Ideas for Revisiting Geometry Proofs in Algebra Class

André Mathurin
Bellarmine College Preparatory (San Jose, CA)

Competent algebra students know whether two lines are parallel or perpendicular by looking at the slopes, but can these students also communicate how slope values connect to geometry-based proofs of parallelism and perpendicularity? Participate in activities that offer ways for algebra students to revisit geometric concepts and proofs.

Contact & Resource Information

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Solve the following system of inequalities: $\begin{cases} x+6>20 \\ 20+x>6 \\ 20+6>x \end{cases}$

Given: A triangle whose sides measure 6, 20, and 23

Prove: The triangle is not a right triangle.

STATEMENTS	REASONS

Preliminaries

- Goals
- Format
- Disclaimers

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  - ✓ *Emphasize the Importance of Justification*
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  - ✓ *Ask an Algebra-Framed Question*
  - ✓ *Seek Out a Geometric Justification*
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- ✷ Disclaimers
  - ✓ *Just Say No to Analytic Geom/Trig*
  - ✓ *Don’t Alert the Newspapers*
  - ✓ *Your Call: Loosey-Goosey or Extreme Rigor*
  - ✓ *Look for the Hidden Gem*

❶ Visualizing SQUARE ROOTS

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Explain how to do it using Geometry.

1st Step: Draw a segment and label it $\overline{AB}$

2nd Step: Construct a line through A that is perpendicular to $\overline{AB}$

3rd Step: On the line, construct $\overline{AC}$ so that it is congruent to $\overline{AB}$

4th Step: Draw the segment $\overline{BC}$

➊ Visualizing SQUARE ROOTS

Is it possible to make a line segment that has the length $\sqrt{n}$ for any positive integer n?

Can you make a segment that has length $\sqrt{3}$?
Explain how to do it using Geometry.

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Justify why the above statement is true.

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Proof By Induction

We've shown that it can be done for up to n=3

Assume that we can make a line segment that has the length $\sqrt{n}$

1st Step: Draw a segment with length $\sqrt{n}$ and label it $\overline{AB}$

2nd Step: Construct a line through A that is perpendicular to $\overline{AB}$

3rd Step: On the line, construct $\overline{AC}$ so that it has length $\sqrt{1}$

4th Step: Draw the segment $\overline{BC}$

By the Pythagorean Theorem, $(\overline{AB})^2 + (\overline{AC})^2 = (\overline{BC})^2$

So $(\sqrt{n})^2 + \left(\sqrt{1}\right)^2 = n + 1 = (\overline{BC})^2$

Thus $\overline{BC} = \sqrt{n+1}$

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IT IS possible to make a line segment that has the length $\sqrt{n}$ for any positive integer n?

