Investigation • The Wave

Name _____ **Period** _____ **Date** _____

You will need: a stopwatch or watch with second hand

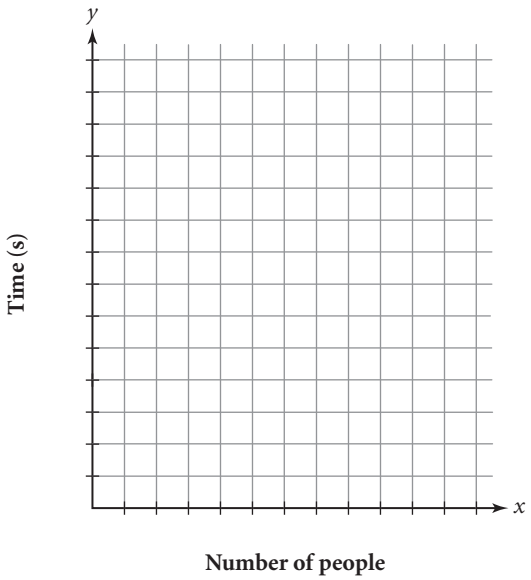
Sometimes at sporting events, people in the audience stand up quickly in succession with their arms upraised and then sit down again. The continuous rolling motion that this creates through the crowd is called "the wave." You and your class will investigate how long it takes different-size groups to do the wave.

Step 1 Using different-size groups, determine the time for each group to complete the wave. Collect at least nine pieces of data of the form (*number of people, time*), and record them in this table.

Number of people	Time (s)

Step 2 Plot the points, and find the equation of a reasonable line of fit. Write a paragraph about your results. Be sure to answer these questions:

- What is the slope of your line, and what is its real-world meaning?
- What are the x- and y-intercepts of your line, and what are their real-world meanings?
- What is a reasonable domain for this equation? Why?



Step 3 Can you use your line of fit to predict how long it would take to complete the wave if everyone at your school participated? Everyone in a large stadium? Explain why or why not.

Investigation • The Wave

With Sample Data

Name _____ **Period** _____ **Date** _____

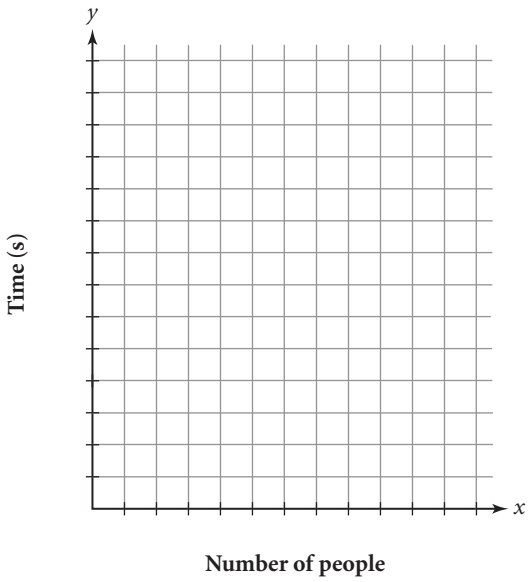
Sometimes at sporting events, people in the audience stand up quickly in succession with their arms upraised and then sit down again. The continuous rolling motion that this creates through the crowd is called "the wave." You and your class will investigate how long it takes different-size groups to do the wave.

Step 1 Using different-sized groups, a mathematics class determined the time for each group to complete the wave. They collected data of the form (*number of people, time*), and recorded them in this table.

Number of people	Time (s)
2	2.1
5	4.4
6	5.2
8	5.8
9	4.7
10	6.7
15	7.5
16	10.4
22	11

Step 2 Plot the points, and find the equation of a reasonable line of fit. Write a paragraph about these results. Be sure to answer these questions:

- What is the slope of your line, and what is its real-world meaning?
- What are the x- and y-intercepts of your line, and what are their real-world meanings?
- What is a reasonable domain for this equation? Why?



Step 3 Can you use your line of fit to predict how long it would take to complete the wave if everyone at the school participated? Everyone in a large stadium? Explain why or why not.

Exercises • Fitting a Line to Data

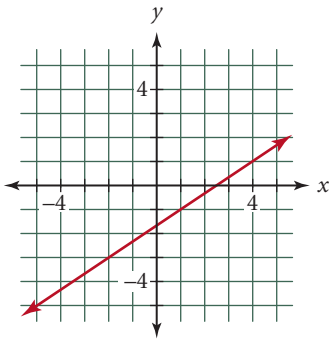
You will need

A graphing calculator for Exercises 7–9.

