Eight Instructional Practices to Promote K-8 Number Sense

National Council of Teachers of Mathematics

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Denver, Colorado

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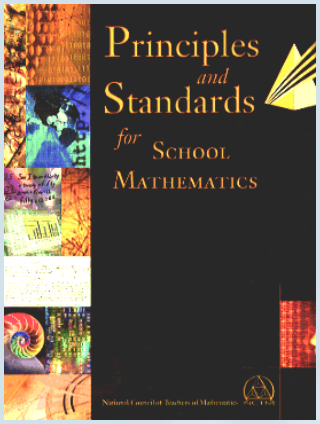
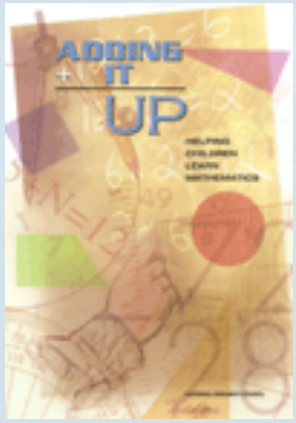
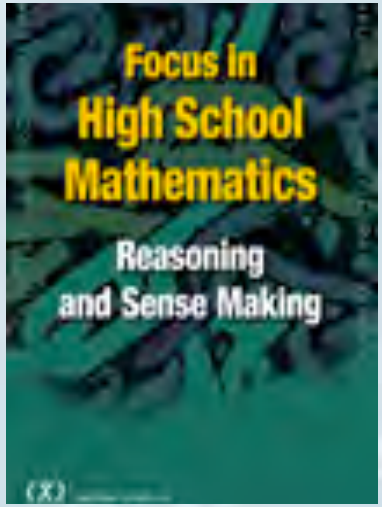
Linking to Standards







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COMMON CORE STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

NCTM Process Standards

- Connections
- Communication
- Problem Solving
- Reasoning and Proof
- Representation

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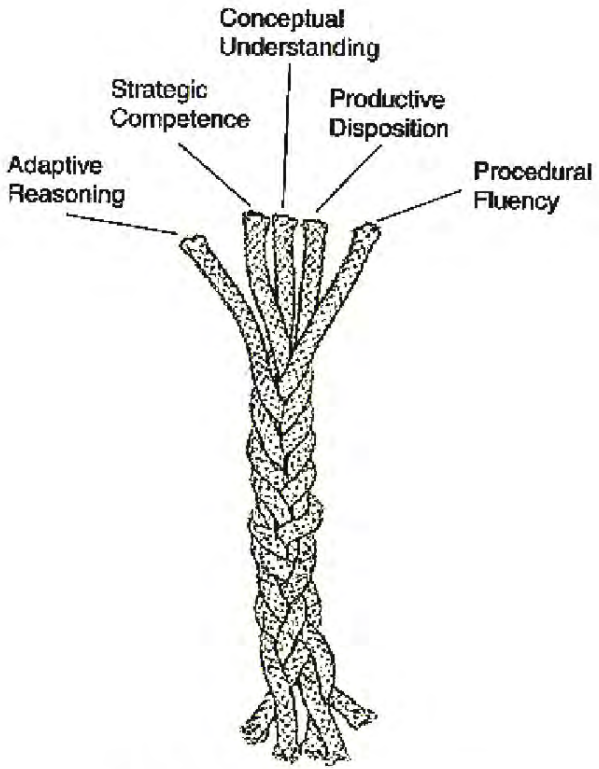
National Research Council's Strands of Proficiency

***Adding It Up*, 2001**



Intertwined Strands of Proficiency

- Adaptive Reasoning
- Strategic Competence
- Conceptual Understanding
- Productive Disposition
- Procedural Fluency

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8 CCSSM Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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Practice 1: Never Say Anything a Kid Can Say

Never Say Anything a Kid Can Say!

