Connecting and Reconnecting to What it Means to Understand Mathematics

Grab your smartphone, tablet, or laptop for this interactive session!

http://bit.ly/18EjHCk



James L. Kratky

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Presentation Overview

- Developing and demonstrating *mathematical* understanding
- Action statements about *understanding*
- Connecting action with the SMP
- Sample tasks to consider

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Warm-up

How do our students:

- Develop their mathematical understanding?
- Demonstrate their mathematical understanding?

(Write down your thoughts in your notes.)

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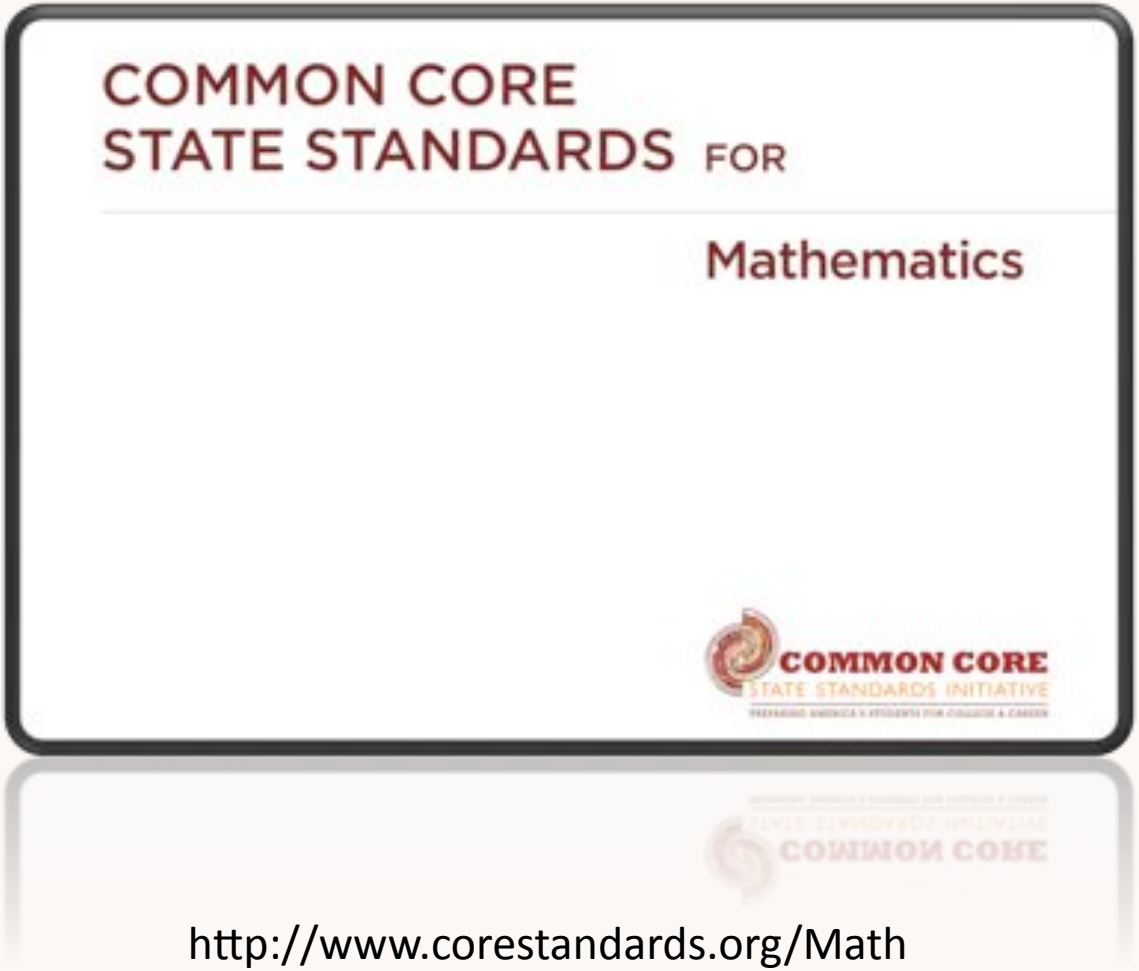
Why ask these questions?



http://www.corestandards.org/Math

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Understanding

"One hallmark of mathematical understanding is the ability to **justify**, in a way appropriate to the student's mathematical maturity, **why a particular mathematical statement is true** or where a mathematical rule comes from"

-CCSSM, p.4

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Example of an Action Statement

Justify why a particular mathematical statement is true.

