NCTM

Exploring Polygon Properties with a Piece of Rope

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Van Hiele Levels of Geometric Understanding

Level 0: Pre-recognition
Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

Level 1: Visualization -- images
A student functioning at this level can recognize and name a shape such as a triangle based upon their knowledge of a triangle image. However, if the picture of a triangle is flipped so the base is at the top and the vertex is pointed downward, this student may not call it a triangle, because it does not exactly match the picture the student has in his or her head. The student is working from mental images, not definitions or properties of the shape.

Level 2: Analysis -- definitions
This level is characterized by student ability to identify characteristics of shapes and correctly use vocabulary related to the characteristics. For example, a student at level two may correctly identify a triangle because it has three sides and three angles. However, the student might not be able to perceive that a figure having 3 angles is sufficient to make the figure a triangle.

Level 3: Abstraction (or informal deduction) -- relationships
Students at this level can recognize relationships between and among properties of shapes and are able to articulate reasons for classifying in certain ways. For example, a student may be able to compare the right angle of a triangle with the right angles of a rectangle. Logical implications (such as "if a quadrilateral is a square, then it is a rectangle") and class inclusions (such as "every rectangle is a parallelogram because it is just a special parallelogram with right angles") can be understood.

Level 4: Deduction
To be at this level, students can construct proofs, and understand the role of axioms and definitions. In order to be successful in high school geometry, students need to be functioning at this level.

Level 5: Rigor
Involves students working in non-Euclidean geometric systems. Symbols without referents can be manipulated according to the laws of formal logic. A student should understand the role and necessity of indirect proof and proof by contrapositive.

Additional Points:

1. The learner cannot achieve one level without passing through the previous levels.

2. Progress from one level to another is more dependent on educational experience than on age or maturation.

3. Certain types of experiences can facilitate or impede progress within a level or to a higher level.

Van Hiele Resources:

Burger, W. F. & Shaughnessy J.M.(1986). Characterising the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education,* 17 (1), 31-48.

Crowley, M. "The van Hiele Model of the Development of Geometric Thought." In M. Lindquist, ed., *Learning and Teaching Geometry, K-12, 1987 Yearbook.* Reston VA: National Council of Teachers of Mathematics.

Fuys, D., Geddes, D. and Tischler, R. (1988) The van Hiele model of thinking in geometry among adolescents. Monograph Number 3, *Journal for Research in Mathematics Education.* Reston VA: National Council of Teachers of Mathematics.

National Council of Teachers of Math (NCTM). (2001) *Navigating through Geometry in Grades 3 – 5.* Reston VA: National Council of Teachers of Mathematics.

National Council of Teachers of Math (NCTM). (2001) *Navigating through Geometry in Prekindergarten – Grade 2.* Reston VA: National Council of Teachers of Mathematics.

Teppo, Anne. "Van Hiele Levels of Geometric Thought Revisited." *Mathematics Teacher,* March 1991, pg 210-221.

van Hiele, Pierre M. "Developing Geometric Thinking through Activities That Begin with Play," *Teaching Children Mathematics 6* (February 1999): 374-378.

van Hiele, Pierre M. (1986) *Structure and Insight. A Theory of Mathematics Education.* Orlando: Academic Press.

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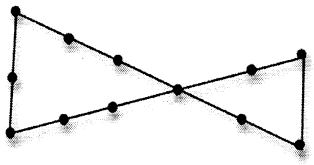
Developing Understanding of Quadrilateral Properties

A. Pre-requisite knowledge: (1) The sum of the angles of a triangle is 180°.
(2) The definition of a polygon. You may use the rope to review this as shown below.

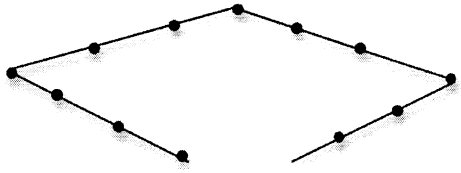
Create groups of 4 people. Give each group a rope or heavy string closed in a loop with 12 equal spaces. For purposes of this activity, all vertices of polygons will be made by holding knots on the rope. This means that side lengths will be natural numbers. As an extension, you may allow side lengths to be fractional.

