Advanced Quantitative Reasoning

Meaningful Mathematics for High School Seniors

Gregory D. Foley
Daniel A. Showalter
Ohio University

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NCTM (2006) says, "Every student should study mathematics every year through high school, progressing to a more advanced level each year."

This talk presents rich problems
- that seniors find engaging,
- that connect a wide range of mathematics, statistics, and modeling, and
- that leverage mathematical action technologies and classroom discourse.

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Advanced Quantitative Reasoning (AQR) is a **course** in mathematics, statistics, and modeling for students who have completed Algebra I, Geometry, and Algebra II—or Integrated Mathematics I–III.

More and more states are requiring students to complete 4 years of high school mathematics (Zinth, 2012). AQR is a 4th-year course designed for the average student.

The AQR **project** is developing and testing student materials and teacher resources (75% done) for such a course. AQR has pilot-tested **textbook** materials for the past four school years and now is field-testing them at 10 high schools in 2012–2013.

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Our society thrives on numbers, yet many high school graduates are ill-equipped to make informed judgments using quantitative information.

Many graduates are not ready for the mathematical and statistical demands of college, with **35.1%** of U.S. college mathematics enrollments in remedial courses: **1.4 million out of 3.9 million** in fall 2010.
(Kirkman, Blair, & Maxwell, 2012)

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Perhaps the worst thing that can happen to a student at the end of his or her secondary mathematics preparation is to enter college not having studied mathematics after a lapse of a year or more.
(Seeley, 2004, p. 24)

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NCTM ***Math Takes Time*** position statement (2006):

- Every student should study mathematics every year through high school, **progressing to a more advanced level each year.**
- All students need to be **engaged** in learning **challenging mathematics.**
- At every grade level, students must have time to become engaged in mathematics that **promotes reasoning** and **fosters communication.**
- Evidence supports the enrollment of high school students in a mathematics course **every year,** continuing beyond the equivalent of a second year of algebra and a year of geometry (Adelman, 1999, 2006).

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The aims of the AQR *springboard* course are

- to **reinforce**, **build on**, and **solidify** the student's working knowledge of middle grades mathematics through Algebra I, Geometry, and Algebra II
- to develop the student's **quantitative literacy** for effective citizenship, for everyday decision making, for workplace readiness, and for postsecondary education
- to develop the student's ability to **investigate** and **solve** substantial problems and to **communicate** with precision
- to **prepare** the student for **postsecondary course work** in STEM and non-STEM fields—and
- *for students who complete the course in the 11th grade*—to **prepare** them to study **AP Statistics**, **AP Computer Sciences**, or **Precalculus** in their senior year of high school.

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Common Core **conceptual categories** for high school mathematics

- **Number & Quantity**
- **Algebra**
- **Functions**
- **Modeling**
- **Geometry**
- **Statistics & Probability**

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Common Core **standards for mathematical practice**

1. **Make sense** of problems and persevere in solving them.
2. **Reason** abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others. **[mathematical communication]**
4. **Model** with mathematics.
5. Use appropriate **tools** strategically.
6. Attend to **precision**.
7. Look for and make use of **structure**.
8. Look for and express **regularity** in repeated reasoning.

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Technology: A key **tool** for learning mathematics, statistics, and modeling

- Technology as **amplifier**, **reorganizer**, and **catalyst** to support learning
- Technology as a set of interconnected representational tools
- Technology as tools for structural exploration of ideas and relationships
- Technology as tools for measurement and for gathering genuine data
- Technology as tools for displaying content and interacting with students and content
- Technology as tools for sharing information: Internet, Moodle, Webinar

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Technology: TI-*n*spire CAS, GeoGebra, . . .