-CCSSM, p. 4

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Challenge: Action Statements

- Action statement(s): How do your students develop mathematical understanding?
- Action statement(s): How do your students show that they understand something in mathematics?
- Submit your action statements to our Google Form!

http://bit.ly/18EjHCk

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Challenge: 3 Action Statements

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Our Results (from this session)!!!

What do your students DO to DEVELOP their mathematical understanding?	How do your students DEMONSTRATE mathematical understanding?
Model with 3D objects.	Create images and models.
Talk math.	
Talk math.	Present arguments and critique arguments of others.
Discuss problems in small groups	Drawings, writings, talk, ...
Practice	Verbally explain
practice problems	taking tests
Ask questions.	Present arguments and critique arguments of others.
Write summaries in their own words.	Present arguments and critique arguments of others.
Try even when the problems are challenging	Verbally explain ideas to other students
Create different representations (graph, story, table, equation...)	Present arguments and critique arguments of others.
explore concepts	explain or justify verbally or written
Derive a rule from a mathematical pattern.	Connect a mathematical concept to a real world situation.
Engage with rich problems	Develop comprehensive solutions to real world problems grounded in mathematical reasoning
defending their answers to peers	completing poster math without help from teacher
Present and/or critique arguments of justifications	Present arguments and critique arguments of others.
	Practice on their own and exploring each mathematical topic, asking questions when not understanding.
	Apply knowledge of mathematical rules to a new (unfamiliar) problem.

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Our Results (Continued)!!!

What do your students DO to DEVELOP their mathematical understanding?	How do your students DEMONSTRATE mathematical understanding?
research a given statistics topic, using the internet, to complete guided notes	
Present and/or critique arguments of justifications	Note: all of the develop statements apply here too!!!
apply their knowledge to a mathematical investigation	
Collaborate with peers to justify conjectures.	Communicate solutions with words, models, and or equations.
Discuss with others how to solve a problem	Present solutions to solving a problem
Interact with mathematical topics in a manner that is relevant to the student.	Discuss topics and be able to defend responses to questioning.
	perform assessments
Persevere through problem solving	Solve by any given means and explain their answer
	apply probability rules to casino games
Solve real life problems	Applying the concept to any new problem
Explore relationships and patterns in a mathematical context	State a conclusion and give support reasons verbally or in written form
inquire further to clarify or deepen their understanding	work together in a project, sharing ideas and knowledge to solve a higher level problem
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Solve real life problems	Applying the concept to any new problem

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1) Make sense of problems and persevere in solving them.

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HW: Repeat for other SMPs

- Isolate action statements in SMPs
- Integrate actions into lesson plans
- Implement instruction to support action

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An Example for Grades K-2

Representing numbers in base ten.

- All you need to know to start: 10 little squares
- This is a 'Quick Images' activity
- Task: how many do you see?

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http://bit.ly/Qm40q9

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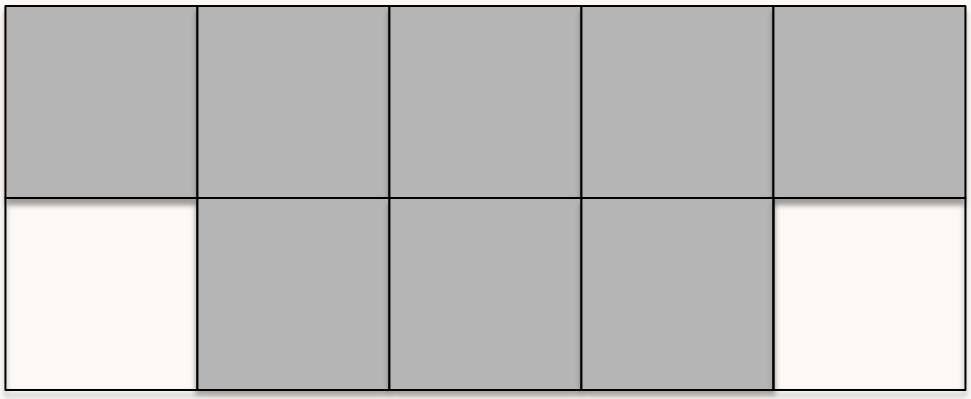
An Example for Grades K-2

Representing numbers in base ten.