Each group should make a polygon with their rope. In an effort to review the definition of a polygon, ask, "What properties does it have?" (in a plane, simple, closed, sides are segments). If students do not mention all four properties, demonstrate the following and ask, "Is this a polygon?"

- Take one of your rope loops and cross it over so it looks something like the figure below. (Not a polygon. To be a polygon, it must be "simple" or having no sides intersect.)



- Use a piece of rope that is not tied into a loop to make a shape that does not close such as the figure below. (Not a polygon. To be a polygon, it must be closed.)



- Form a loose circle with a rope loop and ask if it is a polygon. (Not a polygon. To be a polygon, it must be made of three or more line segments.)

- Try bending a quadrilateral so that it does not lie in a plane – form the quadrilateral on a table, then lift only one vertex off the table. (Not a polygon. To be a polygon, it must lie in one plane or flat surface.)

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B. Properties of Quadrilaterals: Use the following rope activities and questions to guide student exploration. Debrief each question with the class after they have time to explore. Ask students to explain their reasoning and ask if anyone else has a different way of looking at it.

1. Form a quadrilateral with one pair of opposite sides of length 2 and the other pair of length 4. As each person holds a knot (vertex), pull the rope tight enough to hold the sides straight. Give a name to each vertex: A, B, C, D (consecutively). While holding the sides tight, flex the quadrilateral to different positions.
   - *What stays the same?*
   - *What changes?*
   - *What do you notice about opposite sides?*
   - *What is the sum of the interior angles?*
   - *What do you notice about opposite angles?*
   - *When vertex A gets smaller, what happens to vertex B? C? D?*
   - *Do you know a more specialized name for this quadrilateral?*

2. Make angle A into a right angle. (You may use a square corner of a piece of paper to check your accuracy.)
   - *What happens to the other angles?*
   - *Will this always be true when you make one angle of a parallelogram a right angle? How do you know?*
   - *Is it still a quadrilateral?*
   - *Is it still a polygon?*
   - *Is it still a parallelogram? How do you know?*
   - *What other name can you give to this figure now?*
   - *Can you give a definition for "rectangle"?*

3. Make a parallelogram that has four sides equal in length.
   - *What is another name for this parallelogram?*

Flex the figure to different positions.
   - *What stays the same?*
   - *What changes?*
   - *What is the sum of the interior angles?*
   - *What do you notice about opposite angles?*
   - *Is it still a quadrilateral? Is it still a polygon?*

4. Make one of the angles of this rhombus a right angle, checking with your square corner.

- *What happens to the other angles?*
- *Will this always be true when you make one angle of a parallelogram a right angle? How do you know?*
- *Is it still a quadrilateral?*
- *Is it still a parallelogram?*
- *Is it still a polygon?*
- *What other name besides polygon, quadrilateral, parallelogram, and rhombus can you give to this figure now?*
- *Is it still a rectangle? How do you know?*

5. Diagonals: Have one member of your group use a piece of string to show a diagonal of the rectangle.

- *What do you think the definition of "diagonal" is?*
- *How many diagonals does a rectangle have?*
- *Make another quadrilateral that is not a rectangle. How many diagonals does it have?*
- *Can you make a generalization about how many diagonals a quadrilateral has?*
- *Does a diagonal have to be inside the polygon?*

6. Make a concave quadrilateral with a demonstration rope (this is called a dart) and show it to students. Ask them what they think the definition of "concave" is. Then ask them to make a different dart with their own ropes.

- *Can you make a concave quadrilateral with four congruent sides?*
- *How many diagonals does a dart have? Show them with a piece of string.*

7. Make a quadrilateral whose sides are 1-1-5-5 units. Make another whose sides are 2-2-4-4 units. These are examples of a kite.

