Kendall Hunt

Discovering Mathematics

Implementing the Common Core with Discovering Mathematics

COMMON CORE STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

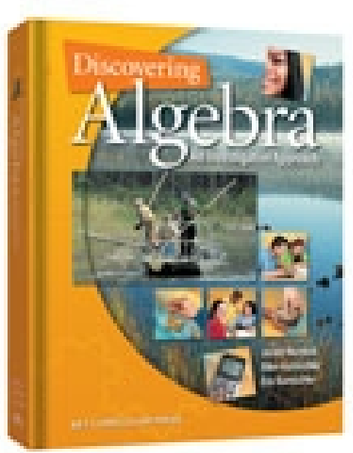
Mathematical Practice Standards

Make sense of problems and persevere in solving them

- Make predictions and determining reasonableness of possible solutions

- Think through different strategies and possible representations needed to find the solution

- Use alternative representations to check solutions



Investigation

Working Out with Equations

Manisha starts her exercise routine by jogging to the gym. Her trainer says this activity burns 215 calories. Her workout at the gym is to pedal a stationary bike. This activity burns 3.8 calories per minute.

First you'll model this scenario with your calculator.

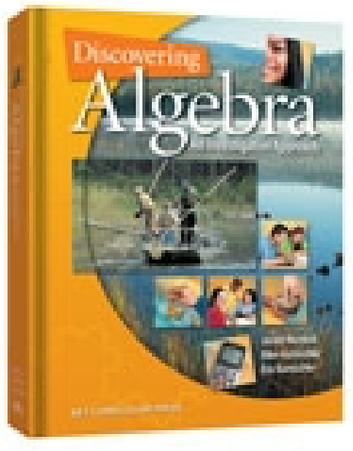
Step 1 Use calculator lists to write a recursive routine to find the total number of calories Manisha has burned after each minute she pedals the bike. Include the 215 calories she burned on her jog to the gym.

Step 2 Copy and complete the table using your recursive routine.

Step 3 After 20 minutes of pedaling, how many calories has Manisha burned? How long did it take her to burn 443 total calories?

Manisha's Workout

Pedaling time (min) x	Total calories burned y
0	215
1	
2	
20	
30	
45	
60	



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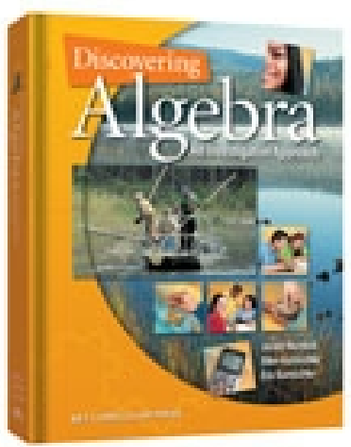
Reason abstractly and quantitatively.

- Contextualize - taking an abstract representation and finding meaning within the context of a problem situation

- Decontextualize - taking a problem situation and developing abstract representations that can be used to find a solution



Investigation

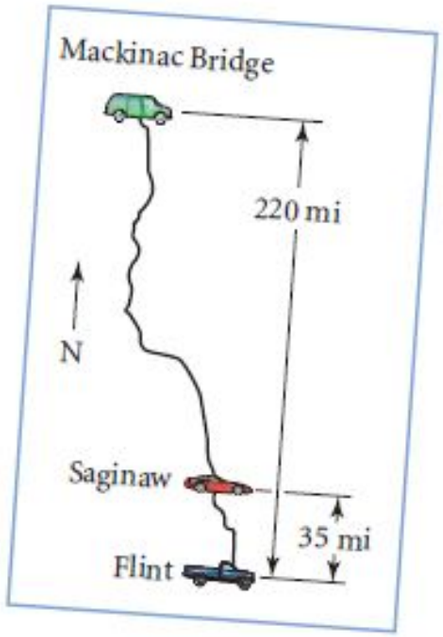
On the Road Again

You will need

- the worksheet On the Road Again Grid

A green minivan starts at the Mackinac Bridge and heads south for Flint on Highway 75. At the same time, a red sports car leaves Saginaw and a blue pickup truck leaves Flint. The car and the pickup are heading for the bridge. The minivan travels 72 mi/h. The pickup travels 66 mi/h. The sports car travels 48 mi/h.

When and where will they pass each other on the highway? In this investigation you will learn how to use recursive sequences to answer questions like these.



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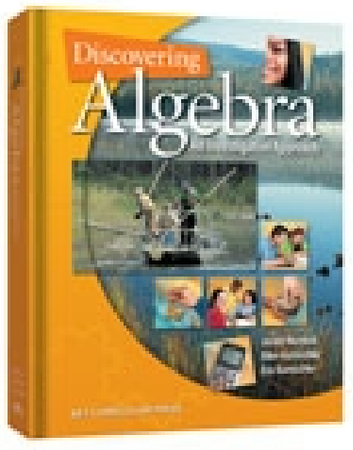

Equation	Situation
$y = 215 + 3.8x$	calories burned in a workout
$y = 321 - 13x$	floor heights in a building
$y = -300 + 15x$	earnings from mowing lawns
$y = 45 + 0.12x$	income from restaurant tabs
$y = 220 - 1.2x$	distance a car is from Flint

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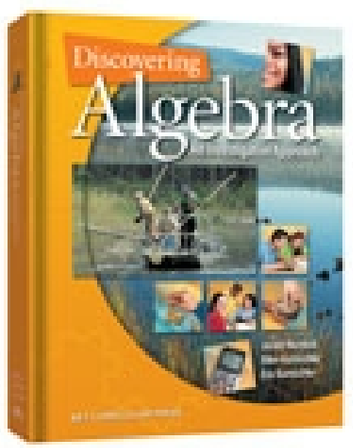
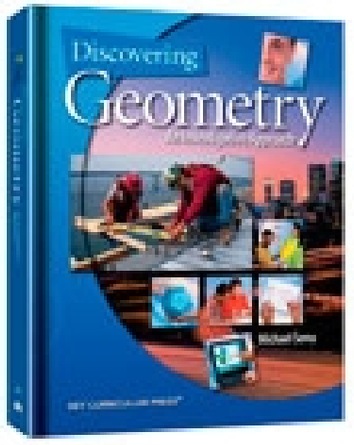


Construct viable arguments and critique the reasoning of others.