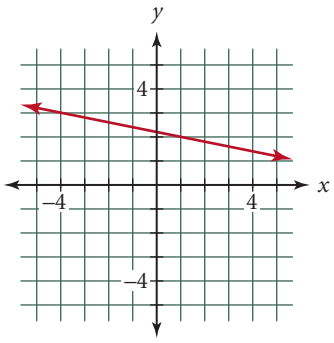
Practice Your Skills

1. Write an equation in point-slope form for each line shown.

a.



b.



2. Write an equation in point-slope form for each line described.

a. slope $\frac{2}{3}$ passing through $(5, -7)$

b. slope -4 passing through $(1, 6)$

c. parallel to $y = -2 + 3x$ passing through $(-2, 8)$

d. parallel to $y = -4 - \frac{3}{5}(x + 1)$ passing through $(-4, 11)$

3. Solve each equation.

a. Solve $u_n = 23 + 2(n - 7)$ for u_n if $n = 11$.

b. Solve $d = -47 - 4(t + 6)$ for t if $d = 95$.

c. Solve $y = 56 - 6(x - 10)$ for x if $y = 107$.

4. Consider the line $y = 5$.

a. Graph this line and identify two points on it.

b. What is the slope of this line?

c. Write the equation of the line that contains the points $(3, -4)$ and $(-2, -4)$.

d. Write three statements about horizontal lines and their equations.

5. Consider the line $x = -3$.

a. Graph it and identify two points on it.

b. What is the slope of this line?

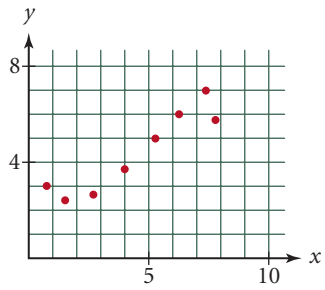
c. Write the equation of the line that contains the points $(3, 5)$ and $(3, 1)$.

d. Write three statements about vertical lines and their equations.

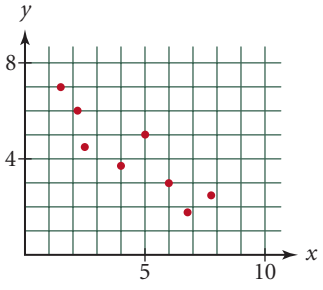
Reason and Apply

6. For each graph below, lay your ruler along your best estimate of the line of fit. Estimate the y-intercept and the coordinates of one other point on the line. Write an equation in intercept form for the line of fit.

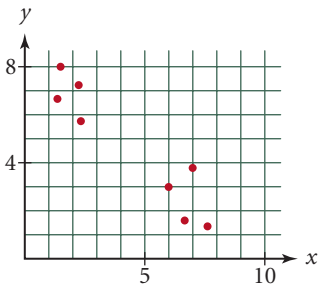
a.



b.



c.



7. **APPLICATION** A photography studio offers several packages to students posing for yearbook photos. Let x represent the number of pictures, and let y represent the price in dollars.

Number of pictures	30	18	15	11
Price ($)	27.00	20.00	17.00	14.00

a. Plot the data, and find an equation of a line of fit. Explain the real-world meaning of the slope of this line.

b. Find the y-intercept of your line of fit. Explain the real-world meaning of the y-intercept.

c. If the studio offers a 50-print package, what do you think it should charge?

d. How many prints do you think the studio should include in the package for a $9.99 special?

8. Use height as the independent variable and length of forearm as the dependent variable for the data collected from nine students.

a. Name a good graphing window for your scatter plot.

b. Write a linear equation that models the data.

c. Write a sentence describing the real-world meaning of the slope of your line.

d. Write a sentence describing the real-world meaning of the y-intercept. Explain why this doesn't make sense and how you might correct it.

e. Use your equation to estimate the height of a student with a 50 cm forearm and to estimate the length of a forearm of a student 158 cm tall.

Height (cm)	Forearm (cm)
185.9	48.5
172.0	44.5
155.0	41.0
191.5	50.5
162.0	43.0
164.3	42.5
177.5	47.0
180.0	48.0
179.5	47.5

9. **APPLICATION** This data set was collected by a college psychology class to determine the effects of sleep deprivation on students' ability to solve problems. Ten participants went 8, 12, 16, 20, or 24 hours without sleep and then completed a set of simple addition problems. The number of addition errors was recorded.

Hours without sleep	8	8	12	12	16	16	20	20	24	24
Number of errors	8	6	6	10	8	14	14	12	16	12

a. Define your variables and create a scatter plot of the data.

b. Write an equation of a line that approximates the data and sketch it on your graph.

c. Based on your model, how many errors would you predict a person to make if she or he hadn't slept in 22 hours?

d. In 9c, did you use interpolation or extrapolation? Explain.

