

# Modeling Tropical Deforestation and Related Atmospheric Carbon Dioxide Levels



**A Project dealing with Biology, Global Economy, Environmental Science, and Regression to Study the Long Term Effects of Climate Change Due to Destruction of the Earth's Resources.**

**Mr. Chris Henderson and Mr. Jay Vick  
East Lawrence High School  
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Chris Henderson

[chenderson@lawrenceal.org](mailto:chenderson@lawrenceal.org)



**Humans Need Oxygen to Survive.** So does every other land-animal on this planet. We inhale the air around us, our lungs sends out the oxygen we need to all parts of our body. Our cells need oxygen just as a car needs gasoline. Oxygen acts as fuel for our cells to do work. We exhale carbon dioxide. This process is called respiration (figure 1) and is part of a larger process called the *Global Carbon Cycle*.

Carbon is part of our environment. It is in all plants. Plants need carbon, along with sunlight and water, to grow roots, stems, limbs, and leaves. Plants absorb carbon dioxide from the air and (through photosynthesis) produce oxygen, which is released back into the air. Humans (and other land animals) eat the plants (or eat other animals which have eaten plants) and ingest carbon into our bodies. All the carbon in our bodies comes ultimately comes from plants. The normal air we inhale contains about 385 parts per million carbon dioxide and the air we exhale contains about ten times as much. The plants absorb the amount we exhale, produce oxygen, and the process begins again. More people would exhale more carbon dioxide which is absorbed by the plants producing more oxygen for the people to breathe. If the process works, then there is no build- up of carbon dioxide in our atmosphere and everything is “just fine-and –dandy”.