- Present justifications for their solutions
- Make and test conjectures
- Use inductive reasoning
- Develop the capacity to build and critique deductive arguments



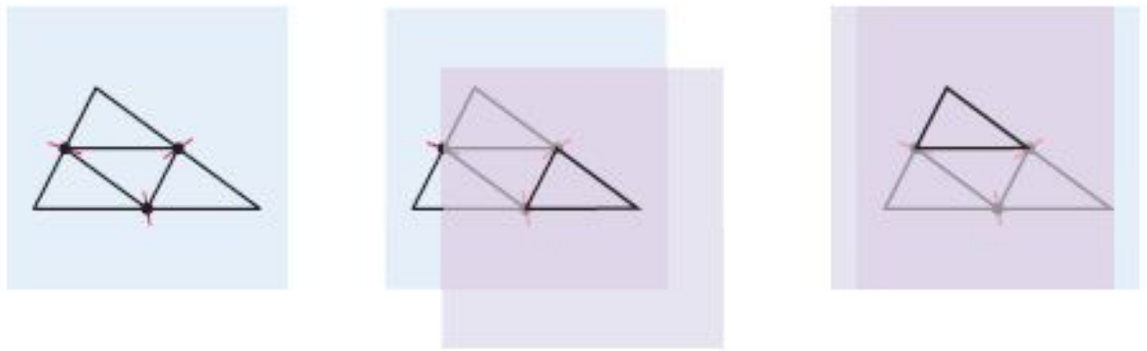
Investigation 1

Triangle Midsegment Properties

You will need

- patty paper
- a straightedge

In this investigation you will discover two properties of triangle midsegments. Each person in your group can investigate a different triangle.



Step 1 Step 2 Step 3

Step 1 Draw a triangle on a piece of patty paper. Pinch the patty paper to locate midpoints of the sides. Draw the midsegments. You should now have four small triangles.

Step 2 Place a second piece of patty paper over the first and copy one of the four triangles.

Step 3 Compare all four triangles by sliding the copy of one small triangle over the other three triangles. Compare your results with the results of your group. Copy and complete the conjecture.

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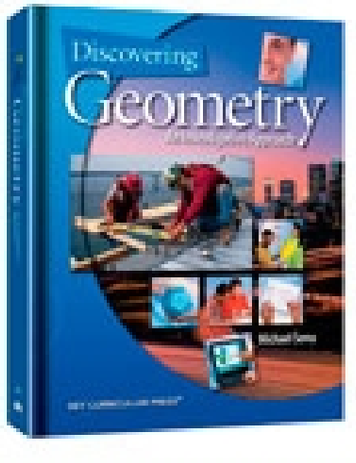
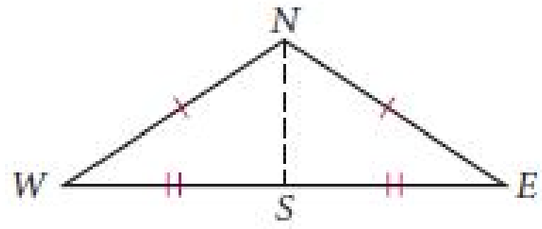


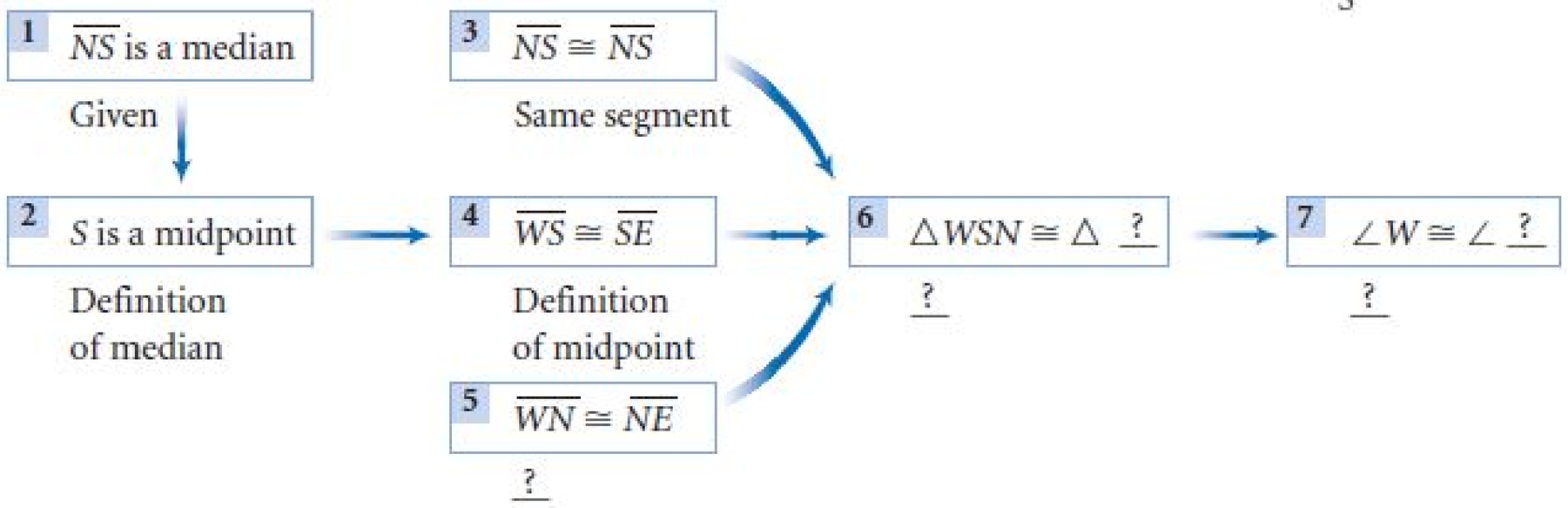
3. Complete this flowchart proof of the Isosceles Triangle Conjecture. Given that the triangle is isosceles, show that the base angles are congruent.