How many did you see?
How do you know?

http://bit.ly/Qm40q9

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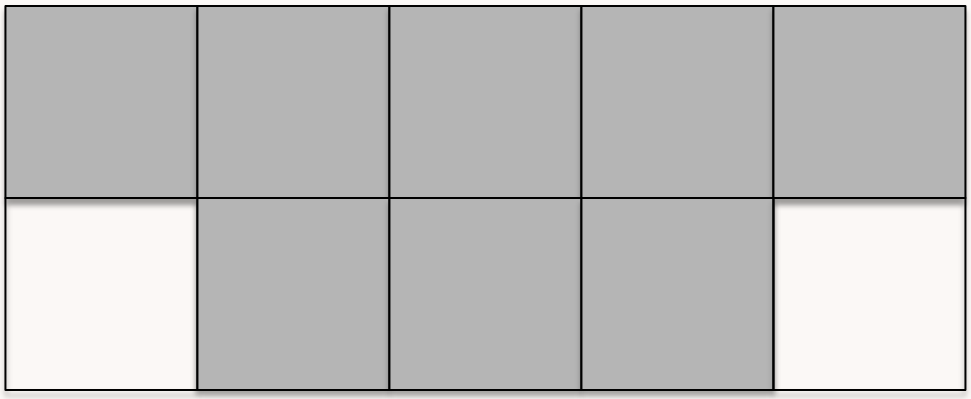
An Example for Grades K-2

Representing numbers in base ten.



How many did you see?
How do you know?

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An Example for Grades K-2

Representing numbers in base ten.

How many did you see?
How do you know?

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An Example for Grades K-2

Representing numbers in base ten.

How many did you see?
How do you know?

CCSS.Math.Content.K.OA.A.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

http://bit.ly/Qm40q9

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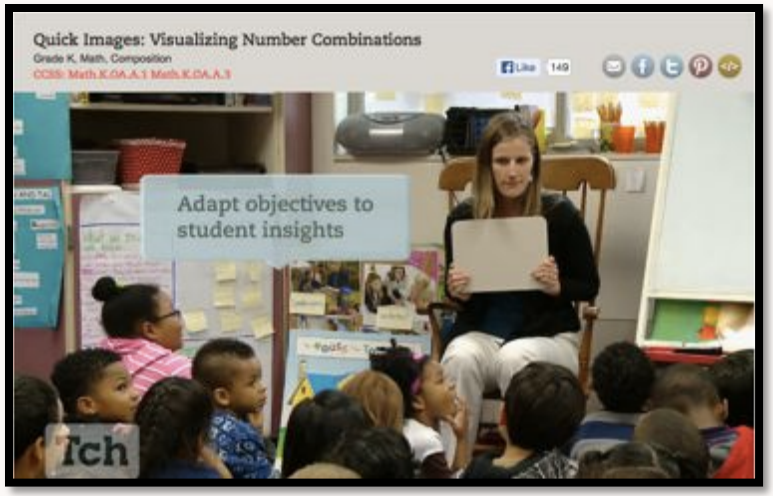
An Example for Grades K-2

Representing numbers in base ten.



https://www.teachingchannel.org/videos/visualizing-number-combinations

http://bit.ly/Qm40q9

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An Example for Grades 3-5

The FOUR operations

http://bit.ly/HhR6Xm

An Example for Grades 3-5

The FOUR operations: Multiplication

When is it okay to use a calculator?