Teacher Notes • Fitting a Line to Data

INVESTIGATION NOTES

Guiding the Investigation

In this investigation students review the point-slope form of a linear equation while experimenting with lines that fit data sets.

You can use the With Sample Data version if you do not wish to conduct the investigation as an activity; however, many students enjoy collecting the data themselves.

Ongoing Assessment

Note students' abilities to graph linear equations. Also, pay attention to their facility at substituting values into the point-slope form of a line and simplifying the resulting equation.

In doing the investigation, students will show their abilities at organizing data and their understanding of linear equations.

Facilitating Student Work

Step 1 The entire class will need the data points, so it's a good idea to involve everyone in establishing a standard technique. Then choose nine different-size groups from the full class. It's best if students about to do the wave sit in a row (in chairs or bleachers) and practice before collecting data.

Steps 2 and 3 Have students complete these steps in groups.

Discussing the Investigation

As students share results, point out that often more than one line of fit is acceptable, as long as students follow the guidelines and can explain their reasoning. **[Ask]** "What are the units of the slope?" [seconds per person] **[Critical Question]** "Why do we want a line of fit for a data set in the first place?" **[Big Idea]** Lines of fit help us make predictions. Use the terms *extrapolation* and *interpolation* for those predictions.

[Big Idea] Help the class see that to find a relationship between the coordinates of all points on the line, we let (x, y) represent any point on the line and decide how x and y can be related by an equation.

[Ask] "How does the point-slope form of a linear equation, $y = y_1 + b(x - x_1)$, compare with the intercept form, $y = a + bx$? Does y_1 equal a?" [No. The value y_1 is not the y-intercept; it's the height of the graph when $x - x_1 = 0$, or $x = x_1$.] Dynamically, the point-slope form shows how far the line moves vertically from y_1 as x moves horizontally from x_1. The intercept form shows how far the line moves vertically from a as x moves horizontally from 0. If students have studied transformations of graphs in geometry, you might point out that the point-slope form is a translation of $y = bx$ to make the line pass through the point (x_1, y_1).

[Critical Question] "Are point-slope equations the same for different data points?" **[Big Idea]** They are the same if and only if all the points lie exactly on the same line. You might want to produce data points that are truly collinear, have students find point-slope equations using various data points, and point out how those equations are equivalent.

Modifying the Investigation

Whole Class Discuss Steps 2 and 3 as a class.

Shortened Use the sample data. Have groups complete Step 2. Discuss Step 3.

One Step Pose this problem: "How can we estimate how long it will take 100 people to do the wave?" Lead the class to design an experiment using different-size groups of students. Agree on a good data set as a class, and then let cooperative groups figure out ways to make the prediction. As students work, you can direct them to use the point-slope form of a linear equation.

Investigation Answers

Investigation

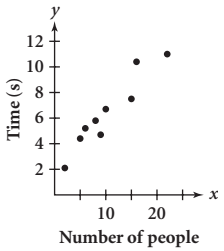
Answers for Steps 1 and 2 will vary. Refer to answers for the sample data to determine the reasonableness of students' answers.

Investigation with Sample Data

Step 1

Number of people	Time (s)
2	2.1
5	4.4
6	5.2
8	5.8
9	4.7
10	6.7
15	7.5
16	10.4
22	11.0

Step 2 Answer for the sample data:



Using the point (5, 4), $\hat{y} = 4 + \frac{6}{13}(x - 5)$. The slope indicates that it takes 0.45 s longer per additional person. The x-intercept is approximately -4.2 and does not have real-world meaning in this context; the y-intercept is 1.9 and indicates the additional time the first person needs to stand and the additional time the last person needs to fully sit down. It might also include reaction time for starting and stopping the stopwatch. The domain could start at 0 people, at 1 person, or at 2 people and increase to the number of students in the entire class.

Step 3 Sample answer: No, for a large group of people, the wave gains momentum and begins to travel faster. So for a large group of people, the data may not be linear.

EXERCISE NOTES

In the exercises students use the point-slope form to find lines of fit and make predictions.

Exercise 2 As needed, remind students that parallel lines have the same slope.

Exercise 2 [Extra Support] Remind students of the relationship between the slopes of parallel lines: The slopes are equal. Caution students who attempt to pull information other than the slope from the equations in part c and d.

Exercises 4 and 5 [Alert] Students might not recall that these are equations of horizontal and vertical lines. As needed, stress that the line with the equation $x = -3$ contains all points whose coordinates (x, y) satisfy the equation—that is, all points that have -3 as the first coordinate.

