 2004

Thank you for coming!

Amy Hillen

ahillen@kennesaw.edu

Kelly Edenfield

kedenfield@carnegielearning.com

The three tasks that were featured in this session were identified under the auspices of the ASTEROID (A Study in Teacher Education: Research on Instructional Design) project (NSF Award #0101799), PI: Margaret S. Smith.

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