All too often, I run into teachers (both preservice and inservice) lamenting that kids are using calculators to compute something simple, like 6 x 7. These teachers express their frustration by threatening to not let kids use any calculators until the kids prove that they know their facts. And there will be no calculators for any simple computations. I understand this thinking but I am not sure it will achieve the desired result - wise calculator use (i.e. phronesis).

http://deltascape.blogspot.com/2012/04/when-is-it-okay-to-use-calculator.html
http://bit.ly/HhR6Xm

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An Example for Grades 3-5

The FOUR operations: Multiplication

The experiment I am most interested in involves computing multiplication facts with and without a calculator. I give pairs of students four worksheets like the one shown below.

How long did it take?

http://deltascape.blogspot.com/2012/04/when-is-it-okay-to-use-calculator.html
http://bit.ly/HhR6Xm

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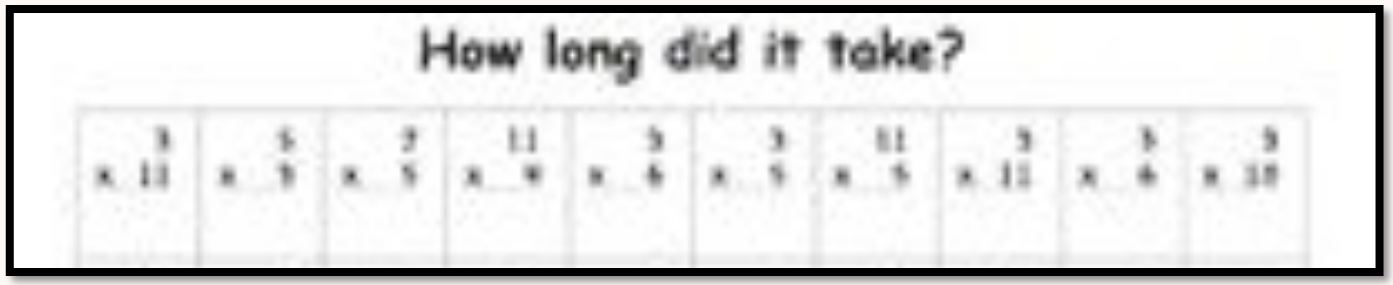
An Example for Grades 3-5

The FOUR operations: Multiplication



http://deltascape.blogspot.com/2012/04/when-is-it-okay-to-use-calculator.html
http://bit.ly/HhR6Xm

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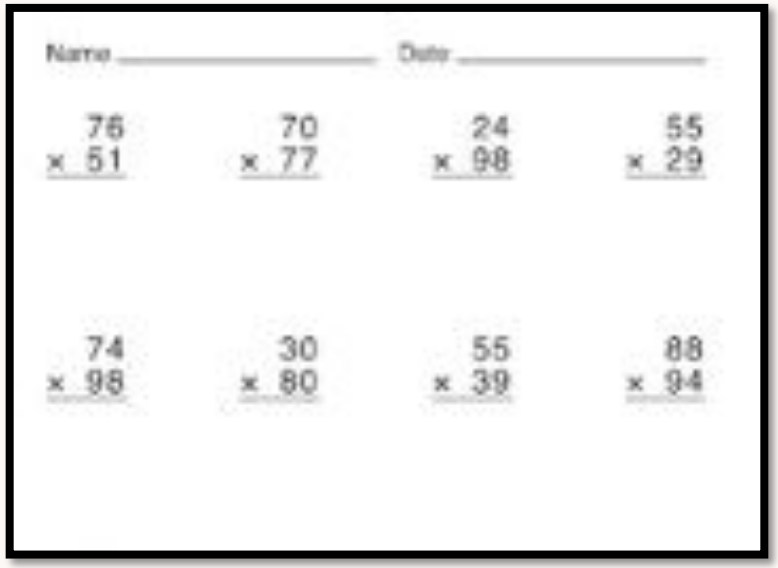
An Example for Grades 3-5

The FOUR operations: Multiplication


Name _____ Date _____

76 × 51	70 × 77	24 × 98	55 × 29
74 × 98	30 × 80	55 × 39	88 × 94

http://deltascape.blogspot.com/2012/04/when-is-it-okay-to-use-calculator.html
http://bit.ly/HhR6Xm

