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Analyzing a Progression of Proportional Reasoning Strategies

NCTM – Denver 2013

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College of Education




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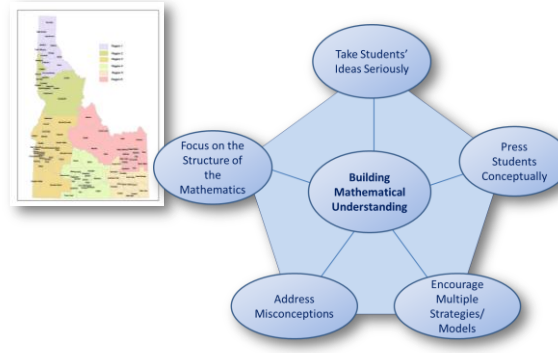
Goals

- Analyze student thinking strategies for proportional reasoning
- Examine a progression of those strategies
- Study and summarize the CCSS related to Proportional Reasoning

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CONTEXT & FRAMEWORK

Mathematical Thinking for Instruction Course
Dr. Jonathan Brendefur

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INTRODUCTION



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Middle School – Procedural Example Problem 1

Solve the proportion:

$$\frac{30}{6} = \frac{b}{7}$$

88% of Pre-Algebra students solved correctly

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Middle School – Application Example Problem 2

At a typical National Football League game, the ratio of males to females in attendance is 3:2. There are 75,000 spectators at an NFL game. How many of the spectators would you expect to be females?

How many Pre-Algebra students do you believe solved this problem correctly?

7% of Pre-Algebra students solved correctly

INTEGRATING TECHNOLOGY INTO MATHS TEACHING

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Goals

Analyze student proportional reasoning

Examine a progression of student strategies

Summarize the CCSS related to Proportional Reasoning

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Anticipating Student Strategies

- Solve the four problems as though you are a 7th grader who does not yet know the cross multiplication algorithm.
- Represent at least one of your solutions visually.

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- Ellie estimates that it takes her 5 hours to walk 8 miles. How many hours would she walk if she walked 48 miles?
- Jane estimates that she takes 8 hours to go 12 miles. How many miles would she walk in 42 hours?
- Quinten is an extreme trail runner and estimates that he takes 3 hours to run 9 miles. How many hours would it take for him to run 24 miles?
- Sierra is also a trail runner. She estimates that she runs 8 miles in 3 hours. If she runs for 2 miles, how long has she run?

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Examining Student Work

- What is similar or different about the way the students solved the problems?
- What in the context or number choice of the problem might have led students to use a particular strategy or model?
- From the student work, what can you say about each student's depth of understanding about proportional relationships? How would you sort them?

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Student Work A

1. Ellie estimates that it takes her 5 hours to walk 8 miles. How many hours would she walk if she walked 48 miles?

hours	5	10	20	30
miles	8	16	32	48

* 30 hours

Student Work B

1. Ellie estimates that it takes her 5 hours to walk 8 miles. How many hours would she walk if she walked 48 miles?

Student Work C

1. Ellie estimates that it takes her 5 hours to walk 8 miles. How many hours would she walk if she walked 48 miles?

5 hr 30 hours
8 mi 48

Student Work D
 2. Jane estimates that she takes 8 hours to go 12 miles. How many miles would she walk in 42 hours?

$$\begin{array}{r} \text{miles} \\ 12 \overline{) 144} \\ \underline{24} \\ 120 \\ \underline{120} \\ 0 \end{array}$$
 (6.3 miles in 4.2 hours)

Student Work E
 2. Jane estimates that she takes 8 hours to go 12 miles. How many miles would she walk in 42 hours?

$$\begin{array}{r} \text{mi} \\ 8 \overline{) 42} \\ \underline{16} \\ 26 \\ \underline{24} \\ 2 \end{array}$$

Student Work F
 2. Jane estimates that she takes 8 hours to go 12 miles. How many miles would she walk in 42 hours?

