Preparing for the Work of Effective Mathematics Instruction

NCTM New Teacher Strand
PK-2, 3-5 Gallery Workshop

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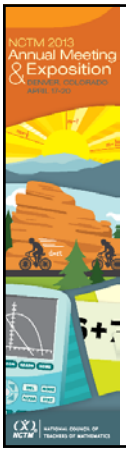
RMC Research Corporation



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Introductions: Raise Hand and Say "That's ME!"

- Pre-service teacher
- 1st year teacher
- 2nd – 3rd year teacher
- Veteran teacher
- Grade level is PK-2
- Grade level is 3-5
- Grade level is 6-12

Lesson Planning

- Use students' mathematical thinking as a critical ingredient in developing key mathematical ideas
- Anticipate what students will do when solving a problem
- Generate questions teachers can ask to promote student learning

Thinking through a Lesson Plan

Protocol Framework

Part 1: Selecting a mathematical task

Part 2: Setting up a mathematical task

Part 3: Supporting students' exploration of task

Part 4: Sharing & discussing task

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Mathematical Tasks

A Critical Starting Point for Instruction

- *Not all tasks are created equal – different tasks will provoke different levels and kinds of student thinking.*

  Stein, Smith, Henningsen, & Silver, 2000

- *The level and kind of thinking in which students engage determines what they will learn.*

  Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997

Worthwhile Mathematical Tasks

According to research studies:

Having the opportunity to work on challenging tasks in a supportive classroom environment translated into substantial learning gains in student thinking, reasoning, problem solving, and communication.

What Makes A Task Worthwhile?

A worthwhile task is a project, question, problem, construction, application, or exercise that **engages students** to reason about mathematical ideas, make connections, solve problems, and develop mathematical skills.

(NCTM)

What Makes A Task Worthwhile?

- Allows for connections to previous knowledge, experiences, and interests
- Incorporates multiple approaches and solutions
- Requires higher-level cognitive demand
- Facilitates reasoning and communicating mathematically

Mathematical Tasks

CONTENT

COGNITIVE DEMAND

Framework for Levels of Cognitive Demand

Lower-Level Demand	*Higher-Level Demand*
Memorization	**Procedures With Connections** • Represented in multiple ways • Focuses on development of deeper concepts
Procedures Without Connections • No connections to concepts or meaning •Focuses on producing correct answers	**Doing Mathematics** •Complex, non-algorithmic • Explore to understand the nature of concepts or relationships

Worthwhile Tasks

1. Individually complete all parts of the task.
2. Work with a partner to compare your work.
3. Look for many ways to solve the task.

Pencils and Erasers

The school store sells pencils and erasers.
Pencils sell for 10¢.
Erasers sell for 5¢.
You have 40¢ to spend.

How many pencils and erasers can you buy?

Tricycles and Wagons

Children go to the park riding their tricycle or in a wagon.
Susie counts 26 wheels at the park today.
Rich counts 8 vehicles at the park today.

How many tricycles and wagons are at the park today?

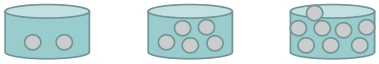
Lizards & Beetles

Lizards have 4 legs. Beetles have 6 legs.
There are lizards and beetles in a container.
Amy counted 36 legs.
Scott counted 8 critters.

How many beetles and lizards are in the container?

Cherries in a Bowl

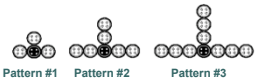
The mother wants to make a cherry pie.
There are 2 cherries in a bowl.
It takes 26 cherries to make a pie.
The mother picks 3 cherries each day and places them in the bowl.



How many days will it take for the mother to have 26 cherries to make the pie?

Buttons Task

Gita plays with her grandmother's collection of black & white buttons.
She arranges them in patterns.
Her first 3 patterns are shown below.