- Software with CAS, Graphing, Interactive geometry, Lists & spreadsheet, Data & statistics
- Linked representations
- Handheld, PC, Mac, and iPad versions
- Linking of hardware: Networking and data transfer
- Teacher-led and interactive instruction
- 3D, Trimble SketchUp, Spherical Easel, Google Earth

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Advanced Quantitative Reasoning **Course Outline**

Core. ***Quick questions***, ***explorations***, ***investigations***, examples, exercises, and increasingly involved projects and presentations

Four parts (units, modules)

- Number and Quantity
- Statistics and Probability
- Modeling with Algebra and Functions
- Modeling with Geometry

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Some AQR number and quantity topics

- Problem-solving strategies
- Quantity, measurement
- Fractions, decimals, percent
- Proportional reasoning
- Quarterback ratings and readability indices
- ID numbers and check digits
- Orders of magnitude (Richter scale) analysis
- Transition matrices
- Probability

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Fermi question

Roughly how many basketballs would it take to circle the Earth at the equator?

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Probability task

Drug testing. Suppose a recent national study indicates that about 3% of high school athletes use steroids and related performance-enhancing drugs. Suppose further that the accuracy of the standard test used is roughly 97%. *That means that 3% of the time, the test returns an incorrect result (either a false positive or a false negative).*

What is the probability that a randomly selected student athlete who tests "positive" is actually a user of performance-enhancing drugs?

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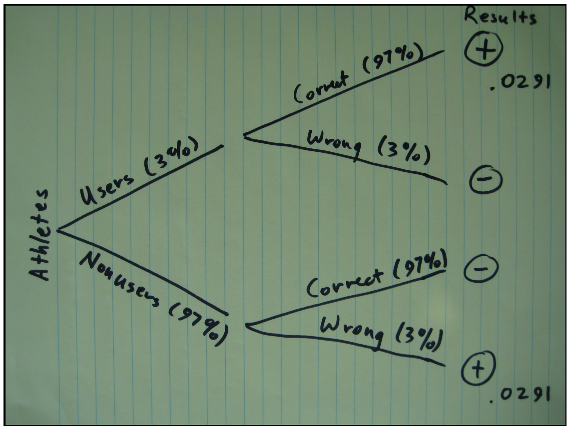

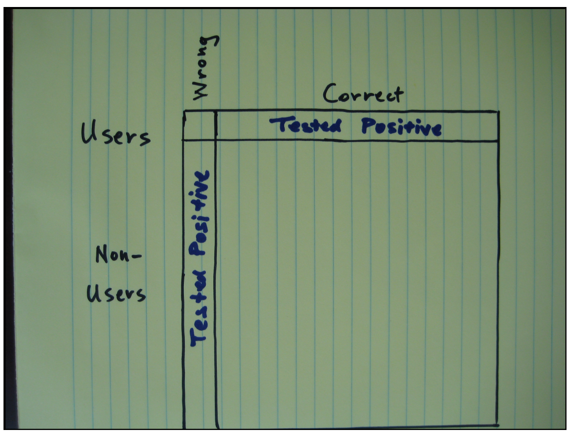

Designing and conducting a statistical study

Is a DoubleStuf® Oreo cookie really double stuffed?

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What concepts and relationships are involved?

Ancient Alligators. Alligators, crocodiles, and their relatives and ancestors are known as **crocodilians.** On Earth for 250 million years, they have survived mass extinctions that killed other animals. The modern American alligator thrives in the coastlands of the Southern states from Texas to the Carolinas. A typical adult male is about 12 ft long and weighs 800 lb.

Suppose that a scientific team excavates an ancient crocodilian with the same proportions (same shape) but twice as long (24 ft) as an adult male American alligator. How much would you expect this ancient beast to have weighed?

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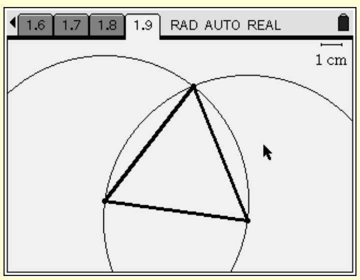

What is the relationship between the area and the side length of an equilateral triangle?