$$\begin{array}{r} \text{hrs} \\ 8 \overline{) 42} \\ \underline{32} \\ 10 \end{array}$$

$$\begin{array}{r} \text{miles} \\ 12 \overline{) 108} \\ \underline{24} \\ 84 \\ \underline{84} \\ 0 \end{array}$$

Student Work G
 3. Quinten is an extreme trail runner and estimates that he takes 3 hours to run 9 miles. How many hours would it take for him to run 24 miles?

$$\frac{3 \text{ hrs}}{9 \text{ mi}} \times \frac{24 \text{ mi}}{1} = 8 \text{ hrs}$$

Student Work H
 3. Quinten is an extreme trail runner and estimates that he takes 3 hours to run 9 miles. How many hours would it take for him to run 24 miles?

$$\frac{3 \text{ hr}}{9 \text{ mi}} \times \frac{24 \text{ mi}}{1} = 8 \text{ hrs}$$

Student Work I
 3. Quinten is an extreme trail runner and estimates that he takes 3 hours to run 9 miles. How many hours would it take for him to run 24 miles?

$$\frac{3 \text{ hrs}}{9 \text{ mi}} \times \frac{24 \text{ mi}}{1} = 8 \text{ hrs}$$

 It would take 8 hours to run 24 miles.

Student Work J
 3. Quinten is an extreme trail runner and estimates that he takes 3 hours to run 9 miles. How many hours would it take for him to run 24 miles?

$$\frac{3 \text{ hr}}{9 \text{ mi}} \times \frac{24 \text{ mi}}{1} = 8 \text{ hrs}$$

Student Work K
 4. Sierra is also a trail runner. She estimates that she runs 8 miles in 3 hours. If she runs for 2 miles, how long has she run?

$$\begin{array}{r} \text{hrs} \\ 3 \overline{) 7.5} \\ \underline{6} \\ 15 \\ \underline{15} \\ 0 \end{array}$$

Student Work L
 4. Sierra is also a trail runner. She estimates that she runs 8 miles in 3 hours. If she runs for 2 miles, how long has she run?

$$8 \div 2 = 4$$

 It takes her 7.5 minutes to run 2 miles.

Student Work M
 4. Sierra is also a trail runner. She estimates that she runs 8 miles in 3 hours. If she runs for 2 miles, how long has she run?

$$\frac{3 \text{ hrs}}{8 \text{ mi}} \times \frac{2 \text{ mi}}{1} = 0.75 \text{ hrs}$$

 2 miles in 45 min.

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Goals

Analyze student proportional reasoning
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 Summarize the CCSS related to Proportional Reasoning

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Composed Unit View

How many blue for 9 orange?

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Composed Unit View

How many blue for 9 orange?
 $2 \times 4.5 = 9$

$6 \times 4.5 = 27$

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Student Work – Composed Unit

Student Work A (Doubling focus)

hours	5	10	20	30
miles	8	16	32	48

* 30 hours

Student Work B (Scaling up)

30 hrs

Student Work D (Scaling up and down)

ways	40	2	42
miles	12	13	63

(63 miles in 47 hours)

Student Work C (Scaling up)

hours $5 \times 6 = 30$ hours

miles $8 \times 6 = 48$

Multiplicative Comparison (within relationship)

How many blue for 9 orange?

Student Work Multiplicative Comparison

Student Work I

$4 + 8 = 12$
 $24 \div 3 = 8$ hours

If would take 8 hours to run 24 miles.

Student Work F

hrs	8	$\times 1.5$	42	$\times 1.5$
miles	12		163	

Graph and Equations

Jane estimates that she takes 8 hours to go 12 miles.

Where do you see...

- Unit rate?
- Composed unit thinking?
- Multiplicative comparison thinking?

$d = 1.5t$

$t = \frac{2}{3}d$

What is the relationship between...

- miles to hours?
- hours to miles?