Exercise 6 You might make worksheets from the Exercise 6 transparency (included with these materials) so that students do not write in their books.

Exercises 6–9 Equations will vary somewhat, depending on which points students choose to be on the lines. Any line that fits the data "by eye" is acceptable if students can explain how they arrived at it.

Exercise 9 **Calculator Note 3A: Entering and Graphing Equations** helps students graph equations on top of scatter plots. They can make predictions by using function tables (**Calculator Note 3B: Function Tables**). **[Extra Support]** Students can use a graphing calculator to create a scatter plot and verify that their equation from part b is accurate.

EXERCISE SOLUTIONS

1. a. Possible answer: $y = 1 + \frac{2}{3}(x - 4)$. The slope of the line is $\frac{1 - (-3)}{4 - (-2)} = \frac{4}{6}$, or $\frac{2}{3}$. It passes through point (4, 1). Any equation using any two of the points $(-2, -3)$, (1, −1), and (4, 1) is correct.

b. Possible answer: $y = 2 - \frac{1}{5}(x - 1)$. The slope of the line is $\frac{2 - 3}{1 - (-4)} = -\frac{1}{5}$. It passes through point (1, 2).

2. a. $y = -7 + \frac{2}{3}(x - 5)$

b. $y = 6 - 4(x - 1)$

c. Parallel lines have the same slope, so the slope is 3. The equation is $y = 8 + 3(x + 2)$.

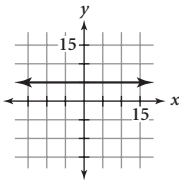
d. Parallel lines have the same slope, so the slope is $-\frac{3}{5}$. The equation is $y = 11 - \frac{3}{5}(x + 4)$.

3. a. $u_n = 23 + 2(11 - 7) = 23 + 2(4) = 23 + 8 = 31$

b. $95 = -47 - 4(t + 6)$, so $142 = -4(t + 6)$, $-35.5 = t + 6$, and $t = -41.5$

c. $107 = 56 - 6(x - 10)$, so $51 = -6(x - 10)$, $-8.5 = x - 10$, and $x = 1.5$

4. a. Answers will vary, but the y-coordinate should always be 5. Possible answers: (0, 5), (1, 5), and (2, 5).

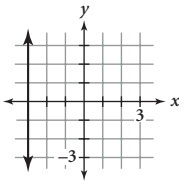


b. Slope $= \frac{5 - 5}{1 - 0} = \frac{0}{1} = 0$

c. $y = -4$

d. Sample answer: Horizontal lines have no x-intercepts (unless the horizontal line is the x-axis, or $y = 0$). They have a slope of 0. All points have the same y-coordinate.

5. a. Answers will vary, but the x-coordinate should always be −3. Possible answers: (−3, 0), (−3, 1), and (−3, 2).



b. Slope $= \frac{1 - 0}{-3 - (-3)} = \frac{1}{0}$; undefined

c. $x = 3$. (*Note:* You can't use the point-slope form because the slope is undefined.)

d. Sample answer: Vertical lines have no y-intercepts (unless the vertical line is the y-axis, or $x = 0$). They have undefined slope. All points have the same x-coordinate.

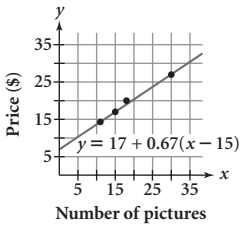
6. Possible answers:

a. The y-intercept is about 1.7. Another point is at about (5, 4.6). The slope of the line is $\frac{1.7 - 4.6}{0 - 5} = \frac{-2.9}{-5} = 0.58$. The equation of the line of fit is $\hat{y} = 1.7 + 0.58x$.

b. The y-intercept is about 7.5. Another point is at about (5, 3.75). The slope of the line is $\frac{7.5 - 3.75}{0 - 5} = \frac{3.75}{-5} = -0.75$. The equation of the line of fit is $\hat{y} = 7.5 - 0.75x$.

c. The y-intercept is about 8.6. Another point is at about (5, 3.9). The slope of the line is $\frac{8.6 - 3.9}{0 - 5} = \frac{4.7}{-5} = -0.94$. The equation of the line of fit is $\hat{y} = 8.6 - 0.94x$.

7. a. $\hat{y} = 17 + 0.67(x - 15)$. The slope means that each picture increases the price by \$0.67.



The sketched line of fit goes through two of the data points, (15, 17) and (30, 27), so you can use them to find the slope.

Slope $= \frac{27 - 17}{30 - 15} = 0.67$

b. Substitute 0 for x and solve for y.