Now you can make an Irrational Ruler

☺

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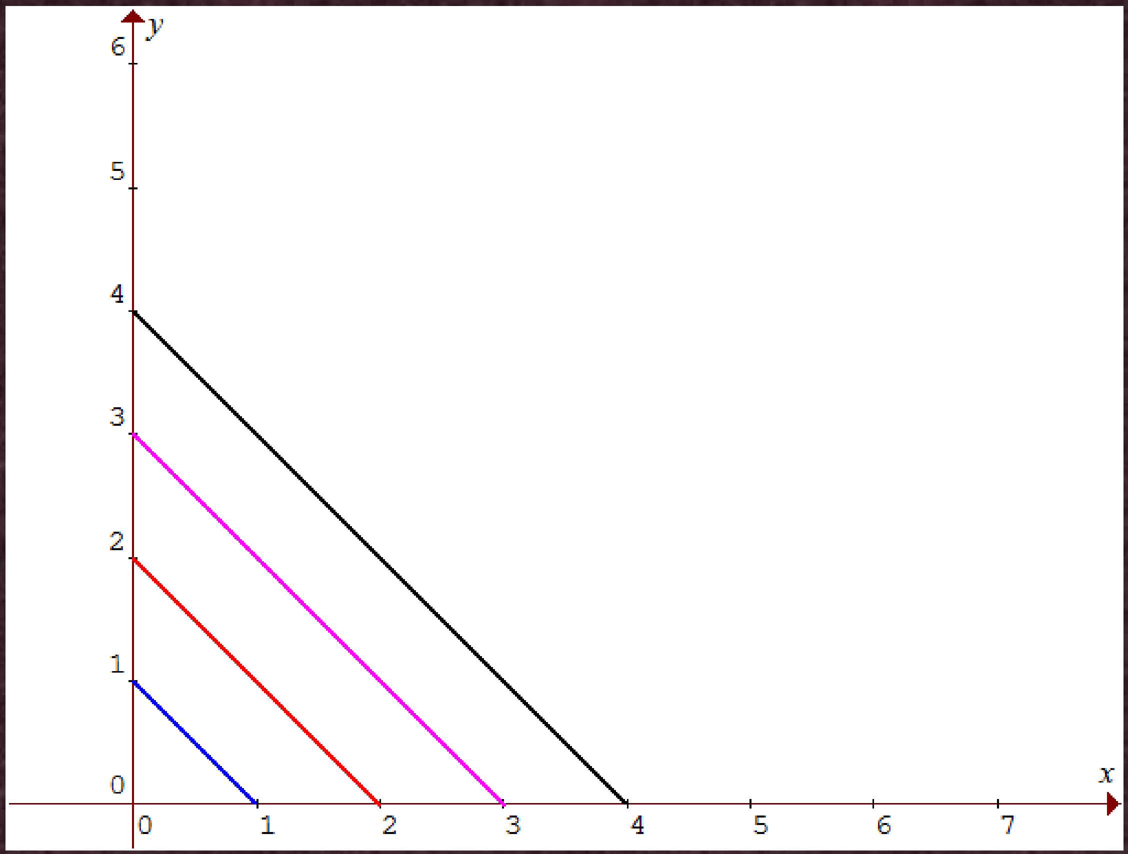

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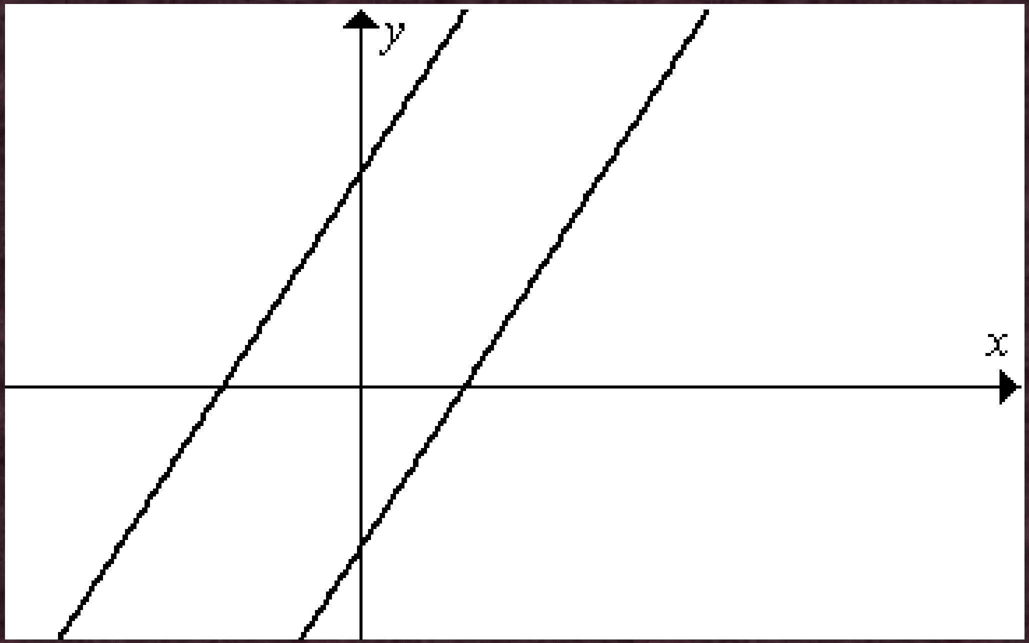
Two lines with the same slope are parallel.
Why is that?