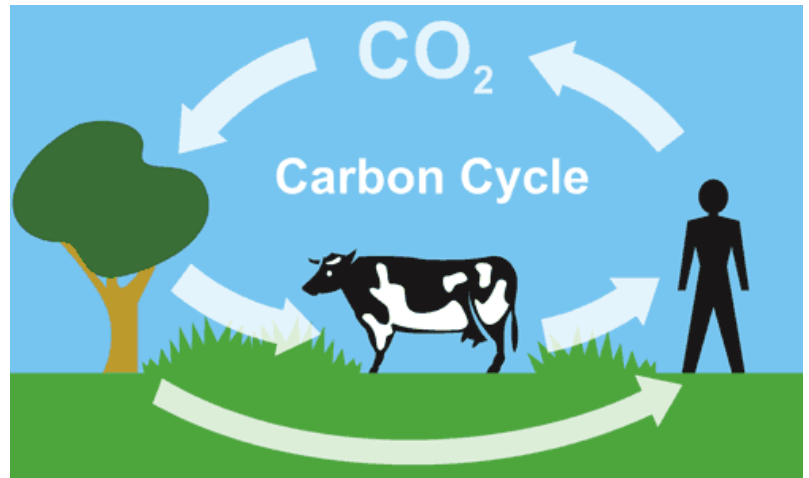


Figure 1 CO2 Respiration Cycle

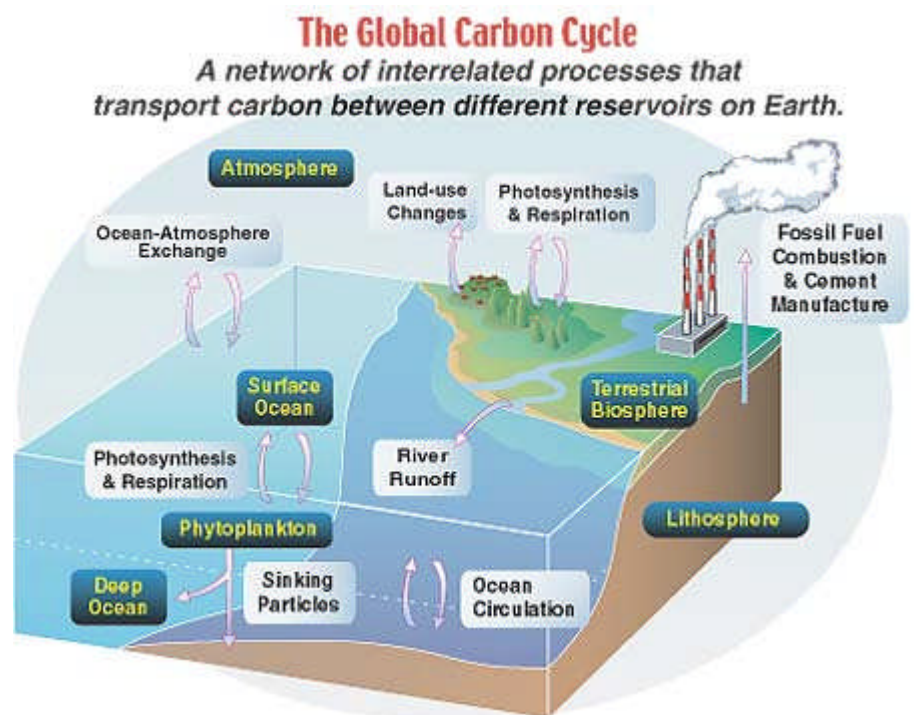


Figure 2 The Global Carbon Cycle  
<http://www.ncdc.noaa.gov/paleo/ctl/clisci1000b.html>

**However, what happens if the world’s supple of plants declines?** With fewer plants, there is less capacity to have carbon dioxide removed from the air. Additionally, as large plants (such as trees) are burned, the carbon dioxide that is stored within them is released into the air. This was not so much a problem up until a century ago because as trees were chopped down and burned (mainly for fuel) there were

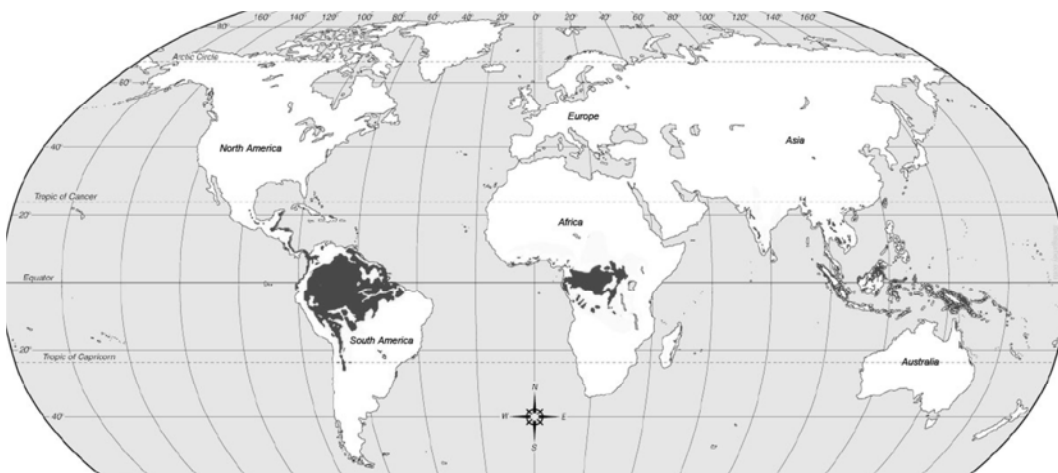
plenty of others to absorb the released carbon dioxide. Now we are faced with a much larger population, cutting and burning trees at a much higher rate with fewer trees remaining. Carbon dioxide begins to build-up in the atmosphere which, among other worries, has been linked to possible global warming. Far and away, the largest “oxygen factories” on the planet are the rainforests.

Rainforests produce nearly one-third (30%) of all the oxygen on our planet. (<http://www.greendiary.com/entry/5-facts-rainforests/>). Because of this, rainforests have often been called the “thermostat of the world”. At the same time, Amazon rain forests have almost one fifth of the fresh water of the entire world and they are very critical to maintain and regulate the earth’s water supply and fresh drinking water. It stores water from the rain such as sponge store water.

It is estimated that originally, the Earth had 6 billion acres (7.8 million square miles) of rainforests. This was approximately 14% of the surface of the Earth. As a repercussion of human exploitation, only 1.6 billion acres (about 2.5 million square miles) now remain-only about 2% of the surface of the Earth. The highest rates of deforestation occurred during the last 60 years. The following maps show the amount of rain forest loss during the past 60 years.



