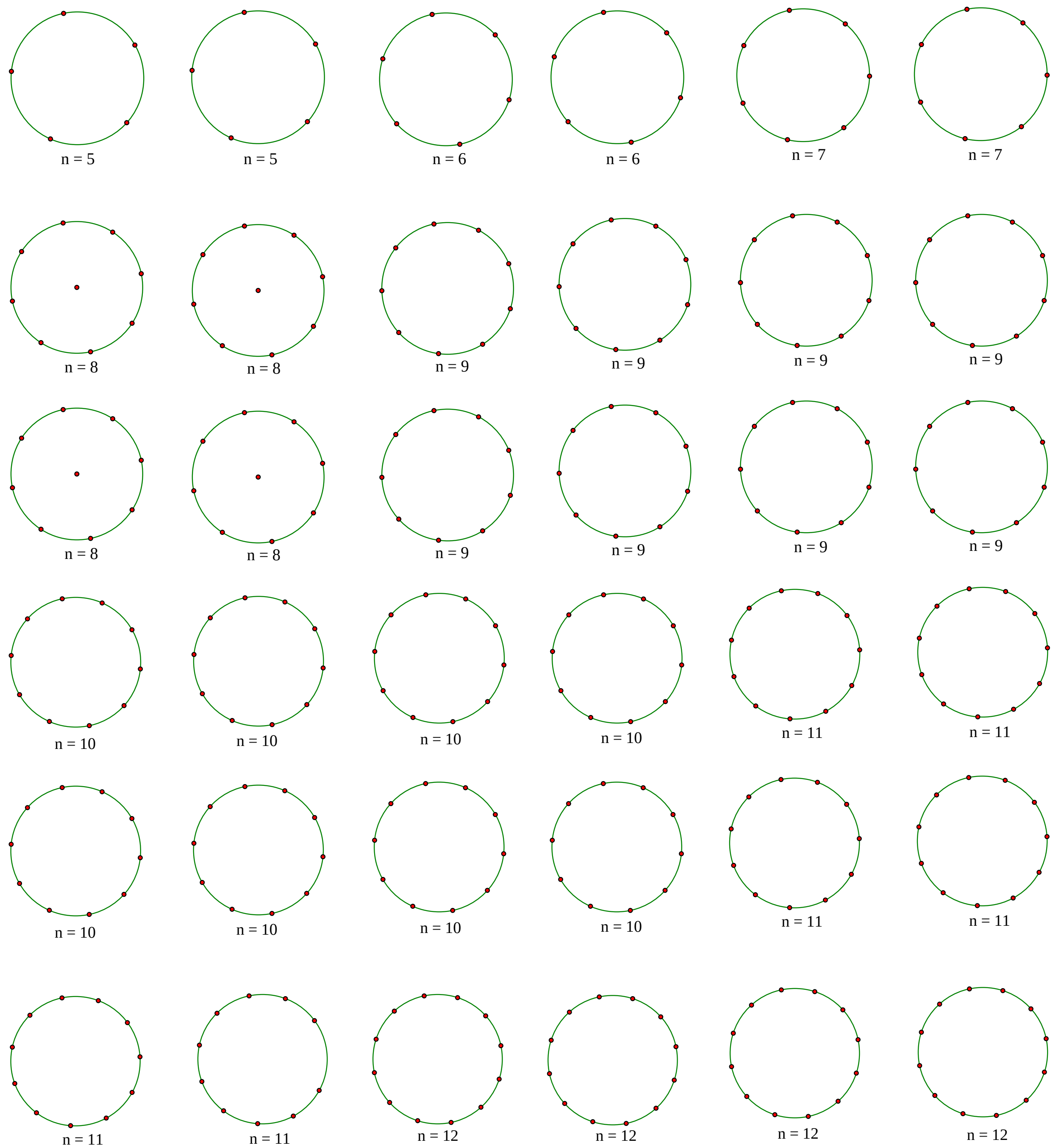
Activity Sheet #1 Presentation #617, Annin/Aguayo,

"Visualizing Patterns: Fibonacci Numbers and 1,000-Pointed Stars"



<u>Activity Sheet #2</u>

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Examine the figures you have drawn on Activity Sheet #1 again. For each value of n (from n = 2 to n = 12), list in the table below all pairs of values of j that are shown next to each figure that is a star. Then indicate how many n-pointed stars you found.

n	Pairs of j values producing stars	Number of n-pointed stars
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

<u>Activity Sheet #3</u> Presentation #617, Annin/Aguayo,

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#1) Circle the numbers below that are relatively prime to n = 20 and determine the number of stars with 20 points.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**

#2) Circle the numbers below that are relatively prime to n = 45 and determine the number of stars with 45 points.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**

#3) How many numbers from 1 to 100 contain a factor of 2 ? ______________

How many numbers from 1 to 100 contain a factor of 5 ? ______________

How many numbers from 1 to 100 contain a factor of both 2 and 5 ? ______________

How many numbers from 1 to 100 are relatively prime to 100 ? ______________

How many 100-pointed stars are there? ____________________

#4) How many numbers from 1 to 1000 contain a factor of 2 ? _____________

How many numbers from 1 to 1000 contain a factor of 5 ? _____________

How many numbers from 1 to 1000 contain a factor of both 2 and 5 ? _____________

How many numbers from 1 to 1000 are relatively prime to 1000 ? ______________

How many 1000-pointed stars are there? ____________________

#5) As time permits, choose some other values of n and determine how many n-pointed stars there are.