Because lines with the same slope don't intersect.
Why?

Because if they have the same slope, they are going the same direction.
OK – but what about the same direction means are they parallel?

They are moving at the same rate, so one never catches up.

❷ How do you KNOW the lines are Parallel?



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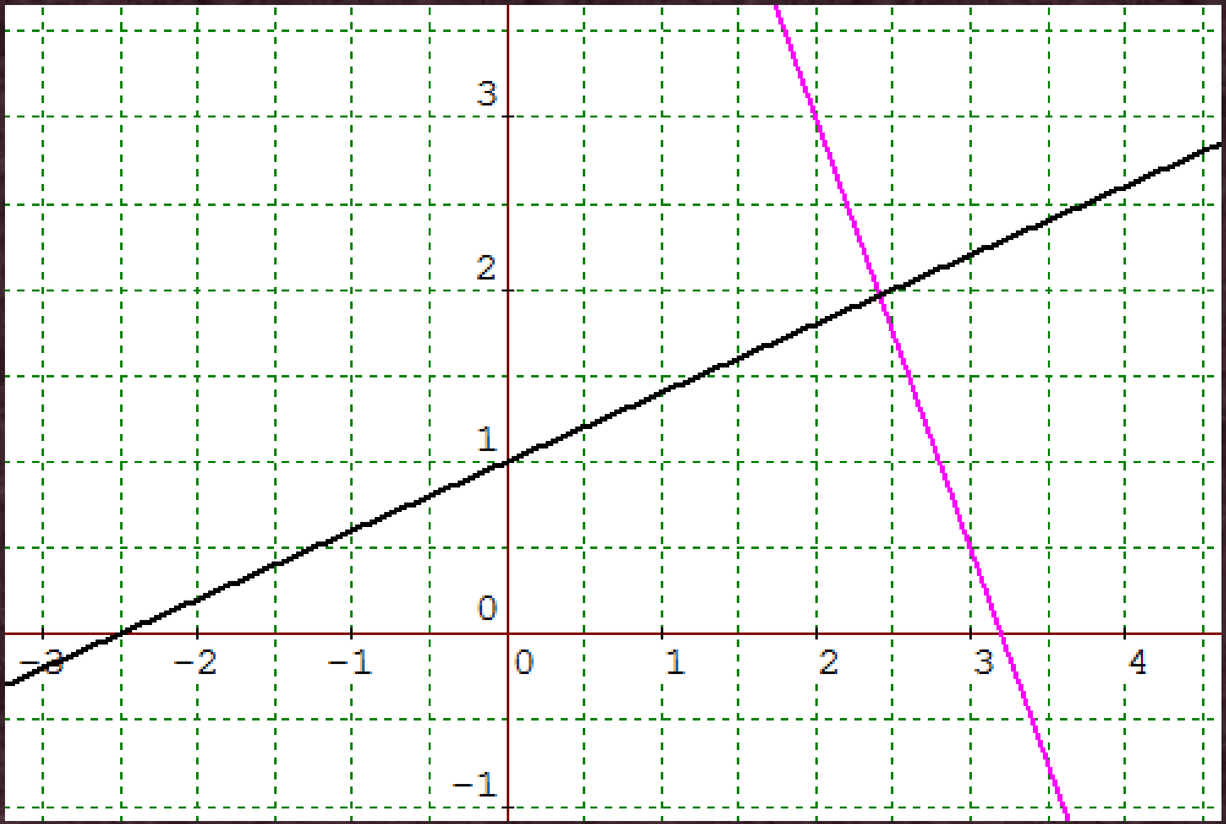

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❹ Classifying Triangles WITHOUT Using a Protractor