**Figure 1 Locations of Rain Forests (1950)**



**Figure 2 Locations of Rain Forests (2011)**

Consider the following facts:

- We are losing Earth's greatest biological treasures just as we are beginning to appreciate their true value. Rainforests once covered 14% of the earth's land surface; now they cover a mere 6% and **experts estimate that the last remaining rainforests could be consumed in less than 40 years.**
- **One and one-half acres of rainforest are lost every second** with tragic consequences for both developing and industrial countries.
- Rainforests are being destroyed because the value of rainforest land is perceived as only the value of its timber by short-sighted governments, multi-national logging companies, and land owners.
- **Nearly half of the world's species of plants, animals and microorganisms will be destroyed or severely threatened over the next quarter century due** to rainforest deforestation.
- **137 plant, animal and insect species become extinct every single day** due to rainforest deforestation. That equates to 50,000 species a year. As the rainforest species disappear, so do many possible cures for life-threatening diseases. Currently, 121 prescription drugs sold worldwide come from plant-derived sources. The U.S. National Cancer Institute has identified 3000 plants that are active against cancer cells. 70% of these plants are found in the rainforest. **25% of the active ingredients in today's cancer-fighting drugs come from organisms found only in the rainforest.** Less than 1% of tropical trees and plants have been tested for possible medicinal value. When the rain forests are gone, so too are possible cures for diseases.
- The Amazon Rainforest has been described as the "Lungs of our Planet" because it provides the essential environmental world service of continuously recycling carbon dioxide into oxygen. **More than 20 percent of the world oxygen is produced in the Amazon Rainforest alone.** 30% of the world's oxygen supply is produced by rainforests.
- Most rainforests are cleared by chainsaws, bulldozers and fires for its timber value and then are followed by farming and ranching operations, even by world giants like Mitsubishi Corporation, Georgia Pacific, Texaco and Unocal.
- In Brazil alone, more than 90 indigenous tribes have been destroyed since the 1900's. With them have gone centuries of accumulated knowledge of the medicinal value of rainforest species. Most medicine men remaining in the Rainforests today are 70 years old or more. Each time a rainforest medicine man dies, it is as if a library has burned down. When a medicine man dies without passing his arts on to the next generation, the tribe and the world loses thousands of years of irreplaceable knowledge about medicinal plants.

<http://www.rain-tree.com/facts.htm>

- One rainforest preserve contains more species of birds than are found in the entire United States
- The Amazon River in South America contains more species of fish than in the entire Atlantic Ocean
- A 25-acre plot of rainforest in Borneo may contain more than 700 species of trees—a number equal to the total tree diversity of North America.
- At least 80 percent of the developed world's diet originated in the tropical rainforest. Its bountiful gifts to the world include fruits like avocados, coconuts, figs, oranges, lemons, grapefruit, bananas, guavas, pineapples, mangos, and tomatoes; vegetables including corn, potatoes, rice, winter squash and yams; spices like black pepper, cayenne, chocolate, cinnamon, cloves, ginger, sugar cane, turmeric, coffee, vanilla, and nuts, including Brazil nuts and cashews.

<http://www.newworldencyclopedia.org/entry/Rainforest>

- **Rainforests act as the world's thermostat** by regulating temperatures and weather patterns.
- One-fifth of the world's fresh water is found in the Amazon Basin.
- **Rainforests are critical in maintaining the Earth's limited supply of drinking and fresh water.**
- **Every second, a slice of rainforest the size of a football field is mowed down.** That's 86,400 football fields of rainforest per day, or over **31 million football fields of rainforest each year.**

<http://www.nature.org/ourinitiatives/urgentissues/rainforests/rainforests-facts.xml>

Several organizations now actively seek to halt, or at least limit, the continued logging of rainforest lands. Some of these organizations (like Greenpeace) have additionally tried to tie companies and corporations to deforestation. These organizations view companies which use products from rainforest lands as the real cause of the problem. Whether these claims are true or not has been completely decided.

One case which briefly got some media attention dealt with soya, which is used mainly to feed animals. Greenpeace, on one occasion, claimed that the fast-food company McDonalds knowingly used soya supplied from the Cargill Corporation and grown on deforested Amazon rainforest lands in the feeding of chickens. McDonald's denied the report but Greenpeace said that they supplied evidences which proved McDonalds to be wrong and linked the soya farmers to the use of slave labors, illegal land grabbing, and massive deforestation. Greenpeace calculated that McDonald's and its suppliers have been responsible for 70,000 km<sup>2</sup> of the Amazon's deforestation in the last three years. Greenpeace has demanded that fast food companies eliminate soya trade and any meat products that are associated with the Amazon rainforest. (Greenpeace. 2006. *We're Trashin' It: How McDonald's is Eating up the Amazon*. Amsterdam: Greenpeace). Some news agencies have also picked up and carried the reports (see <http://www.foxnews.com/story/0,2933,190947,00.html>) While this report may or may not be so, it should also be understood that other organizations which support the rainforests have not always been so quick to make such claims. With this in mind, the student is encouraged to read the research for themselves but to not automatically jump to any conclusions about corporate intentions. Again, the above claims have never been completely accepted.