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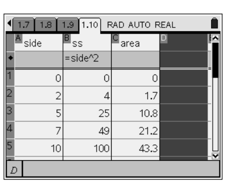
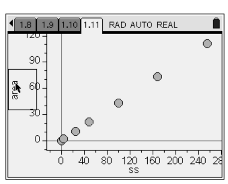
Data for the Side Length and Corresponding Area of an Equilateral Triangle

Side length *s*	0	2	5	7	10	13	16
Area *A*	0	1.7	10.8	21.2	43.3	73.2	110.9





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Modeling activity: Zipf's law

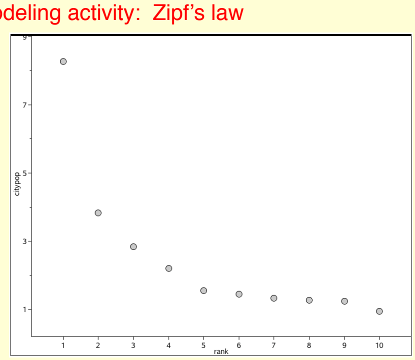
"In a given country . . . , the largest city is always about twice as big as the second largest, and three times as big as the third largest, and so on" (Strogatz, 2009).

How accurate is this model for city population in the United States?

2010 U.S. Census Bureau Data
(in millions of persons)

1	New York	8.175
2	Los Angeles	3.793
3	Chicago	2.696
4	Houston	2.099
5	Philadelphia	1.526
6	Phoenix	1.446
7	San Antonio	1.327
8	San Diego	1.307
9	Dallas	1.198
10	San Jose	0.946

Modeling activity: Zipf's law



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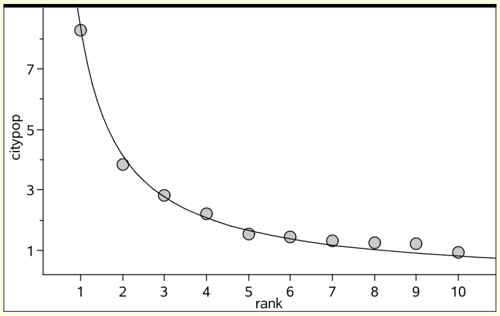

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Modeling longitude and latitude with fruit

(a) Identify two **antipodal points** as the north and south poles. Mark and label the two points as *N* and *S*.

(b) Draw an arc from the north pole to the south pole. Label it as 0°longitude. This is the **prime meridian.**

(c) Locate and mark the midpoint of the prime meridian. Mark the midpoints for 4 or 5 other meridians (lines of longitude). Draw in the great circle that represents the equator.

(d) With the aid of a globe, an atlas, or the Internet—locate, mark, and label points for London, England; Quito, Ecuador; and your home city or county on the orange.

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Geometric modeling: Spherical geometry

If you are traveling along a great-circle shortest path from Athens, Texas to Athens, Greece will you pass closer to Athens, Georgia, USA or Athens, Ohio?

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Advanced Teacher Capacity: An associated professional development initiative

- Two-week summer institutes (60 contact hours).
- Two daylong follow-up workshops plus online support
- **TPACK:** technology, pedagogy, & content knowledge
- **Technology:** TI-*n*spire (additional tools for Modspar)
- **Pedagogy:** Tasks, tools, and talk
- **QUANT:** Statistics and probability
- **Modspar:** Modeling and spatial reasoning

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QUANT: Quantifying uncertainty and analyzing numerical trends

- Words of statistics; measurement and data collection
- Formulating statistical questions and designing of statistical studies
- Data analysis and descriptive statistics
- Combinatorics, random processes, and probability, including conditional probability
- Using data, probability, and distributions to justify conclusions and to make decisions

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Modspar: Modeling and spatial reasoning

- What is modeling?
- Discrete dynamical systems: Finite differences, difference equations, web plots
- Recursively and explicitly defined functions
- General proportional model and reexpressing data
- Modeling with polar and parametric equations
- Three-dimensional geometry and modeling
- Spherical geometry and modeling

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