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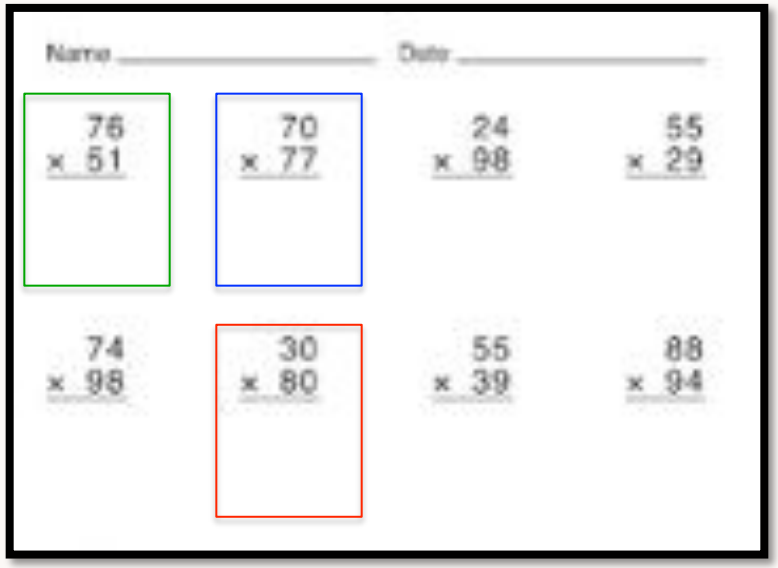
An Example for Grades 3-5

The FOUR operations: Multiplication



http://deltascape.blogspot.com/2012/04/when-is-it-okay-to-use-calculator.html
http://bit.ly/HhR6Xm

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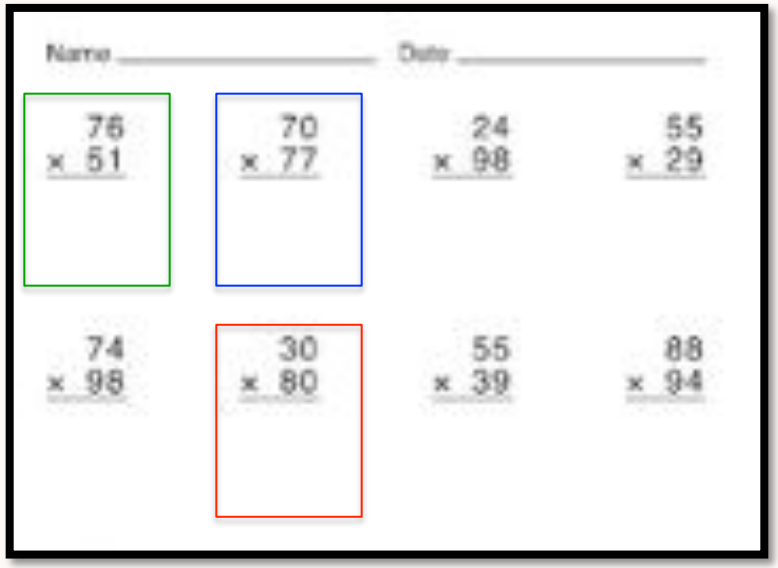
An Example for Grades 3-5

The FOUR operations: Multiplication

$$\begin{array}{rrr} & 3 & 0 \\ \times & 8 & 0 \\ \hline & & \end{array}$$



http://deltascape.blogspot.com/2012/04/when-is-it-okay-to-use-calculator.html
http://bit.ly/HhR6Xm