1. Draw Pattern 4 next to Pattern 3.
2. How many white buttons does Gita need for Pattern 5 and Pattern 6? Explain how you figured this out.
3. How many buttons in all does Gita need to make Pattern 11? Explain how you figured this out.
4. Gita thinks she needs 69 buttons in all to make Pattern 24. How do you know that she is **NOT** correct? How many buttons does she need to make Pattern 24?

Worthwhile Tasks

Question:
What cognitive demand level would you give to the mathematical task?

Justify your thinking.

Part 1: Selecting a Mathematical Task

- What are your goals (*content and mathematical practices*) for the lesson?
- In what ways does the task build on students' prior knowledge and experience?
- What are all the ways the task can be solved?
- What challenges might the task present to struggling or ELL students?

Selecting Tasks Mathematics Content

K-8 Domains Progression

Domains	K	1	2	3	4	5	6	7	8
Counting and Cardinality									
Operations and Algebraic Thinking									
Number and Operations in Base Ten									
Number and Operations - Fractions									
Ratios and Proportional Relationships									
The Number System									
Expressions and Equations									
Functions									
Geometry									
Measurement and Data									
Statistics and Probability									

Selecting Tasks: Mathematical Practices

"The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students."

The Mathematical Practices describe what *students* should be *doing* as they learn the mathematics content standards.

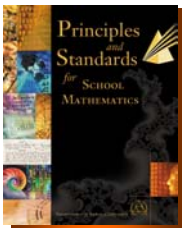
Selecting Tasks: Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Underlying Frameworks for Mathematical Practices

National Council of Teachers of Mathematics

5 **Process** Standards
- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representations



NCTM (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

NCTM Process Standards & Standards for Mathematical Practices

NCTM Process Standards	Standards for Mathematical Practices
Problem Solving	1. Make sense of problems and persevere in solving them. 5. Use appropriate tools strategically.
Reasoning and Proof	2. Reason abstractly and quantitatively. 3. Critique the reasoning of others. 8. Look for and express regularity in repeated reasoning.
Communication	3. Construct viable arguments.
Connections	6. Attend to precision. 7. Look for and make use of structure.
Representations	4. Model with mathematics.

Part 1: Selecting a Mathematical Task

MATHEMATICAL GOALS

What are the mathematical goals for this task?

What do you want students to know and understand about mathematics as a result of this task?

Part 1: Selecting a Mathematical Task

Previous Knowledge:

- In what ways does the task build on students' previous knowledge?
- What definitions and/or concepts do students need to begin work on task?

Part 1: Selecting a Mathematical Task

Challenges for Students:

- What particular challenges might the task present to struggling students?
- How will you address these challenges?

Thinking through a Lesson Plan

Protocol Framework

Part 1: Selecting a mathematical task

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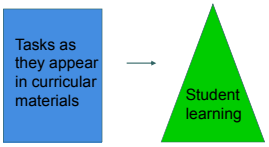
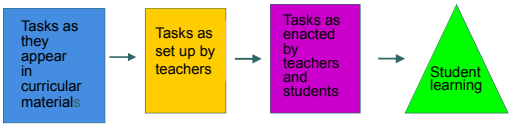
The Nature of Tasks Used in the Classroom …

Will Impact Student Learning!



But, WHAT TEACHERS DO with the tasks matters too!

The Mathematical Tasks Framework



Stein, Grover & Henningsen (1996)
Smith & Stein (1998)
Stein, Smith, Henningsen & Silver (2000)

PART 2: SETTING UP A MATHEMATICAL TASK

- What are expectations for students?
- What resources or tools will students have to use in their work?
- How will students work (individual,pairs)?
- How will students record and report their work?
- How will we introduce students to the activity?

FACTORS THAT SUPPORT HIGH-LEVEL DEMAND COGNITIVE TASKS

- Sufficient time to explore – not too much, not too little.
- Scaffolding of student thinking and reasoning by asking thought-provoking questions that preserve task complexity.
- Sustained press for justifications, explanations, and/or meaning.
- Students are provided means of monitoring their own progress.
- Teacher and/or students model high-level thinking and reasoning.