STEVEN C. REINHART

AFTER EXTENSIVE PLANNING, I presented what should have been a masterpiece lesson. I worked several examples on the overhead projector, answered every student's question in great detail, and explained the concept so clearly that surely my students understood. The next day, however, it became obvious that the students were totally confused. In my early years of teaching, this situation happened all too often. Even though observations by my principal clearly pointed out that I was very good at explaining mathematics to my students, knew my subject matter well, and really seemed to be a dedicated and caring teacher, something was wrong. My students were capable of learning much more than they displayed.

Implementing Change over Time

THE LOW LEVELS OF ACHIEVEMENT of many students caused me to question how I was teaching, and my search for a better approach began. Making a commitment to change 10 percent of my teaching each year, I began to collect and use materials and ideas gathered from supplements, workshops, professional journals, and university classes. Each year, my goal was simply to teach a single topic in a better way than I had the year before.

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Before long, I noticed that the familiar teacher-centered, direct-instruction model often did not fit well with the more in-depth problems and tasks that I was using. The information that I had gathered also suggested teaching in nontraditional ways. It was not enough to teach better mathematics; I also had to teach mathematics better. Making changes in instruction proved difficult because I had to learn to teach in ways that I had never observed or experienced, challenging many of the old teaching paradigms. As I moved from traditional methods of instruction to a more student-centered, problem-based approach, many of my students enjoyed my classes more. They really seemed to like working together, discussing and sharing their ideas and solutions to the interesting, often contextual, problems that I posed. The small changes that I implemented each year began to show results. In five years, I had almost completely changed both *what* and *how* I was teaching.

The Fundamental Flaw

AT SOME POINT DURING THIS METAMORPHOSIS, I concluded that a fundamental flaw existed in my teaching methods. When I was in front of the class demonstrating and explaining, I was learning a great deal, but many of my students were not! Eventually, I concluded that if my students were to ever really learn mathematics, *they* would have to do the explaining, and *I*, the listening. My definition of a good teacher has since changed from "one who explains things so well that students understand" to "one who gets students to explain things so well that they can be understood."

Getting middle school students to explain their thinking and become actively involved in classroom discussions can be a challenge. By nature, these

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Practice 2: Ask clarifying questions

- What do you know so far?
- What are you wondering about?
- What have you tried so far?
- Can you draw a picture of what is happening?
- Can you try this with simpler numbers?
- What tools might you use to help you?
- Is your answer reasonable? How do you know?

Practice 2: Ask clarifying questions

Classroom discussions

- Who would like to share their thinking?
- What was the first thing your eyes saw, or your brain did?
- Convince me.
- How did you figure that out?
- Billy, can you tell us where you got that 5?
- How many people solved it the same way as Billy?

Practice 2: Ask clarifying questions

Classroom discussions

- Who used another strategy?
- How is your strategy like Billy's? How is it different?
- Will your strategy always work? Can you find a time when it doesn't work?

AVOID

- Questions that can be answered by yes or no
- Asking "Do you understand?"

- *www.**pbs**.org/teachers/_files/pdf/TL_**Math**Card.pdf*

Practice 3: Focus on Strategic Thinking

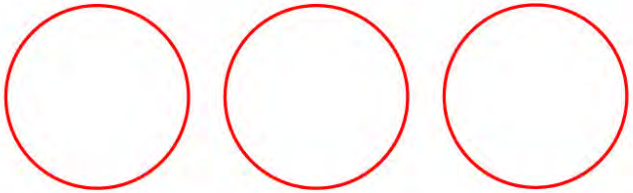
	Level D Grade 3	Level E Grade 4	Level F Grade 5	Level G Grade 6
1. Make a Table	💡	💡	💡	💡
2. Look for a Pattern	💡	💡	💡	💡
3. Make a Drawing or Diagram	💡	💡	💡	💡
4. Make a Model	💡	💡	💡	💡
5. Guess and Check	💡	💡	💡	💡
6. Act It Out	💡	💡	💡	💡
7. Make or Use a Graph	💡	💡	💡	💡
8. Make an Organized List	💡	💡	💡	💡
9. Account for All Possibilities		💡	💡	💡
10. Solve a Simpler Problem		💡	💡	💡
11. Work Backwards		💡	💡	💡
12. Write an Expression or Equation			💡	💡
13. Identify a Subgoal			💡	💡
14. Generalize				💡
15. Change Your Point of View				💡