Given: $\triangle NEW$ is isosceles, with $\overline{WN} \cong \overline{EN}$ and median $\overline{NS}$

Show: $\angle W \cong \angle E$



Flowchart Proof



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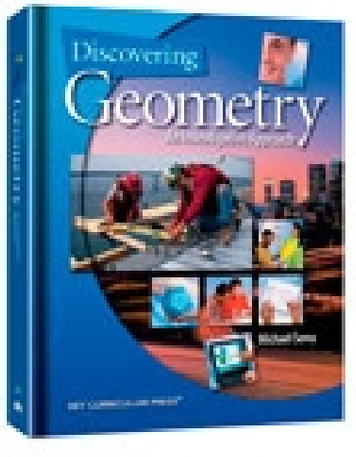
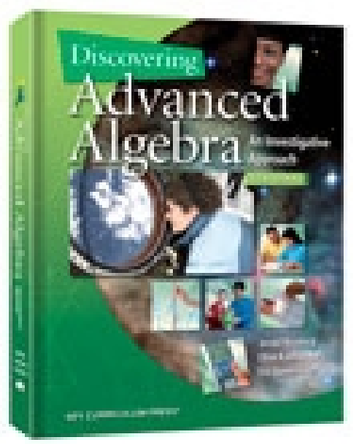


Model with mathematics.

- Create mathematical models to represent relationships
- Know when approximate models are appropriate and when precision is required
- Use different representations to express a model
- Determine limitations of a given model



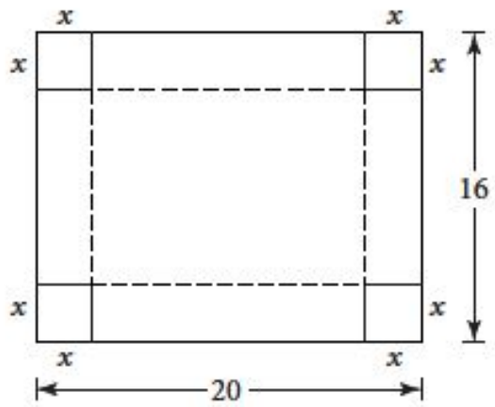
Investigation

The Box Factory

You will need

- graph paper
- scissors
- tape (optional)

What are the different ways to construct an open-top box from a 16-by-20–unit sheet of material? What is the maximum volume this box can have? What is the minimum volume? Your group will investigate this problem by constructing open-top boxes using several possible integer values for x.



Procedure Note

1. Cut several 16-by-20–unit rectangles out of graph paper.
2. Choose several different values for x.
3. For each value of x, construct a box by cutting squares with side length x from each corner and folding up the sides.

Step 1 Follow the Procedure Note to construct several different-size boxes from 16-by-20–unit sheets of paper. Record the dimensions of each box and calculate its volume. Make a table to record the x-values and volumes of the boxes.

Step 2 For each box, what are the length, width, and height, in terms of x? Use these expressions to write a function that gives the volume of a box as a function of x.

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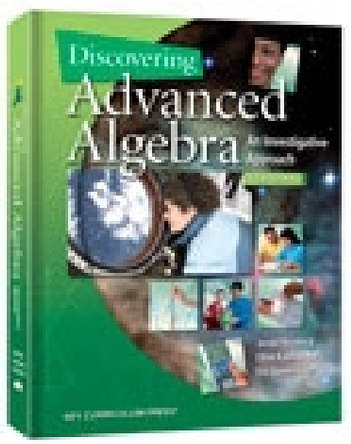