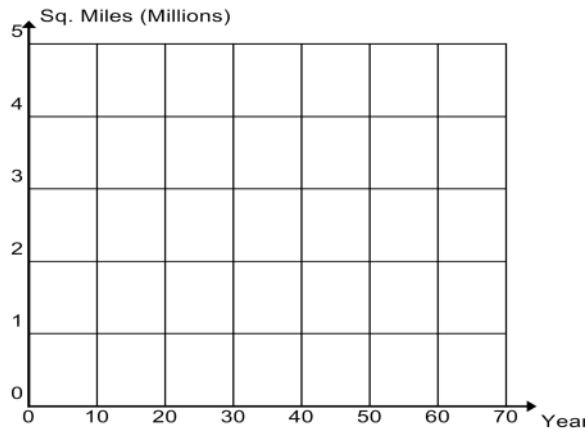
The following table shows the amount of tropical rainforests remaining in the world\*:

Year	Number of Square Miles (in millions)
1955	4.25
1965	3.86
1975	3.71
1985	3.17
1995	2.70
2005	2.50
2012	2.41

\* adapted from *Earth Algebra: College Algebra with Applications to Environmental Issues*; 2<sup>nd</sup> Ed. By Schaufele et al and *Gaia: An Atlas of Planet Management* Anchor Press

**Problem Set 1:**

1. Create a scatter plot of the data above. Be sure to label each axis. Let the horizontal axis be years since 1955. In other words, let 1950 = year 0, 1955 = year 5 and so on.



2. Use your calculator to enter the data and find a linear model that best-fits the data. Write the linear model in the space below.
3. What is the value of “r”, the correlation coefficient? The calculator can determine the value of r or you can use the formula

$$r = \frac{1}{n-1} \sum \left( \frac{x_i - \bar{x}}{s_x} \cdot \frac{y_i - \bar{y}}{s_y} \right).$$

Remember, the correlation coefficient seeks to explain the strength of the linear relationship between the two variables. What does this value for r seem to reveal about the linear relationship between these variables?

4. Using the linear model, predict the size of the world’s rainforests in 2015. Round your answer to the nearest hundredths.
5. Using the linear model, find the year when the rainforests will disappear.
6. What is the rate of deforestation of rainforest according to the linear model?

7. Does the linear model seem to accurately predict the answer to # 4 (above)? Look at the right-hand end of your graph to answer the question. What (if anything) appears to be wrong with this model?
8. One way, besides considering the correlation coefficient “r”, used to examine the accuracy of a model is to look at the **total error**. In order to do this, we make a table and include columns for predicted values and differences:

Year	Actual Number of Square Miles (in millions)	Predicted Number of Square Miles (in millions)	Differences (Residuals)
0 (1955)	4.25		
10 (1965)	3.86		
20 (1975)	3.71		
30 (1985)	3.17		
40 (1995)	2.70		
50 (2005)	2.50		
57 (2012)	2.41		
<b>Error</b>			

Find the predicted value by substituting each year into the model. Find each difference by subtracting **Actual Square Miles – Predicted Square Miles**. The difference in the actual y-value of a point and the y-value shown on the regression line is often called a **residual**. Add the residuals to get the total error. Fill in the table above.

9. A better way to find the amount of error is to use **Least Squares**. This method is better since differences may be positive or negative whereas squares are always positive.

Year	Number of Square Miles (in millions)	Predicted Number of Square Miles (in millions)	Difference	(Difference) <sup>2</sup>
0 (1955)	4.25			
10 (1965)	3.86			
20 (1975)	3.71			
30 (1985)	3.17			
40 (1995)	2.70			
50 (2005)	2.50			
57 (2012)	2.41			
Sum of the Squares of the Differences				

One way to think about which model would be “the best” is to use the sum of the squares of the differences. The model with the smallest sum of the squares of the differences, or “**Least Squares**”, can be thought of as the better model. Fill in the chart above for the linear model.