- *How would you define "kite"?*
- *What appears to be true about the diagonals of a kite?*

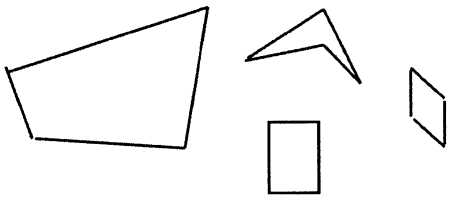
8. Try to make a trapezoid with your rope.

- *What properties of a trapezoid can you name?*

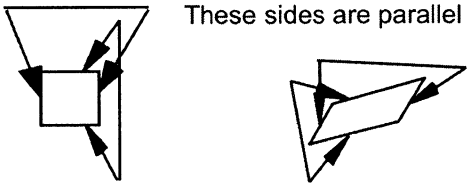
9. Give out the quadrilateral definitions sheet. Discuss each definition. Have students create an example of each as you discuss. Note that the orientations of the drawings may not be what is stereotypically shown in textbooks.

Types of Quadrilaterals

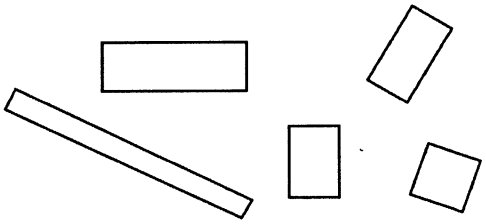
A **quadrilateral** is a **four**-sided polygon.



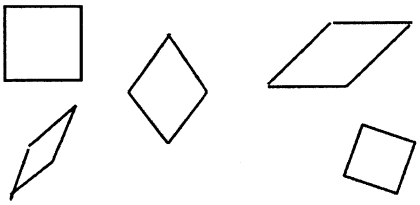
A **parallelogram** is a quadrilateral with both pairs of **opposite** sides **parallel**.



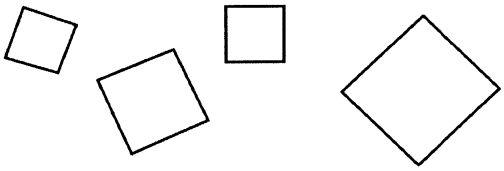
A **rectangle** is a quadrilateral with **four right angles**.



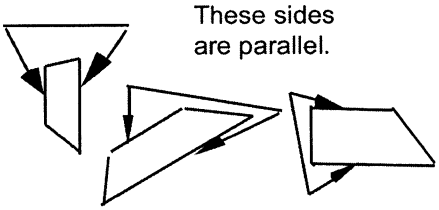
A **rhombus** is a quadrilateral with **four sides congruent**.



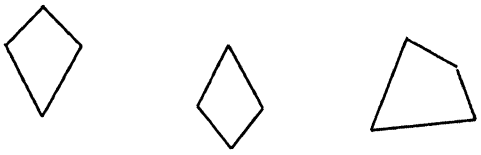
A **square** is a quadrilateral with **four right angles** and **four congruent sides.**



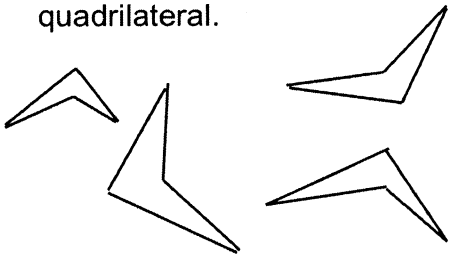
A **trapezoid** is a quadrilateral with **exactly one** pair of parallel sides.



A **kite** is a convex quadrilateral with two distinct pairs of **adjacent congruent sides**.



A **dart** is a **concave** quadrilateral.



Note: variations occur in international definitions of trapezoid and kite. You may see the following variations: A trapezoid is a quadrilateral with at least one pair of parallel sides. A kite can have all four sides congruent.

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