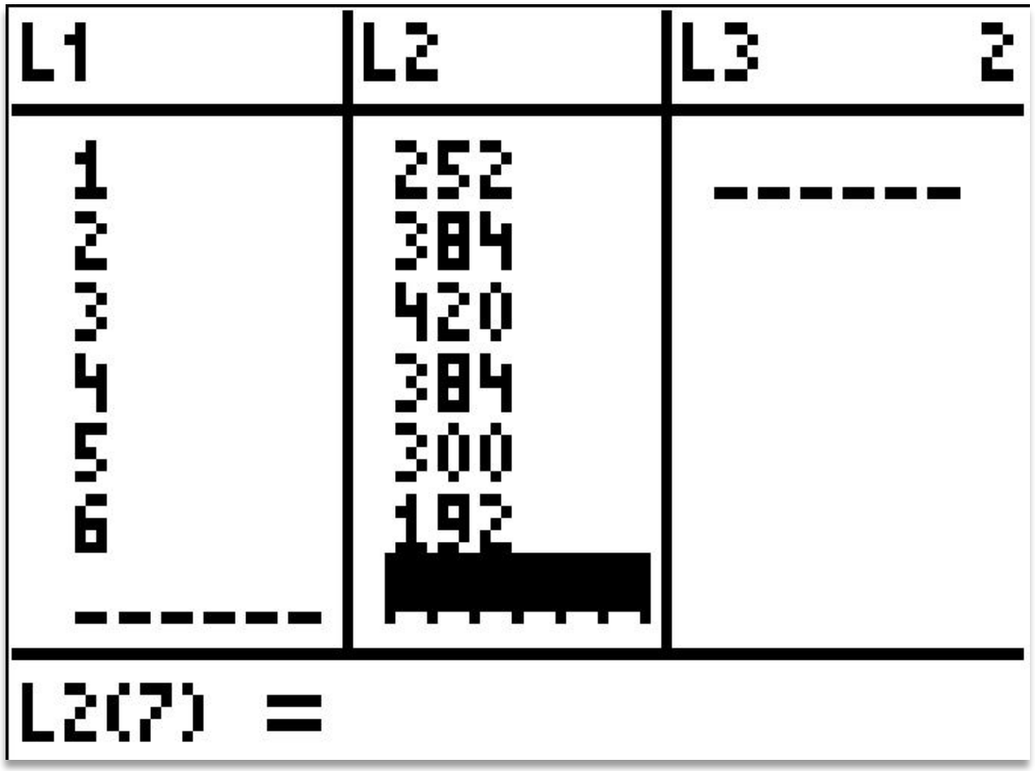

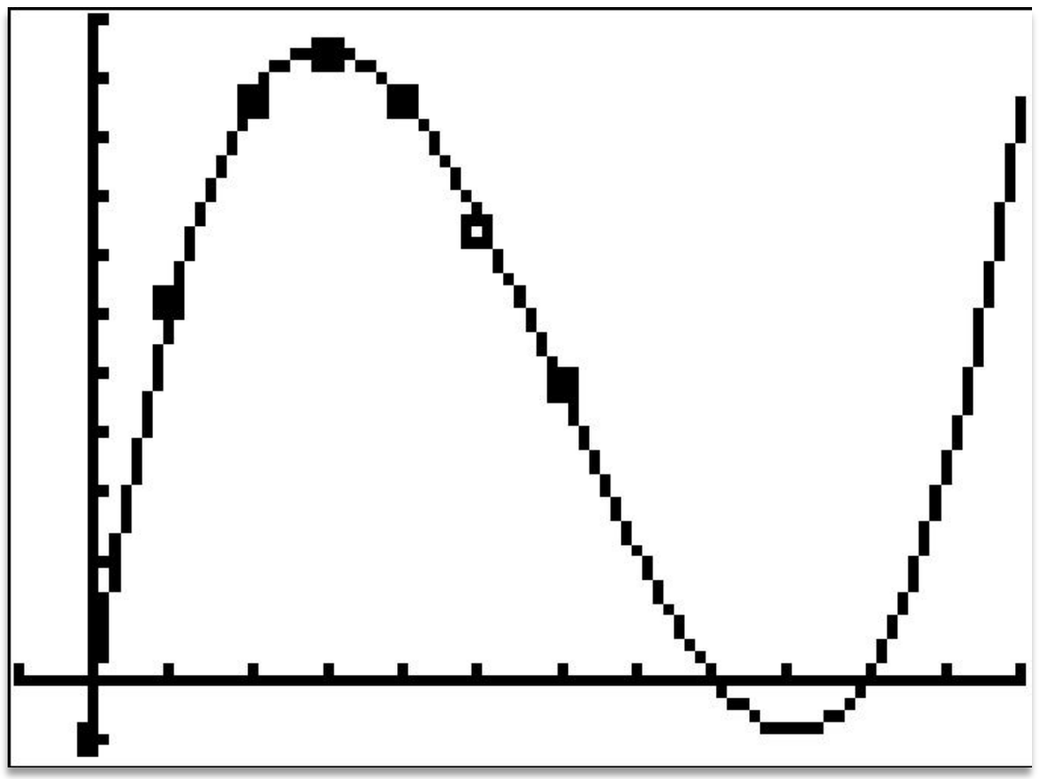

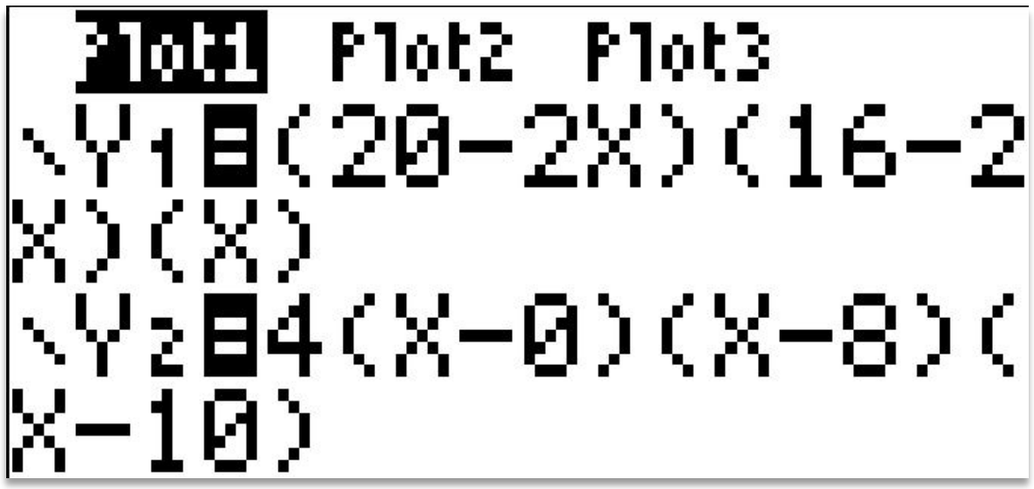

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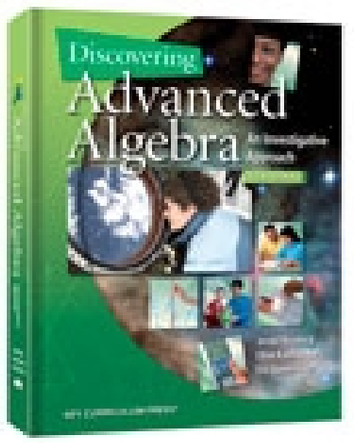
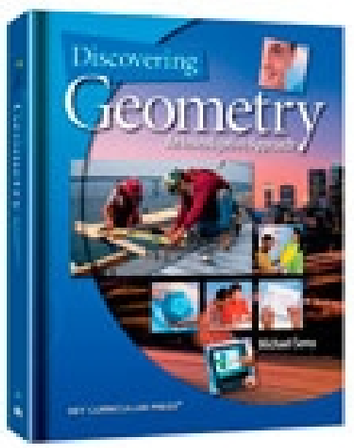


Use appropriate tools strategically.

- Know not just "how" to use technology, but why
- Use technology as an inquiry tool
- Graphing calculators, Sketchpad, Internet resources, ruler, compass, paper and pencil



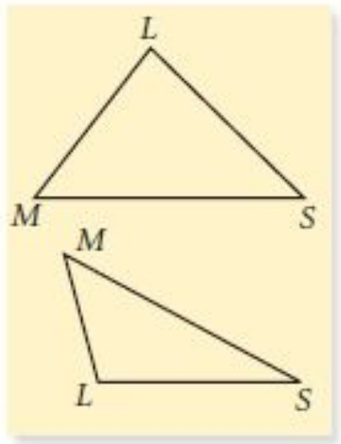
Investigation 2

Where Are the Largest and Smallest Angles?

You will need

- a ruler
- a protractor

Each person should draw a different scalene triangle for this investigation. Some group members should draw acute triangles, and some should draw obtuse triangles.



Step 1 Measure the angles in your triangle. Label the angle with greatest measure $\angle L$, the angle with second greatest measure $\angle M$, and the smallest angle $\angle S$.

Step 2 Measure the three sides. Label the longest side l, the second longest side m, and the shortest side s.

Step 3 Which side is opposite $\angle L$? $\angle M$? $\angle S$?

Discuss your results with others. Write a conjecture that states where the largest and smallest angles are in a triangle, in relation to the longest and shortest sides.

Side-Angle Inequality Conjecture C-21

In a triangle, if one side is longer than another side, then the angle opposite the longer side is ?.