Strategic Thinking

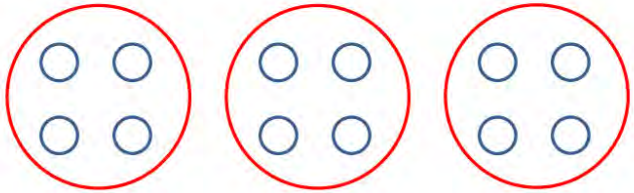
- Use multiple representations
  - Concrete (materials)
  - Pictorial (semi-concrete)
  - Semi-abstract (number lines, models)
  - Abstract (symbolic)

Concrete/Pictorial

e 1a



e 1b



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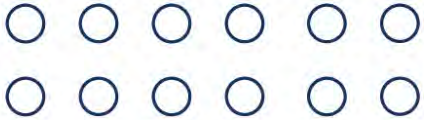
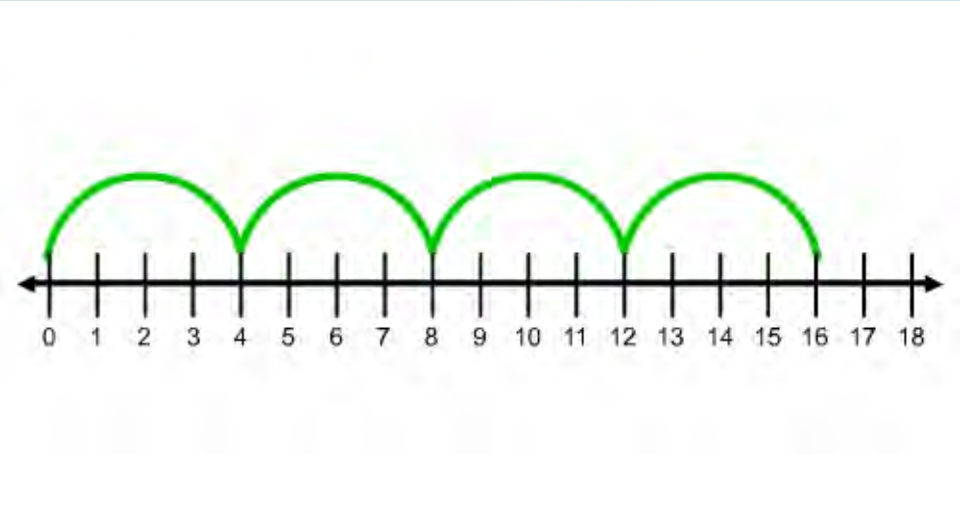
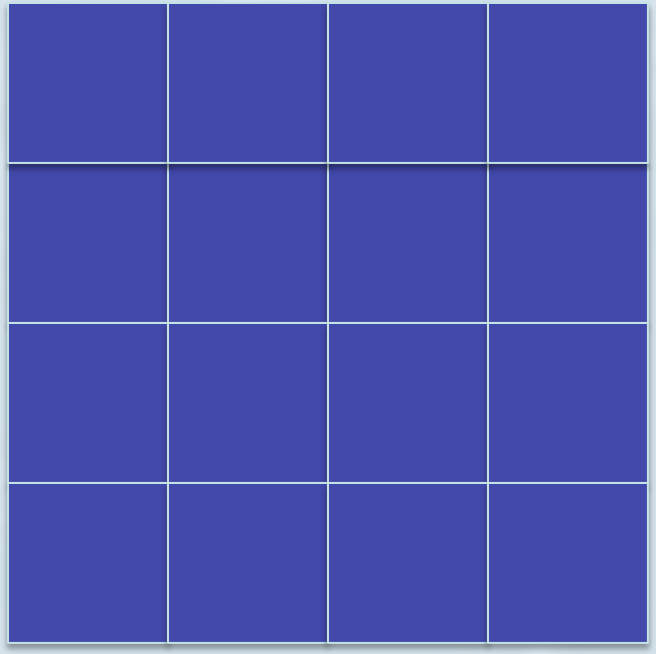


Semi abstract





Abstract

$3 \times 4 = 12$

"3 groups of 4 is the same as 12"

Practice 4: Give students opportunities to estimate

Tell all you know about the solutions to these problems

16×0.05

39.5×0.95

$436.2 \div 0.63$

$82.5 \div 1.2$

Practice 5: Make connections explicit

$$9 \div 3$$

$$\frac{9}{10} \div \frac{3}{10}$$

Practice 6: Incorporate daily review into lessons

- Number Talks….
  - http://insidemathematics.org
  - http://mathperspectives.com/num_talks.html

What is a Number Talk?