How do ratio tables emphasize both composed unit and multiplicative thinking?

Goals

Analyze student proportional reasoning

Examine a progression of student strategies

Summarize the CCSS related to Proportional Reasoning

Progression in the CCSS

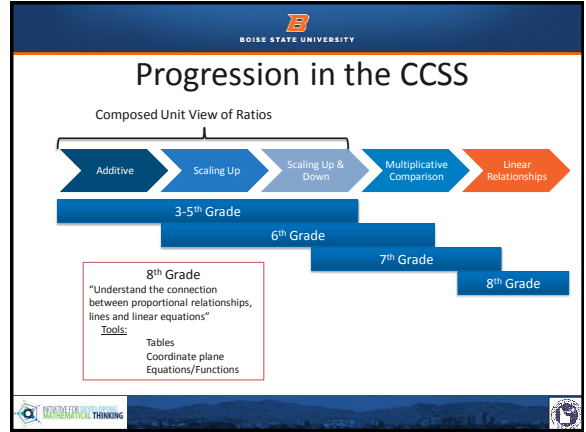
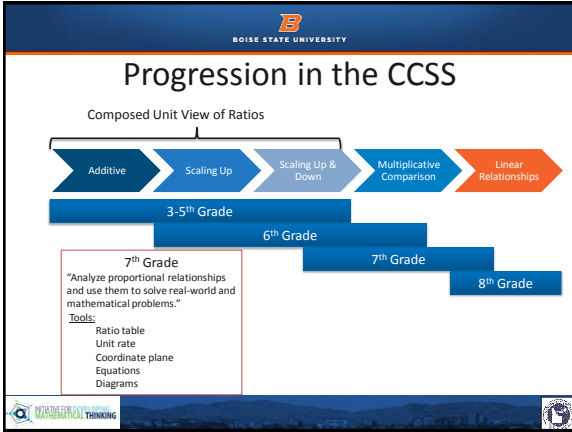
Composed Unit View of Ratios

6th Grade

"Understand ratio concepts and use ratio reasoning to solve problems"

Tools:

- Ratio table
- Coordinate plane
- Bar model or number line
- Unit rate
- Equations



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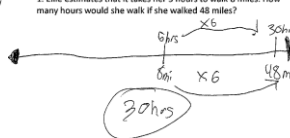
The MTI project is sponsored by the Idaho Legislature and the State Department of Education.

STUDENT WORK CARDS TO PRINT

1. Ellie estimates that it takes her 5 hours to walk 8 miles. How many hours would she walk if she walked 48 miles?
1. Jane estimates that she takes 8 hours to go 12 miles. How many miles would she walk in 42 hours?
1. Quinten is an extreme trail runner and estimates that he takes 3 hours to run 9 miles. How many hours would it take for him to run 24 miles?
1. Sierra is also a trail runner. She estimates that she runs 8 miles in 3 hours. If she runs for 2 miles, how long has she run?

Student Work A
 1. Ellie estimates that it takes her 5 hours to walk 8 miles. How many hours would she walk if she walked 48 miles?

$$\begin{array}{r} \text{hours} \ 5 \ | \ 10 \ 20 \ 30 \\ \text{miles} \ 8 \ | \ 16 \ 32 \ 48 \\ \hline \end{array}$$
 * 30 hours

Student Work B
 1. Ellie estimates that it takes her 5 hours to walk 8 miles. How many hours would she walk if she walked 48 miles?

 30 hrs

Student Work C
 1. Ellie estimates that it takes her 5 hours to walk 8 miles. How many hours would she walk if she walked 48 miles?

$$\begin{array}{l} 5 \times 6 = 30 \text{ hours} \\ 8 \times 6 = 48 \end{array}$$

Student Work D
 2. Jane estimates that she takes 8 hours to go 12 miles. How many miles would she walk in 42 hours?