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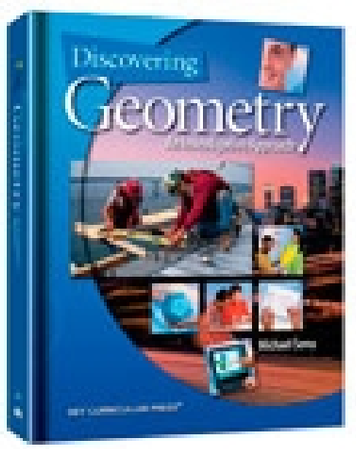
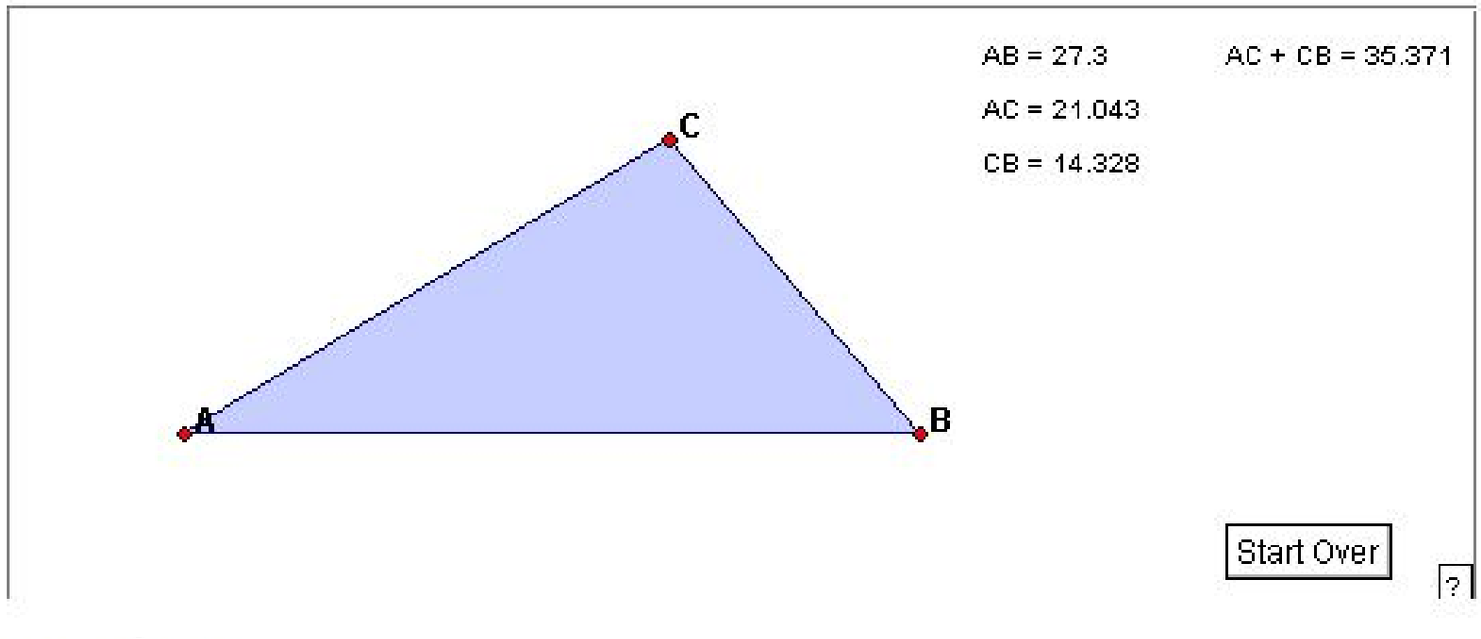


Sketch

This sketch shows triangle *ABC*, the lengths of the sides, *AB*, *AC*, and *CB*, and the sum *AC* + *CB*. Drag any of the vertices to change the shape of the triangle. Observe how the measurements are related as you change the triangle.



Investigate

1. Initially, how is the sum *AC* + *CB* related to the length of side *AB*?
2. Does this relationship change when you drag the vertices of the triangle?
3. Find a location for point *C* for which *AC* + *CB* = *AB*. Describe what happens to the triangle at this point.

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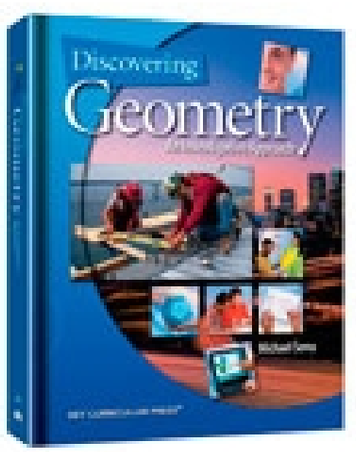
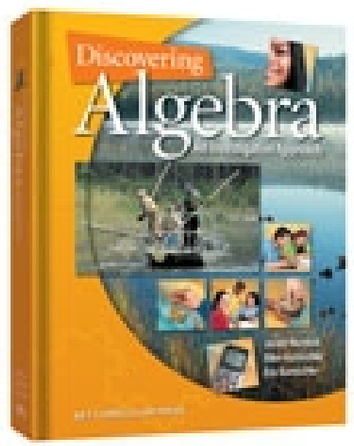


Attend to precision.

- Accurate use of terminology
- Pays attention to units
- Use appropriate amount of precision



Investigation

Rocket Science

A model rocket blasts off and its engine shuts down when it is 25 m above the ground. Its velocity at that time is 50 m/s. Assume that it travels straight up and that the only force acting on it is the downward pull of gravity. In the metric system, the acceleration due to gravity is 9.8 m/s^2. The quadratic function $h(t) = \frac{1}{2}(-9.8)t^2 + 50t + 25$ describes the rocket's **projectile motion.**

Known as the father of rocketry, Robert Hutchings Goddard fired the first successful liquid-fueled rocket in 1926. Learn more about Goddard at www.keymath.com/DA .