- A short, ongoing daily routine that provides students with meaningful ongoing practice with computation

- a powerful tool for helping students develop computational fluency because the expectation is that they will use number relationships and the structures of numbers to add, subtract, multiply and divide.

Why Number Talks?

- Develop Number Sense
- Develop Fluency
- Encourage Strategic Thinking
- Use Various Representations

Six step format of a number talk

1. Teacher presents a task.
2. Students have time to figure out the answer.
3. Students share their answers.
4. Students share their thinking.
5. The class agrees on the correct answer.
6. The steps are repeated for additional tasks.

The teacher….

- Provides a safe environment where each child's thinking is valued.
- Selects groups or strings of problems that allow access to all children.
- Selects problems that intentionally highlight mathematical concepts.
- Values everyone's thinking, focusing on how children get their answers.
- Shifts the focus from, "See what I see," to "What do YOU (the child) see?"
- Records, clarifies, restates.
- Realizes that it if the children don't get it, then it is the teacher's responsibility to figure out their misconceptions or lack of proficiency and to begin instruction at that point.

Number Talk with Fractions

$$5 - 2\frac{3}{5}$$

Practice 7: Use rich tasks

1) $\frac{2}{3} + \frac{1}{7} =$

2) $\frac{1}{3} + \frac{1}{12} =$