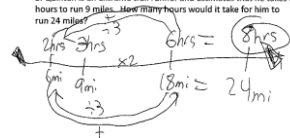
$$\begin{array}{r} \text{miles} \ 12 \ | \ 24 \ 36 \ 48 \\ \text{hrs} \ 8 \ | \ 16 \ 24 \ 32 \ 40 \ 48 \\ \hline \end{array}$$
 63 miles in 48 hours

Student Work E
 2. Jane estimates that she takes 8 hours to go 12 miles. How many miles would she walk in 42 hours?

$$\begin{array}{r} \text{mi} \ 12 \ | \ 42 \\ \text{hrs} \ 8 \ | \ 42 \\ \hline \end{array}$$

Student Work F
 2. Jane estimates that she takes 8 hours to go 12 miles. How many miles would she walk in 42 hours?

$$\begin{array}{r} \text{hrs} \ 8 \times 1.5 \ | \ 42 \times 1.5 \\ \text{miles} \ 12 \ | \ 63 \end{array}$$

Student Work G
 3. Quinten is an extreme trail runner and estimates that he takes 3 hours to run 9 miles. How many hours would it take for him to run 24 miles?

 8 hrs

Student Work H
 3. Quinten is an extreme trail runner and estimates that he takes 3 hours to run 9 miles. How many hours would it take for him to run 24 miles?

$$\begin{array}{r} 3 \text{ hr} \times \\ 9 \text{ mi} \ 24 \\ \hline \end{array}$$

$$\begin{array}{r} 24 \\ 9 \overline{) 24.00} \\ \underline{18} \\ 60 \\ \underline{54} \\ 60 \\ \underline{60} \\ 0 \end{array}$$

$$\begin{array}{r} 24 \\ 9 \overline{) 24.00} \\ \underline{18} \\ 60 \\ \underline{54} \\ 60 \\ \underline{60} \\ 0 \end{array}$$

Student Work I
 3. Quinten is an extreme trail runner and estimates that he takes 3 hours to run 9 miles. How many hours would it take for him to run 24 miles?

$$\begin{array}{l} 24 \div 3 = 8 \text{ hours} \\ \text{It would take 8 hours to run 24 miles.} \end{array}$$

Student Work J
 3. Quinten is an extreme trail runner and estimates that he takes 3 hours to run 9 miles. How many hours would it take for him to run 24 miles?

$$\begin{array}{r} \text{hrs} \ 3 \ | \ 9 \ 18 \\ \text{mi} \ 9 \ | \ 27 \ 36 \ 45 \\ \hline \end{array}$$
 24 miles in 8 hours

Student Work K
 4. Sierra is also a trail runner. She estimates that she runs 8 miles in 3 hours. If she runs for 2 miles, how long has she run?

$$\begin{array}{r} \text{hrs} \ 3 \ | \ 75 \\ \text{mi} \ 8 \ | \ 2 \\ \hline \end{array}$$
 4

Student Work L
 4. Sierra is also a trail runner. She estimates that she runs 8 miles in 3 hours. If she runs for 2 miles, how long has she run?

$$\begin{array}{r} 8 = 2 \times 4 \\ 4 \times 3 = 12 \\ \underline{12} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \end{array}$$
 It takes her 75 minutes to run 2 miles

Student Work M
 4. Sierra is also a trail runner. She estimates that she runs 8 miles in 3 hours. If she runs for 2 miles, how long has she run?

$$\begin{array}{r} \text{hrs} \ 3 \ | \ 15 \ 20 \\ \text{mi} \ 8 \ | \ 12 \\ \hline \end{array}$$
 2 miles in 45 min.

Student Work N
 4. Sierra is also a trail runner. She estimates that she runs 8 miles in 3 hours. If she runs for 2 miles, how long has she run?

$$\begin{array}{r} \text{hrs} \ 0 \ 2 \ 4 \\ \text{mi} \ 0 \ 0.25 \ 1.5 \ 3 \end{array}$$