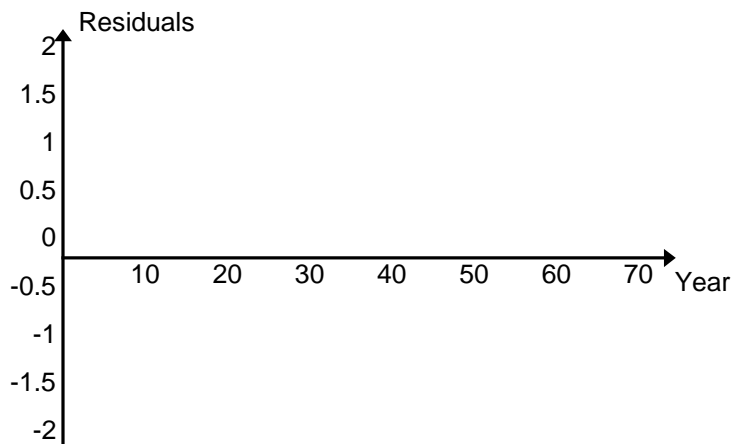
The calculator seems to find the equation by “magic” by how does it actually arrive at the values of “a” and “b”? Most calculators have programmed into them a formula which finds the “least squares regression equation”. The formula used is:

$$\hat{y} = ax + b \quad \text{where } a = r \frac{s_y}{s_x} \text{ and } b = \bar{y} - a\bar{x}$$

Here  $\hat{y}$  stands for the predicted (not the actual) value for y given an x-value.

10. Find the values of  $r$ ,  $s_x$ ,  $s_y$ ,  $\bar{x}$ , and  $\bar{y}$ . Use these and verify the equation given to you by your calculator as the linear model for the number of millions of square miles of rainforest remaining on Earth.
  
11. Even though using “least squares” can be effective in determining how well a model fits data there are still some problems which may arise causing the model to not be so good after all. Explain how a model may have a low value for the sum of the squares of the differences but yet may not be adequate in describing the data or in providing future (predictive) outcomes.
  
12. One way often used to accept or reject the linear model is to graph the residuals. If a pattern on the residual graph can be seen then the linear model is usually rejected. However, if the residuals seem to be randomly scattered then the linear model is accepted. On a residual graph, the horizontal axis corresponds to the linear model.

On the blank graph below, graph the residuals and tells whether or not the linear model is supported for the data?





When data cannot be well-fit using a linear model then the use of other models should be examined. A graphing calculator can typically find these for you but then you may ask the question “how is this done?” Often, this is accomplished by **linearizing the data**. Linearizing the data means that the data is transformed so that the scatter plot becomes more linear. Very often this is a trial-and-error process but the following can be helpful.

Look at the original scatter plot. If the scatter plot seems to resemble the graph of :

- a) a logarithmic function then use the transformation  $(\ln x, y)$
- b) an exponential function then use the transformation  $(x, \ln y)$
- c) a power function then use the transformation  $(\ln x, \ln y)$
- d) a quadratic function then use the transformation  $(x, \sqrt{y})$

Once the data is linearized then find the linear model (for the linearized data) and use inverse functions to rewrite it in the appropriate form (logarithmic, exponential, etc).

This also how correlation coefficients are found for non-linear model. The data is linearized, a linear model is determined from the linearized data, and the correlation coefficient is then found using the linearized model.

13. Use your calculator to find an exponential model that best-fits the data. Write the model below. (Texas Instruments calculators and Casio calculators will possibly display this model differently.)
14. Using the exponential model, predict the size of the rainforests in 2015.
15. Using the exponential model, when will the rainforests disappear? To do this you need to set your equation equal to 0. What happens when you try to solve the equation?
16. Think of a simple way to get around this problem then try working again. In what year would the model predict the rainforests disappear?

17. The graph of this function “curves” and isn’t linear. This means that the rate of change (that is, slope) is changing all along the graph. However, you can approximate slope by taking two points close together and finding the slope of a line segment connecting the two points.