$y = 17 + 0.67(0 - 15) = 6.95$

In addition to the cost per picture, there is a fixed fee of \$6.95.

c. $17 + 0.67(50 - 15) = \$40.45$

d. Substitute 9.99 for y and solve for x.

$$17 + 0.67(x - 15) = 9.99$$
$$0.67(x - 15) = -7.01$$
$$x - 15 \approx -10.46$$
$$x \approx 4.54$$

The studio should include four or five prints. Four prints would make more of a profit, but five would be a better promotional discount.

8. a. Answers will vary. Possible answer: $145 \leq x \leq 200$, $40 \leq y \leq 52$.

b. $\hat{y} = 50.5 + 0.26(x - 191.5)$, or $\hat{y} = 0.26x + 0.71$. Using the points (191.5, 50.5) and (165, 43.5) from the sketched line, you can find the slope.

Slope $= \frac{50.5 - 43.5}{191.5 - 165} = 0.26$

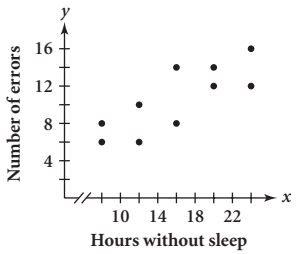
The equation of the line of fit is $\hat{y} = 50.5 + 0.26(x - 191.5)$.

c. On average, a student's forearm length increases by 0.26 cm for each additional 1 cm of height.

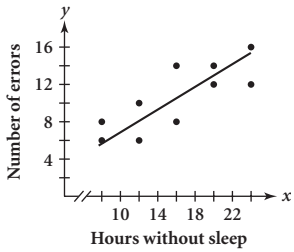
d. The y-intercept is meaningless because a height of 0 cm will not predict a forearm length of 0.71 cm. The domain should be specified.

e. 189.58 cm; 41.79 cm. Substitute 50 for $\hat{y}$ and solve for x. $50 = 0.26x + 0.71$, so $0.26x = 49.29$, and $x \approx 189.58$. Next, substitute 158 for x and solve for $\hat{y}$. $\hat{y} = 0.26(158) + 0.71 = 41.79$

9. a. Let x represent the number of hours without sleep, and let y represent the number of errors.



b. Possible answer: $\hat{y} = 3 + 0.5x$

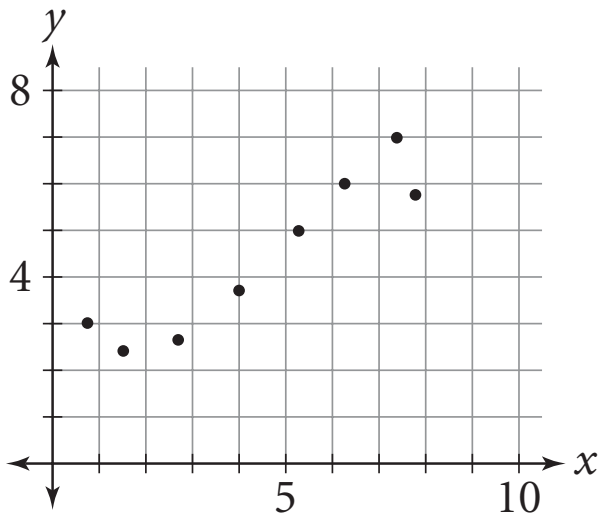


c. 14 errors; $0.5(22) + 3 = 14$

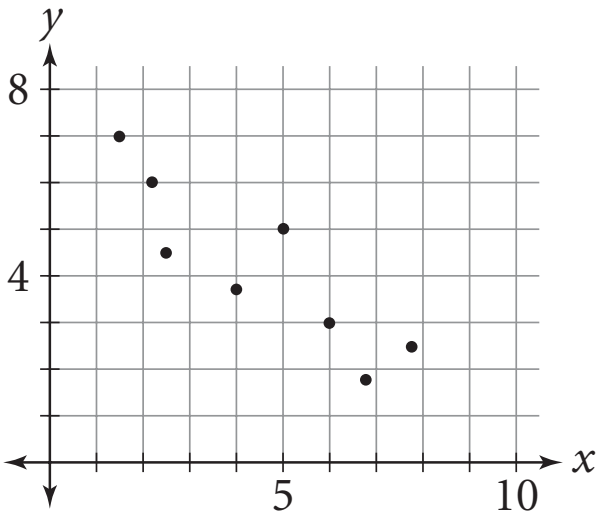
d. Interpolation, because you found a value between those given in the table

Exercise 6

a.



b.



c.