THINKING THROUGH A LESSON PLAN

Protocol Framework

Part 1: Selecting a mathematical task

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FACTORS THAT IMPEDE HIGH-LEVEL DEMAND OF COGNITIVE TASKS

- Problematic aspects of task become routinized.
- Emphasis shifts from meaning, concepts, or understanding to correctness/completeness of answer.
- Not enough time to wrestle with demands of task or too much time is allowed (classroom management).
- Students not held accountable for high-level products or processes.

PART 3: SUPPORTING STUDENTS' EXPLORATION

As students work, what questions do we ask to:

- Help a group get started or make progress on the task?
- Focus students' thinking on the key mathematical ideas in task?
- Assess students' understanding of key mathematical ideas, problem-solving strategies, or representations?
- Advance students' understanding of mathematical ideas?
- Encourage *all* students to share their thinking with others or to assess their understanding of their peers' ideas?

Part 3: Supporting Students' Exploration

How will you ensure students remain engaged in task?

- What assistance will you give to a student who becomes frustrated?
- How will you extend task to provide additional challenge to those who finish before others?
- What will you do if students focus on non-mathematical aspect of activity?

Part 3: Supporting Students' Exploration

Additional support for students

- Hint Cards
  - Definition of terms in pictorial form
  - Question that connects prior learning to new task
  - Way to modify task
- Think Beyond Cards
  - Modify rules for greater challenge

Thinking through a Lesson Plan

Protocol Framework

Part 1: Selecting a mathematical task

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Part 4: Sharing and Discussing the Task

- In what ways will the order in which solutions are presented help develop students' understanding of the mathematical ideas of the task?
- What specific questions will you ask so students will:
  - Make sense of the mathematical ideas to learn?
  - Expand on and/or question the solutions shared?
  - Make connections among the strategies presented?
  - Look for patterns?
  - Begin to form generalizations?

Facilitating Productive Discussions

Establish appropriate expectations for student participation.

- Students always supply justifications for why their thinking makes sense.
- Students provide counterexamples.
- Teacher asks other students to repeat peers' responses.
- Classmates pose questions to student making presentation.

Facilitating Productive Discussions

Teacher decides which aspect will be focus of discussion.

- Use of various strategies
  - Decide if valid, reasonable for context of problem
  - Compare for similarities (differences) or efficiency
- Student errors
- Generate different ideas with which to grapple

Facilitating Productive Discussions

Teacher encourages all students to communicate their thinking in writing.

- Create a representation on paper (words, picture, diagram, table)
- Record key discussion points
- Scaffold process until students are able to write independently.
  - Whole class
  - Partner
  - Individual

Thinking through a Lesson Plan

Useful Tool: Plan, Teach, Reflect

- **Think deeply about specific lesson in how to advance students' mathematical understanding**
- **Shift emphasis to student thinking rather than teacher action**
- **Collaboration with other teachers**

Thinking through a Lesson Plan

A problem-solving atmosphere
allows all students at different levels
to develop their understanding
of mathematics
by engaging
in relatable problem contexts
while also deepening
their understanding of mathematics
concepts.

Thinking through a Lesson Plan

- Choose problems purposefully.
- Plan questions/support ahead to develop students' understanding of mathematics.
- Expect all students to verify/justify their strategy and answer.
- Choose which strategies and their sequence to share in large group sharing.
- Establish expectations and purposes for listening and understanding classmates' strategies.

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Thinking through a Lesson Protocol

(*adapted from template from Peg Smith, University of Pittsburgh*)

Part 1: Selecting a Mathematical Task

What are your goals (content and mathematical practices) for the lesson?

What do you want students to know as a result of the lesson?

What do you want student to be able to do as a result of the lesson?

In what ways does the task build on students' previous knowledge, life experiences, and culture?