Using the Exponential Model	Approximate Rate of Change
from year 0 to year 10	
from year 10 to year 20	
from year 20 to year 30	
from year 30 to year 40	
from year 40 to year 50	
from year 50 to year 57	

According to this model does the rate seem to be increasing or decreasing?

18. Does this model seem to predict the outcome accurately? GIVE A REASON to justify your answer. Complete the table below to find the sum of the squares of the differences using the exponential model.

Year	Number of Square Miles (in millions)	Predicted Number of Square Miles (in millions) Using Exponential Model	Difference	(Difference) <sup>2</sup>
0 (1955)	4.25			
10 (1965)	3.86			
20 (1975)	3.71			
30 (1985)	3.17			
40 (1995)	2.70			
50 (2005)	2.50			
57 (2012)	2.41			
Sum of the Squares of the Differences				

19. Use your calculator to find a logistic model that fits the data. Write the model below.

16. Using the logistic model, calculator the expected number of square miles remaining in 2015.

16. Using the logistic model, in what year would you expect to see the rainforests disappear?

17. How well does this model fit the data? Reset the view window of your calculator so that  $x\text{-max} = 100$  and  $y\text{-min} = 0$ . Graph the logistic model and look at the right-end of the graph. Describe the right-end nature of the graph of this model. How well do you think the graph will describe future rainforest amounts?

18. Complete the table to approximate the rates of change using the logistic model

Using the Logistic Model	Approximate Rate of Change
from year 0 to year 10	
from year 10 to year 20	
from year 20 to year 30	
from year 30 to year 40	
from year 40 to year 50	
from year 50 to year 57	

19. Complete the table for the logistic model:

Year	Number of Square Miles (in millions)	Predicted Number of Square Miles (in millions) Using Logistic Model	Difference	(Difference) <sup>2</sup>
0 (1955)	4.25			
10 (1965)	3.86			
20 (1975)	3.71			
30 (1985)	3.17			
40 (1995)	2.70			
50 (2005)	2.50			
57 (2012)	2.41			
Sum of the Squares of the Differences				

20. Determine a logarithmic model that fits the data.

21. Using the logarithmic model, calculate the predicted amount of rainforest remaining in 2015?

22. Using the logarithmic model, in what year will be rainforest disappear?

23. Complete the table for the logarithmic model:

Year	Number of Square Miles (in millions)	Predicted Number of Square Miles (in millions) Using Logarithmic Model	Difference	(Difference) <sup>2</sup>
0 (1955)	4.25			
10 (1965)	3.86			
20 (1975)	3.71			
30 (1985)	3.17			
40 (1995)	2.70			
50 (2005)	2.50			
57 (2012)	2.41			
Sum of the Squares of the Differences				

24. Think about the correlation coefficients, residuals, and least-square values for each model. Which of these would be most likely the best model to describe the amount of rainforest lands remaining on Earth? Give a reason for your answer.

## Reasons for Rainforest Deforestation

There are 3 main reasons for rainforest loss

1. cutting down trees for to use the land for agriculture/farming
2. clearing land for logging and fuel
3. clearing land for raising cattle

The first of these reasons is really stupid! It is a proven fact that rainforest land, when cleared, is very poor farm soil. Governments, such as the one in Brazil, know this and yet still encourage farm expansion into rainforest areas. Once cleared, the soil is very prone to erosion and mudslides.

Clearing the land of trees for fuel presents altogether another problem. Rainforest trees act as giant sponges soaking up huge amounts carbon dioxide. When the trees are chopped down, they no longer rid the air of carbon dioxide but they no longer produce oxygen. Additionally, the carbon dioxide stored up with the tree is released into the atmosphere when the tree is burned.

Combining these two on large scales can have drastic impacts on the environment. Not only are staggering amounts of carbon dioxide released into the air but the soil quickly erodes making forest regeneration extremely difficult.