3) $\frac{1}{9} + \frac{1}{10} =$

4) $\frac{1}{4} + \frac{1}{9} =$

5) $\frac{1}{3} + \frac{1}{10} =$

6) $\frac{1}{3} + \frac{1}{9} =$

7) $\frac{1}{4} + \frac{1}{5} =$

8) $\frac{5}{6} + \frac{1}{10} =$

9) $\frac{1}{8} + \frac{1}{9} =$

10) $\frac{2}{7} + \frac{3}{10} =$

11) $\frac{1}{7} + \frac{1}{10} =$

12) $\frac{1}{6} + \frac{1}{12} =$

13) $\frac{1}{7} + \frac{1}{9} =$

14) $\frac{5}{6} + \frac{1}{8} =$

15) $\frac{1}{6} + \frac{1}{10} =$

16) $\frac{1}{7} + \frac{1}{8} =$

17) $\frac{1}{8} + \frac{1}{10} =$

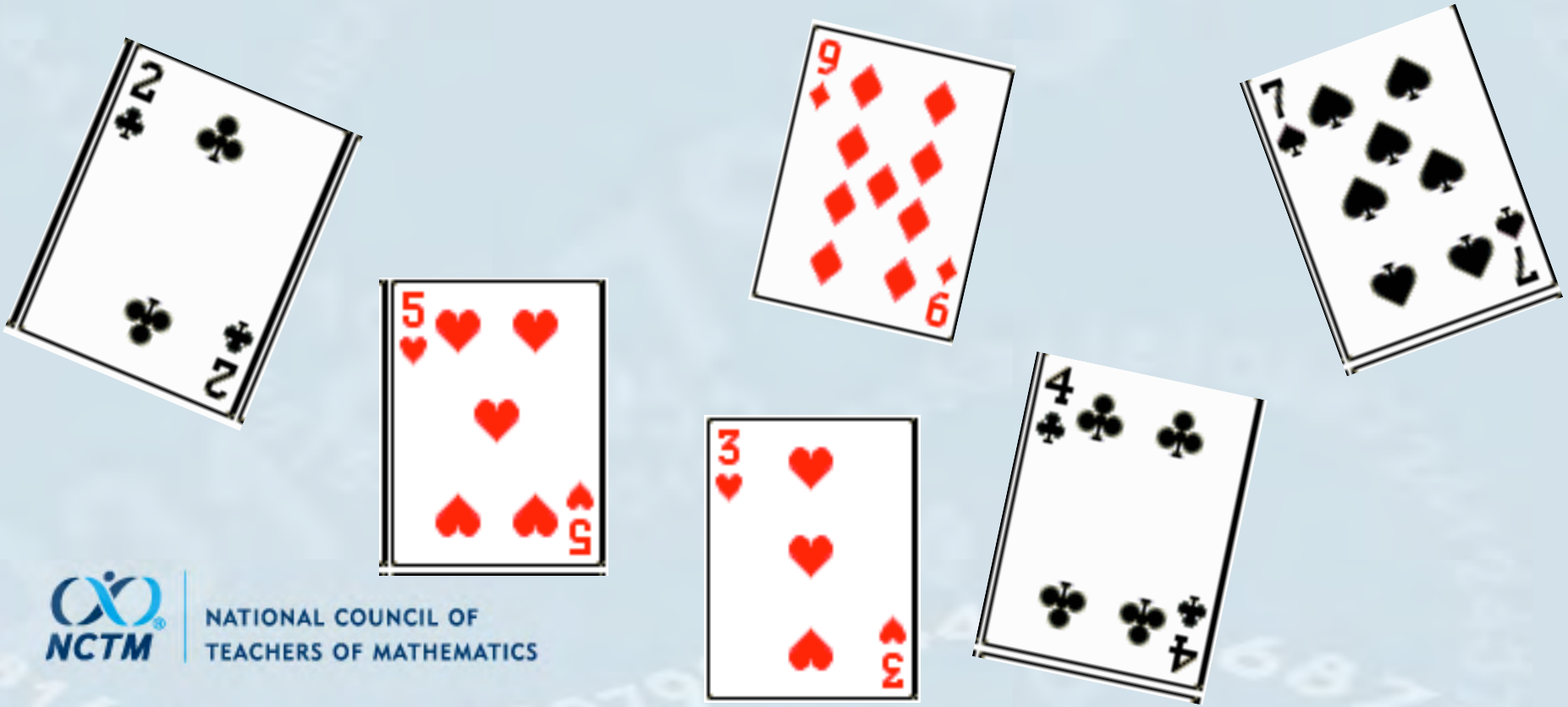
18) $\frac{1}{6} + \frac{1}{9} =$

19) $\frac{5}{7} + \frac{2}{9} =$

20) $\frac{1}{7} + \frac{7}{10} =$

Close to 1

Use 4 of the cards below to make 2 fractions whose sum is close to 1.



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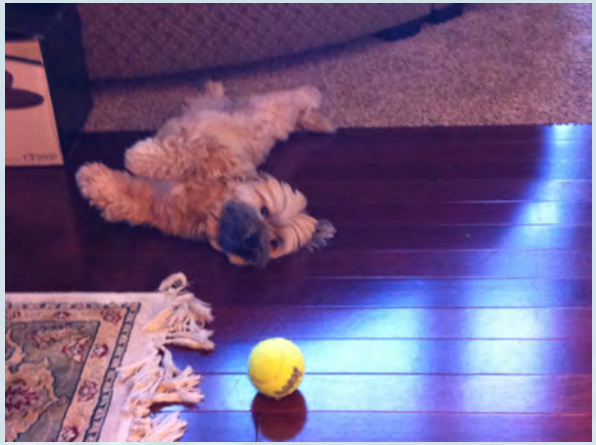
A Perimeter Task….

Show all of the rectangular puppy pens using 16 feet of fencing without using any fractions or decimals.



Practice 8: Imbed formative assessment all parts of your lesson

- Warm up activity
- Number talks
- Listen to student responses and analyze student work during class
- Quick write
- Exit slips

(see NCTM research brief at www.nctm.org)

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As we work to improve our teaching practice, we should remember…

It's not just about teaching better mathematics, it is really about teaching mathematics better!