What questions will you ask to help students access their prior knowledge and relevant experiences?

What definitions, concepts, or ideas do students need to know to begin work on the task?

What are all the ways the task can be solved?

Which of these methods do you think your students will use?

What misconceptions might students have?

What errors might students make?

What particular challenges might the task present to struggling students or ELL students?

How will you address these challenges?

Part 2: Setting Up a Mathematical Task

What are your expectations for students as they work on and complete this task?

What resources or tools will students have to use in their work that will give them entry into, and help them reason through, the task?

How will the students work—independently, in small groups, in pairs—to explore this task?

How long will they work individually or in groups or pairs?

Will students be partnered in a specific way? If so, in what way?

How will students record and report their work?

How will you introduce students to the activity so as to provide access to *all* students while maintaining the cognitive demands of the task?

How will you ensure that students understand the context of the problem?

What will you hear that lets you know students understand what the task is asking them to do?

Part 3: Supporting Students' Exploration of the Task

As students work independently or in small groups, what questions will you ask to:

- Help a group get started or make progress on the task?
- Focus students' thinking on the key mathematical ideas in the task?
- Assess students' understanding of key mathematical ideas, problem-solving strategies, or the representations?
- Advance students' understanding of the mathematical ideas?
- Encourage *all* students to share their thinking with others or to assess their understanding of their peers' ideas?

How will you ensure that students remain engaged in the task?

- What assistance will you give or what questions will you ask a student (or group) who becomes quickly frustrated and requests more direction and guidance in solving the task?
- What will you do if a student (or group) finishes the task almost immediately? How will you extend the task so as to provide additional challenge?
- What will you do if a student (or group) focuses on non-mathematical aspects of the activity (e.g., spends most of their time making a poster of their work?)

Part 4: Sharing and Discussing the Task

Which solution paths do you want to have shared during the class discussion?

In what order will the solutions be presented? Why?
In what ways will the order in which solutions are presented help develop students' understanding of the mathematical ideas that are the focus of your lesson?

How will you orchestrate the class discussion so that you accomplish your mathematical goals?

What specific questions will you ask so that students will:

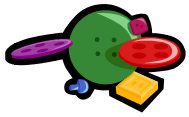
1. make sense of the mathematical ideas that you want them to learn?
2. expand on, debate, and question the solutions being shared?
3. make connections among the different strategies that are presented?
4. look for patterns?
5. begin to form generalizations?

How will you ensure that over time, ***each*** student has the opportunity to share his or her thinking and reasoning with peers?

What will you see or hear that lets you know that *all* students in the class understand the mathematical ideas that you intended for them to learn?

What will you do tomorrow that will build on this lesson?

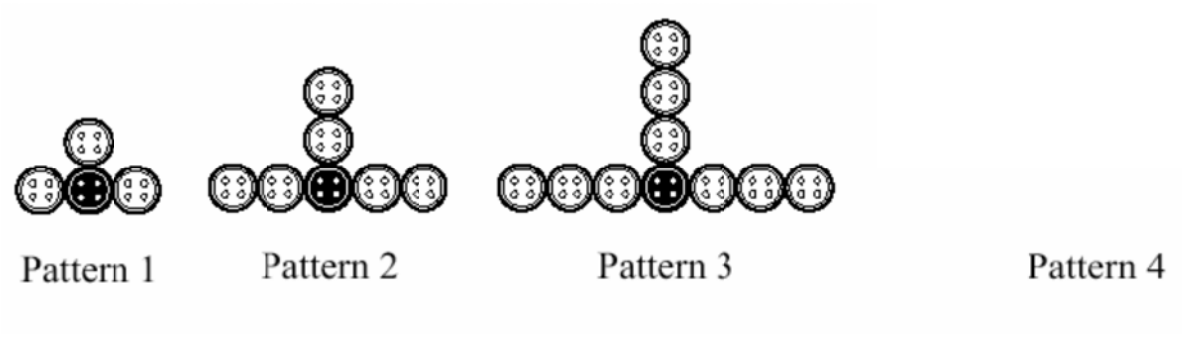
Buttons



Gita plays with her grandmother's collection of black and white buttons.