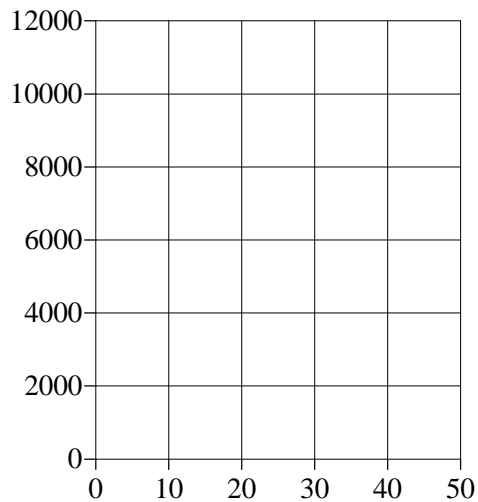
Look at the table below:

Year	Square miles cut for logging	Square miles cut for agriculture /development	Square miles cut for ranching
1960	1,698.8	8,532.9	1,158.3
1970	4,594.6	14,633.3	3,557.97
1980	8,494.2	18,996.2	5,250.9
1990	16,216.3	22,934.5	6,409.3
2000	8,096.56	26,313.82	10,120.7
2005	8,262.58	26,853.6	10,328.2

### Problem Set 2:

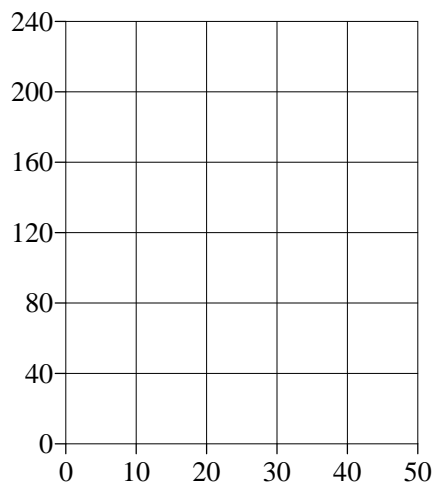
1. Go to the internet and find the number of square miles that Lawrence County, AL covers.
2. What was the total number of square miles of rainforest cut in 2005?
3. How many times the size of Lawrence County was this amount?
4. According to this table, since 1960 what is the fastest growing cause for rainforest deforestation? Though not as much of the rainforest has been cut down for cattle grazing (as of yet) as for the other areas, the rate is obviously higher.

Enter the data for cattle grazing into your calculator and draw a scatterplot of the data. Let List 1 be years (it's best to let this be measured in years since 1960...so 1960 would be year 0, 1970 would be year 10,...and let List 2 be the number of square miles cut down for cattle grazing). Draw the scatter plot on the graph below.



5. Use your calculator and find the exponential model that best fits the data
6. Using the model, find the rate of growth of cattle grazing?
7. The chart below shows the amount of fast-food hamburgers sold in the US each year since 1960. Keep List 1 and enter the number of hamburgers sold (in billions) in List 2. Make a scatterplot of the data. Draw the scatterplot on the graph below.

Year	1960	1970	1980	1990	2000	2005
Numbers of Fast-Food Hamburgers Sold (in Billions)	32	71	104	138	220	232



8. If you ignore the y-values, what do you notice about the two scatter plots?

9. Use your calculator and find an exponential model that best-fits the scatterplot in # 7.
10. From this model, what is the rate of growth of fast-food hamburgers in the US?
11. What do you notice about the rate of growth in # 6 and the rate of growth in # 10?

## Carbon Dioxide Emissions and Atmospheric Build-Up

There are several gases in the atmosphere that are classified as “greenhouse gases”. By far the largest type of these gases is carbon dioxide (CO<sub>2</sub>). Carbon Dioxide makes up about 50% of all the greenhouse gases. CO<sub>2</sub> is naturally present in the atmosphere and in moderate amounts is beneficial. However, large amounts of CO<sub>2</sub> have been possibly linked to drastic changes in the Earth’s climate. ***What follows is controversial.*** There are many people who preach “global warming”. Others strictly deny that man has anything to do with affecting the climate and any changes are the result of natural cycles. After studying this you might be able to reach your own opinions.

There are many sources that influence the release of carbon dioxide into the atmosphere. Here are just a few that directly add CO<sub>2</sub> in the atmosphere:

- Gasoline and diesel powered vehicles
- Emissions from any manufacturing or power plant that uses any type of fossil fuel.
- Cutting down and burning large amounts of trees or forested areas

However, there are many other factors that indirectly influence CO<sub>2</sub> build-up. Some are:

- Sitting in an air-conditioned room
- Eating a hamburger
- Buying anything made from rainforest wood (such as teak wood)
- Going anywhere in a car, truck, bus, train, or plane
- Turning on electric lights/watching television/playing video games/charging a cell phone
- Using and then disposing (instead of recycling) anything made of plastic
- Burning logs in a fireplace

It is obvious that, in today’s world, many of these are unavoidable. It is also obvious that some of these factors influence CO<sub>2</sub> build-up more than others. It is ridiculous to argue that all of these activities should be halted. The real problem is to find more efficient ways to perform them.