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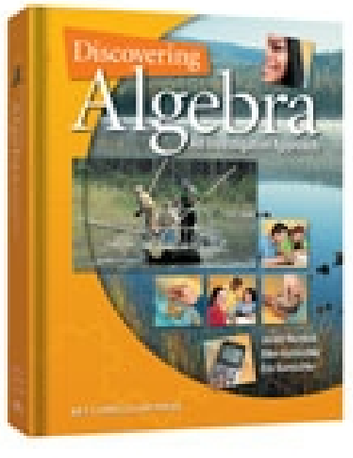
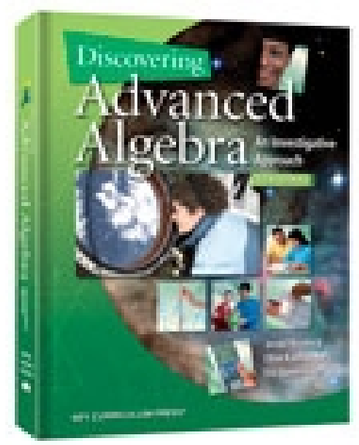


Look for and make use of structure.

- Finds patterns in mathematical representations
- Number sense
- Uses known structures and patterns to build proof



Investigation

Properties of Logarithms

Step 1 Use your calculator to complete the table. Record the values to three decimal places.

Step 2 Look closely at the values for the logarithms in the table. Look for pairs of values that add up to a third value in the table. For example, add log 2 and log 3. Where can you find that sum in the table?

Record the equations that you find in the form log 2 + log 3 = log ?. (*Hint*: You should find at least six equations.)

Step 3 Write a conjecture based on your results from Step 2.

Step 4 Use your conjecture to write log 90 as the sum of two logs. Do the same for log 30 and log 72. Then use the table and your calculator to test your conjecture.

Complete the following statement:
log a + log b = log ?.

Log form	Decimal form
Log 2	0.301
Log 3	
Log 5	
Log 6	
Log 8	
Log 9	
Log 10	
Log 12	
Log 15	
Log 16	
Log 25	
Log 27	

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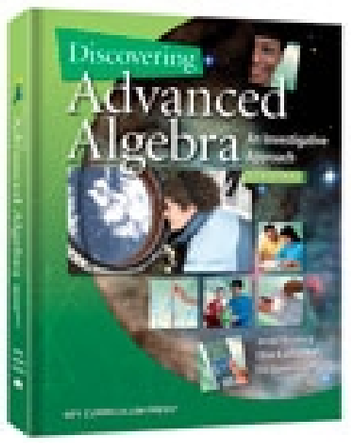
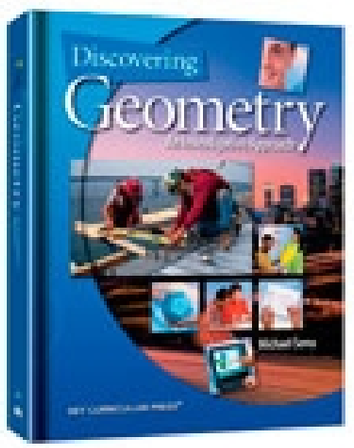


Look for and express regularity in repeated reasoning.

- Sees repeated patterns to connect recursive sequences to explicit functions
- Inductively examines relationships to determine properties and definitions
- Applies reasoning from one concept to another



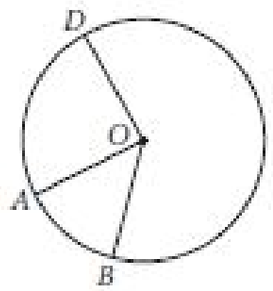
Investigation 1

Defining Angles in a Circle

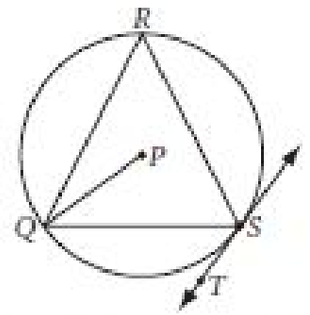
Write a good definition of each boldfaced term. Discuss your definitions with others in your group. Agree on a common set of definitions as a class and add them to your definition list. In your notebook, draw and label a figure to illustrate each term.

Step 1

Central Angle



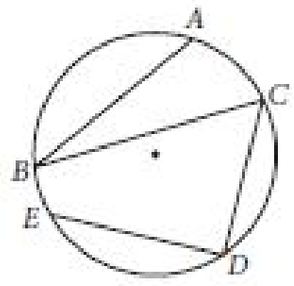
$\angle AOB$, $\angle DOA$, and $\angle DOB$ are central angles of circle O.



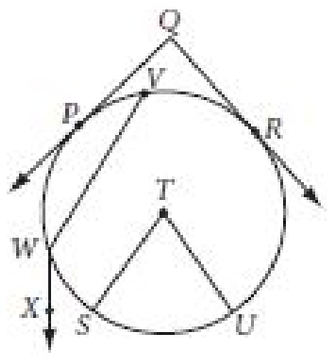
$\angle PQR$, $\angle PQS$, $\angle RST$, $\angle QST$, and $\angle QSR$ are not central angles of circle P

Step 2

Inscribed Angle



$\angle ABC$, $\angle BCD$, and $\angle CDE$ are inscribed angles.



$\angle PQR$, $\angle STU$, and $\angle VWX$ are not inscribed angles.

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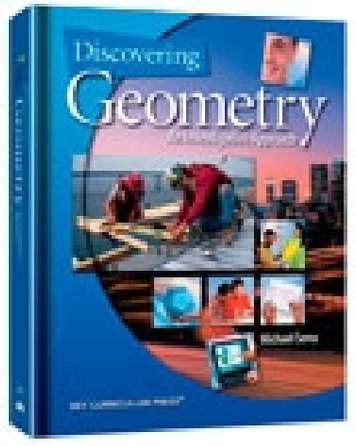


Learning Not Memorizing

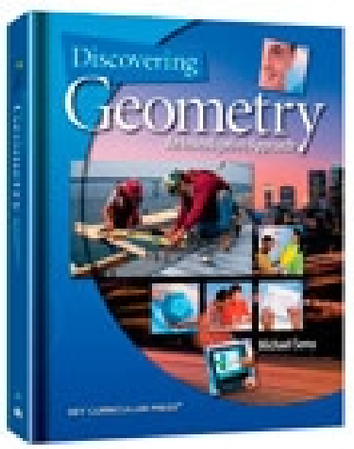
Level 0: ***Visual***

Level 1: ***Descriptive/Analytic***

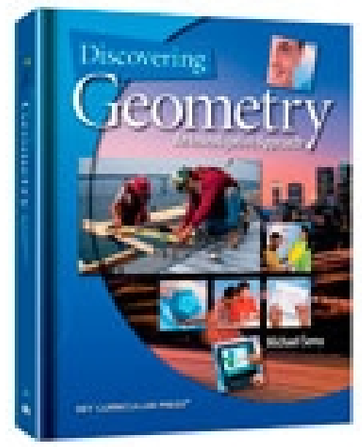
Level 2: ***Abstract/Relational***

Level 3: ***Formal Deduction/Proof***

Level 4: ***Rigor***



What shape is this?