She arranges them in patterns.

Her first three patterns are shown below.



1. Draw Pattern 4 next to Pattern 3.

2. How many white buttons does Gita need for Pattern 5 and Pattern 6.

   Pattern 5 _____ Pattern 6 _____

3. How many buttons in all does Gita need to make Pattern 11? _____

   Explain how you figured this out.

4. Gita thinks she needs 69 buttons in all to make Pattern 24.
   How do you know she in NOT correct?

   How many buttons does she need to make Pattern 24?

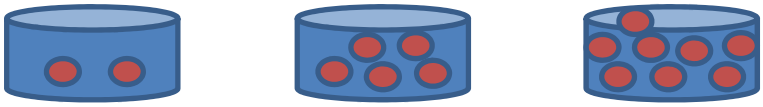
Cherries in a Bowl

The mother wants to make a cherry pie when she sees 2 cherries in a bowl.

It takes 26 cherries to make a pie.

The mother will pick 3 cherries each day and place them in the bowl.

How many days will it take for the mother to have 26 cherries to make the pie?



Start **Day 1** **Day 2**

Tricycles and Wagons

Children go to the park riding on their tricycle or in a wagon.

Susie counts 26 wheels at the park today.

Rich counts 8 seats at the park today.

How many tricycles and wagons are at the park today?

Lizards and Beetles

Lizards have 4 legs. Beetles have 6 legs.

There are lizards and beetles in a container.

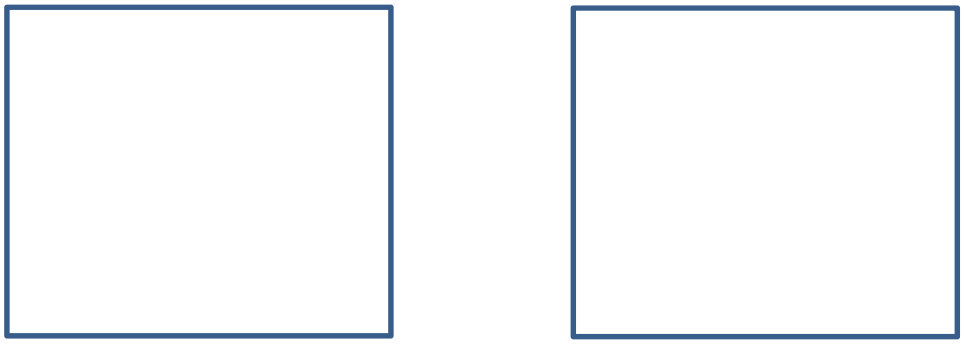
Amy counts 36 legs.

Scott counts 8 critters.

How many lizards and how many beetles are in the container?

SHARING TWO BROWNIES

Three children want to share these two brownies so they each get the same amount.



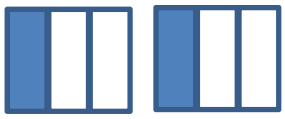
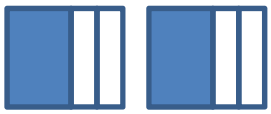
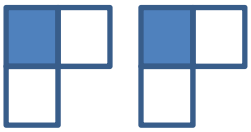
Using the brownies above, show one way the children can share the brownies fairly.

Using fractions, tell how much brownie each child should get. Be ready to justify your answer.

Draw two more square brownies and show a different way the three children can share fairly. Tell how much brownie each child should get with your new solution.

Repeat the last step. See how many *different* solutions you can think of.

Sharing Two Brownies Task: Misconceptions

Solution	Representation	Label fractional parts
A		1/3 + 1/3
B		
C		1/2 + 1/3
D		1/3 + 1/3
E		2/3
F		1/4 + 1/4