CO<sub>2</sub> levels are described in “parts per million”, which is abbreviated as **ppm**. This seems very small, and it is. But very small amounts of a greenhouse gas can be devastating.

The National Oceanic and Atmospheric Administration (NOAA) maintain several research laboratories around the globe. The Mauna Loa Observatory in Hawaii has been collecting air samples and recording CO<sub>2</sub> levels for years. The chart below shows the data:

Year	CO <sub>2</sub> Level in ppm
1965	320
1970	326
1980	338
1990	354
1995	361
2000	370
2010	392

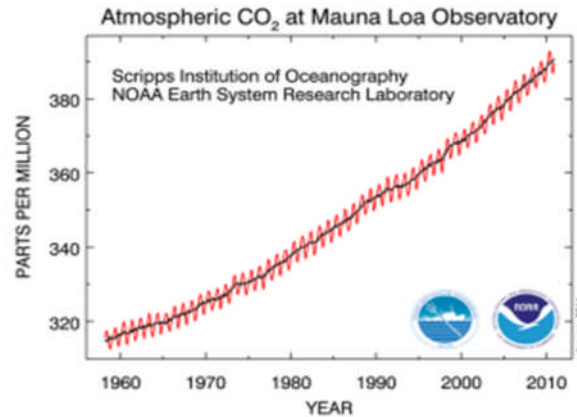


Figure 3 [www.esrl.noaa.gov/gmd/](http://www.esrl.noaa.gov/gmd/)

The fact then is that there is more and more CO<sub>2</sub> in the atmosphere than at any other time in our history.

### Problem Set 3

- Find a model that best fits the data. It is obvious that the best model will probably be linear, quadratic, exponential, or logistic. Find all 4, decide which will be the best-fit, and write the equation of the model below.
- Science Daily ran an 2009 article stating some researchers claim that “projections show CO<sub>2</sub> levels rising as high as 600 or even 900 parts per million in the next century if no action is taken to reduce carbon dioxide” [www.sciencedaily.com/releases/2009/10/091008152242.htm](http://www.sciencedaily.com/releases/2009/10/091008152242.htm)  
Based on your model, and using the year 2109 for the year, is this prediction reasonable?
- Another article (<http://www.sciencedaily.com/releases/2011/10/111025090351.htm>) states that the rate of CO<sub>2</sub> build-up is increasing at 2 ppm per year. Based on the model from #1, is this accurate?
- Go to <http://co2now.org/> and find what the most current level of CO<sub>2</sub> in ppm. Using 2013 for the year, does the model reasonably predict this level of CO<sub>2</sub>?



5. The air you breathe is dirtier than the air I breathed when I was your age. To find a percent of increase/decrease use the following formula:

$$\frac{Y_f - Y_0}{Y_0} \times 100 \quad \text{where } Y_f \text{ is the final quantity and } Y_0 \text{ is the original quantity}$$

What is the percent of increase of CO<sub>2</sub> in the air from 1980 (I was 22) and 2010?

6. The following is a quote from the NOAA (National Atmospheric and Oceanic Administration) National Climatic Data Center:

*“An estimate from the tropical ocean, far from the influence of ice sheets, indicates that the tropical ocean may warm 5°C for a doubling of carbon dioxide. The paleo data provide a valuable independent check on the sensitivity of climate models, and the 5°C value is consistent with many of the current coupled climate models.”* <http://www.ncdc.noaa.gov/paleo/globalwarming/temperature-change.html>

As ocean temperatures rise so do water levels. This is because water expands as it gets warmer. Also, warmer temperatures mean more melting of polar ice fields. A rise of even a few inches can cause flooding along coastlines. Considering that most of the world major cities and economic centers are usually located on coastal areas this has the potential to be devastating.

An oceanic temperature change of +5° C is expected to produce a rise of 6-8 meters in ocean levels. This would result in flooding all major coastal cities in the United States, roughly 25% of the total US population. <http://pubs.usgs.gov/fs/fs2-00/> The human and economic results would be catastrophic.

Use the model above and predict the times when this might happen?