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Directions:

You are being asked to arrange tiles using only two types of tiles. One type of tile is of length 1 by 1 and the other type of tile is twice as long (1 by 2 tile).

Objective: Before you begin to tile you must discover a way to look at all possible arrangements of length 1 by *n*, where *n* is the length of the space available to tile.

First, how many different ways can you arrange tiles if you are only required to arrange them to make 1 by 1 tile arrangement?

Second, how many different ways can you arrange tiles if you are only required to arrange them to make 1 by 2 tile arrangement?

Next, how many different ways can you arrange tiles if you are only required to arrange them to make 1 by 3 tile arrangement?

Finally, how many different ways can you arrange tiles if you are only required to arrange them to make 1 by 4 tile arrangement?

What do you predict will be the number of different ways you can arrange tiles for a 1 by 5? Or 1 by 6? Or 1 by *n*?

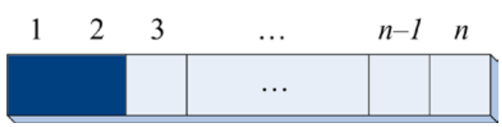
Consider looking at the number of different ways to arrange tiles of length 1 by n using a table. We let $f(n)$ represent the number of different arrangements possible given the length to be used to arrange tiles is 1 by n.

$f(n)$	Number of different ways	Picture of Arrangements
$f(1)$		
$f(2)$		
$f(3)$		
$f(4)$		
$f(5)$		
$f(6)$		
…	…	…
$f(n)$		

Consider a combinatorial interpretation, how many ways to arrange tiles of 1 by n length with 1 by 1 tiles and 1 by 2 tiles?

If the first tile is a 1 by 1 tile, then there are $f(n\text{-}1)$ ways to complete the arrangement.

If the first tile is a 1 by 2 tile, then there are $f(n\text{-}2)$ ways to complete the arrangement.



Hence,

$$f(n) = f(n\text{-}1) + f(n\text{-}2).$$

The Fibonacci sequence is the list of numbers where the first two numbers $f(0) = 1$ and $f(1) = 1$ generated the following list by adding the two previous numbers in your list. For example, the list of the Fibonacci numbers:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, …

From the previous table, we express a given entry of the table as a linear combination of the two previous entries in the same column.

$f(n)$	Value	Linear Combination		
$f(0)$	1	Previous Entry		2^{cd} to Previous Entry
$f(1)$	1			
$f(2)$	2	1(1)	+	1(1)
$f(3)$	3	1(2)	+	1(1)
$f(4)$	5	1(3)	+	1(2)
$f(5)$	8	1(5)	+	1(3)
…				
$f(n)$		$1*f(n-1)$	+	$1*f(n-2)$

Now express the linear combination to generate our list of the Fibonacci numbers if we were to arrange them using two columns filling the first row with the first two numbers, second row with the next two numbers, and so on.

$f(n)$	Value	Linear Combination		
$f(0)$	1			
$f(2)$	2			
$f(4)$	5	__ (2)	+	__ (1)
$f(6)$	13	__ (5)	+	__ (2)
$f(8)$	34	__ (13)	+	__ (5)
$f(10)$	89	__ (34)	+	__ (13)
…				
f(n)		__$*f(n-2)$	+	__$*f(n-4)$

$f(n)$	Value	Linear Combination		
$f(1)$	1			
$f(3)$	3			
$f(5)$	8	__ (3)	+	__ (1)
$f(7)$	21	__ (8)	+	__ (3)
$f(9)$	55	__ (21)	+	__ (8)
$f(11)$	144	__ (55)	+	__ (21)
…				
$f(n)$		__ $*f(n-2)$	+	__ $*f(n-4)$

Consider expressing the linear combination when arranging the Fibonacci numbers in three columns filling the first row with the first three numbers, second row with the next three numbers, and so on.

$f(n)$	Value	Linear Combination		
$f(0)$	1			
$f(3)$	3			
$f(6)$	13	__ (3)	+	__ (1)
$f(9)$	55	__ (13)	+	__ (3)
$f(12)$	233	__ (55)	+	__ (13)
$f(15)$	987	__ (233)	+	__ (55)
…				
$f(n)$		__*$f(n\text{-}3)$	+	__*$f(n\text{-}6)$

We have expressed the linear combinations when arranging the Fibonacci numbers in one column, two columns, and three columns. Now, do we see a pattern in these three cases to predict the next linear combination when we arrange the Fibonacci numbers in the case of four columns? Or more cases?

Results			
Number of Columns	Linear Combination		
1	__*f(n-1)	+	__*f(n-2)
2	__*f(n-2)	+	__*f(n-4)
3	__*f(n-3)	+	__*f(n-6)
4	__*f(n-4)	+	__*f(n-8)
5	__*f(n-5)	+	__*f(n-10)
6	__*f(n-6)	+	__*f(n-12)
7	__*f(n-7)	+	__*f(n-14)

Carefully study the linear combinations in this table above as a function of the number of columns, which we will subsequently denote by *k*. We observe that a pattern arises from the coefficients. We describe this pattern:

The k^{th} row for the coefficients of $f(n\text{-}k)$	
The k^{th} row for the coefficients of $f(n\text{-}2k)$	

We can now take our results from the table above and our description of the patterns that arise from the coefficients, to generalize all our linear combination to one recurrence relation for every $n>2k$, where $k\geq 1$:

$$f(n)=\_\_\_\_\_\_\_ f(n-k)+\_\_\_\_\_\_\_ f(n-2k)$$