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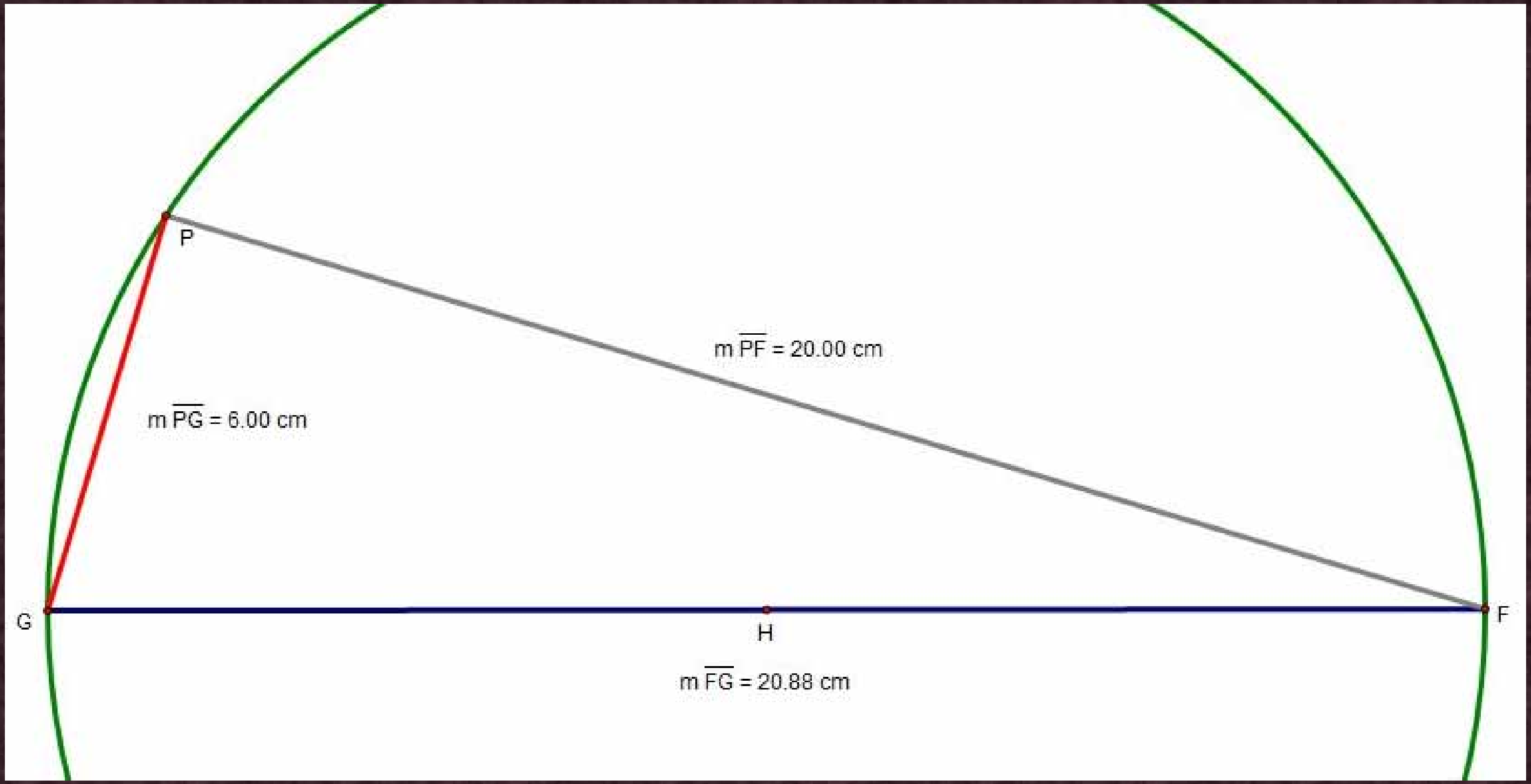