Match the response to the van Hiele level

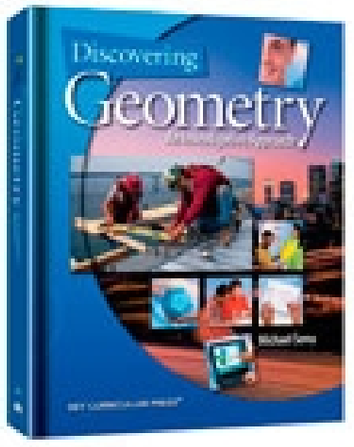
- It is a rectangle because it is a quadrilateral with four right angles, the opposites are parallel, consecutive sides are perpendicular, etc…

- I can prove it is a rectangle if it’s a parallelogram with one right angle

- I know it’s a rectangle if it’s a parallelogram with four right angles

- It is a rectangle because it looks like a door

- Well, if I draw that rectangle on a sphere

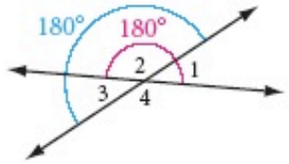


Paragraph Proof

Lesson 2.5: Angle Relationships

Developing Proof You used inductive reasoning to discover both the Linear Pair Conjecture and the Vertical Angles Conjecture. Are they related? If you accept the Linear Pair Conjecture as true, can you use deductive reasoning to show that the Vertical Angles Conjecture must be true?

You can see from the diagram that the sum of the measures of angles 1 and 2 is equal to the sum of the measures of angles 2 and 3 because they are both linear pairs. Because angle 2 is the same in both sums, angle 1 must equal angle 3. To write a deductive argument, go through this logic one step at a time.



Deductive Argument

For any linear pair of angles, their measures add up to 180°.

$m\angle 1 + m\angle 2 = 180°$

$m\angle 2 + m\angle 3 = 180°$

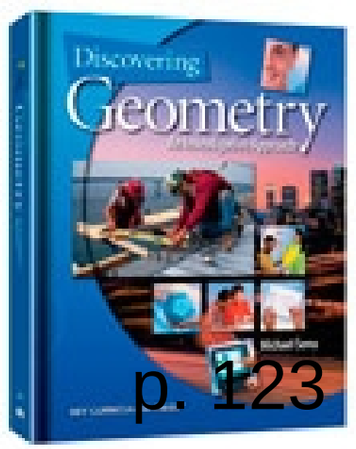
Since both expressions on the left equal 180°, they equal each other.

$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$

Subtract $m\angle 2$ from both sides of the equation.

$m\angle 1 = m\angle 3$

Vertical angles 1 and 3 have equal measures, so they are congruent. ■



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Flowchart Proof

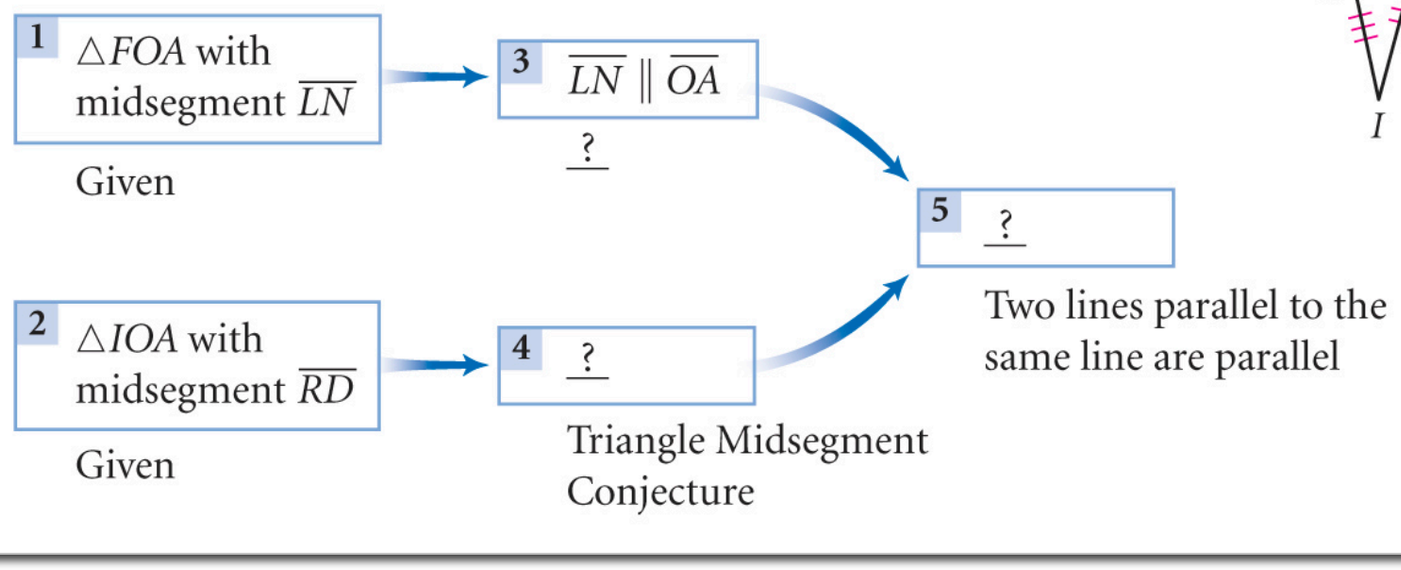
Lesson 5.4: Properties of Midsegments

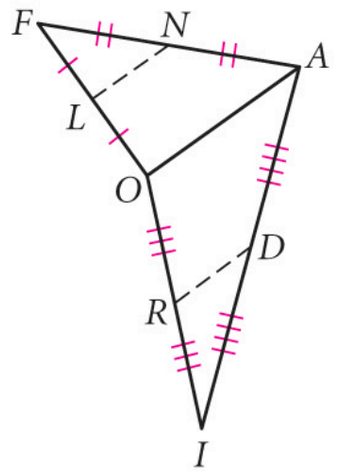
8. Copy and complete the flowchart to show that $\overline{LN} \parallel \overline{RD}$.