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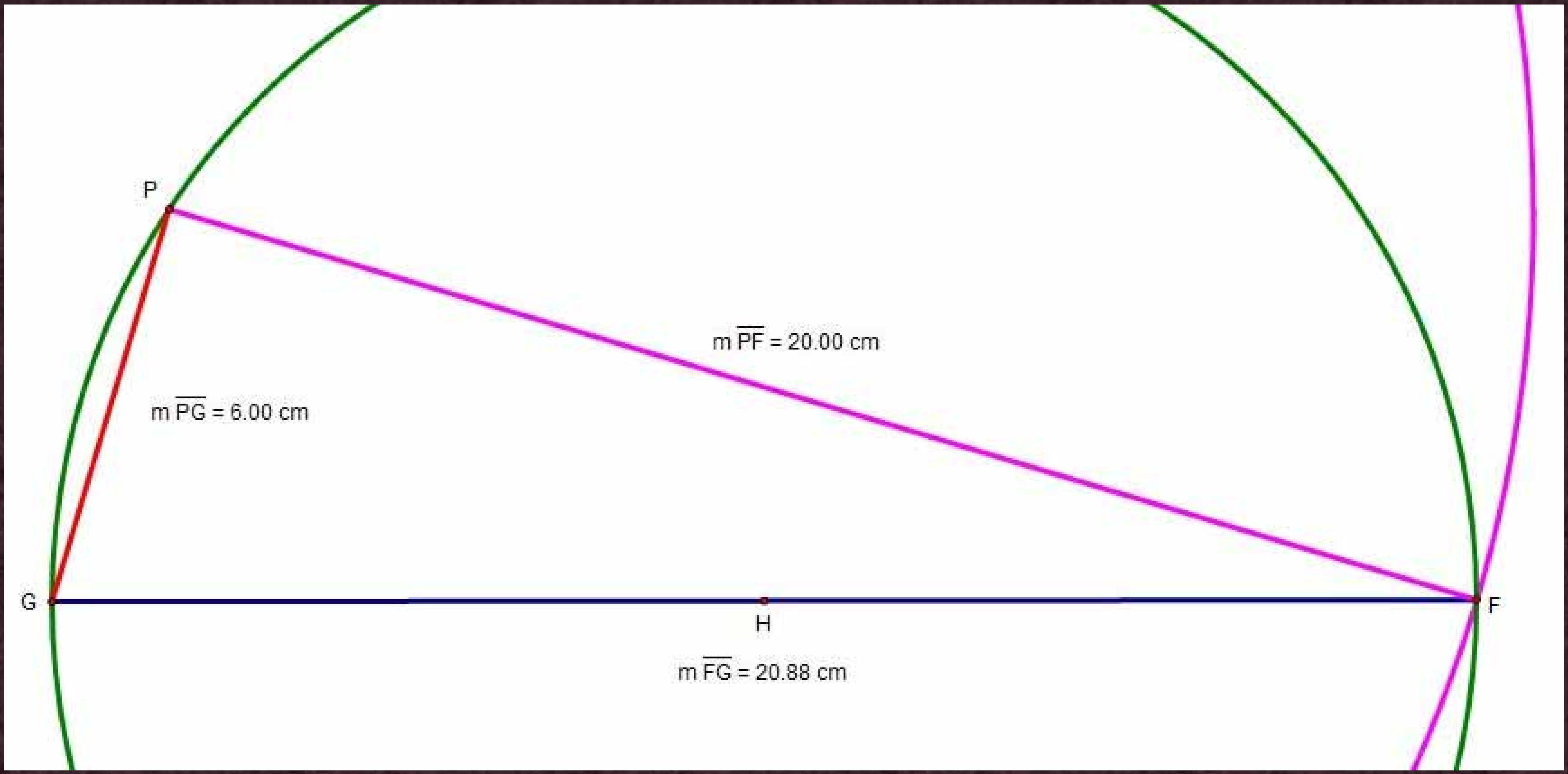

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