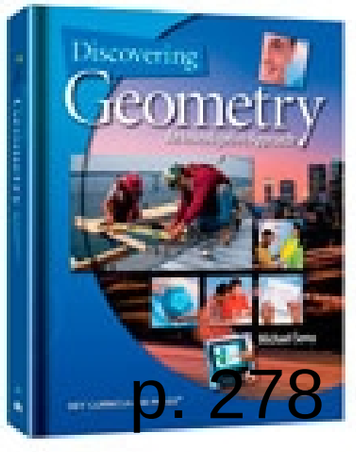
Given: Midsegment $\overline{LN}$ in $\triangle FOA$
Midsegment $\overline{RD}$ in $\triangle IOA$

Show: $\overline{LN} \parallel \overline{RD}$

Flowchart Proof







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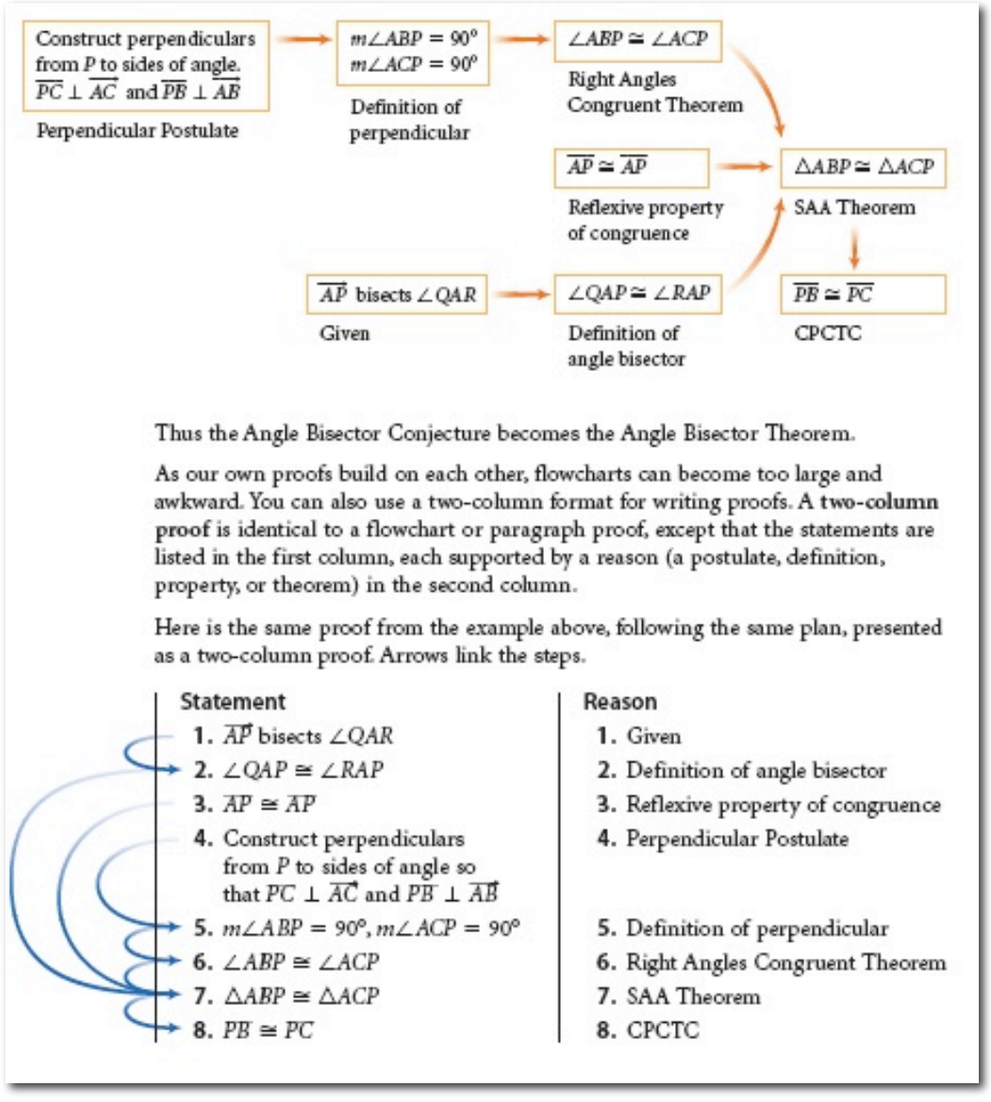
Flowchart to Two-Column

Lesson 13.3: Triangle Proofs

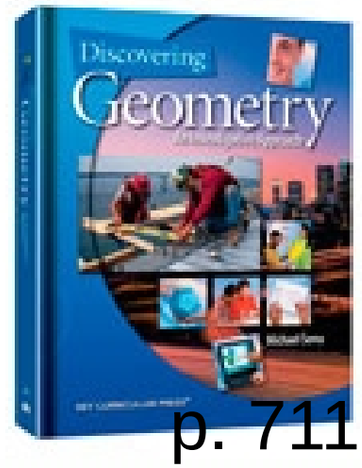


Thus the Angle Bisector Conjecture becomes the Angle Bisector Theorem.

As our own proofs build on each other, flowcharts can become too large and awkward. You can also use a two-column format for writing proofs. A **two-column proof** is identical to a flowchart or paragraph proof, except that the statements are listed in the first column, each supported by a reason (a postulate, definition, property, or theorem) in the second column.

Here is the same proof from the example above, following the same plan, presented as a two-column proof. Arrows link the steps.

Statement	Reason
1. $\overline{AP}$ bisects $\angle QAR$	1. Given
2. $\angle QAP \cong \angle RAP$	2. Definition of angle bisector
3. $\overline{AP} \cong \overline{AP}$	3. Reflexive property of congruence
4. Construct perpendiculars from P to sides of angle so that $\overline{PC} \perp \overline{AC}$ and $\overline{PB} \perp \overline{AB}$	4. Perpendicular Postulate
5. $m\angle ABP = 90°, m\angle ACP = 90°$	5. Definition of perpendicular
6. $\angle ABP \cong \angle ACP$	6. Right Angles Congruent Theorem
7. $\triangle ABP \cong \triangle ACP$	7. SAA Theorem
8. $PB \cong PC$	8. CPCTC



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