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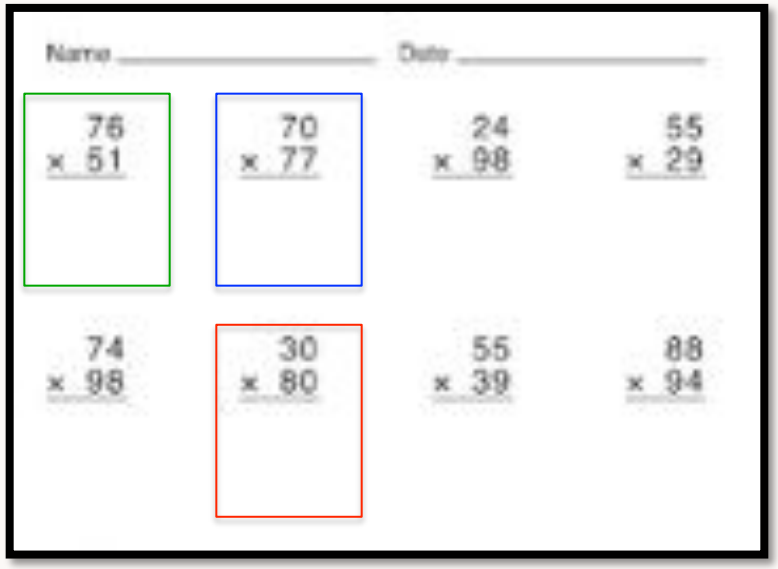
An Example for Grades 3-5

The FOUR operations: Multiplication

		3	0
	x	8	0
			0
		0	0
		0	0
2	4	0	0



CCSS.Math.Content.3.NBT.A.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

http://deltascape.blogspot.com/2012/04/when-is-it-okay-to-use-calculator.html
http://bit.ly/HhR6Xm

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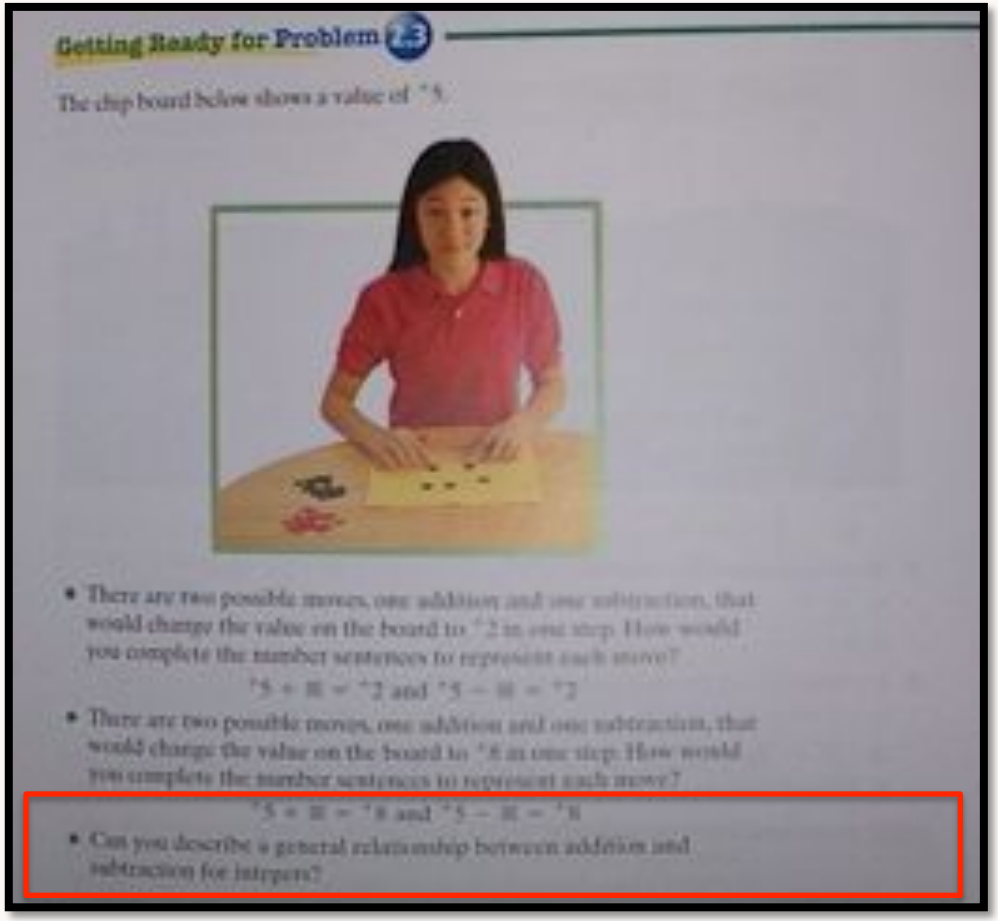
An Example for Grades 6-8

Moving towards notion of variables.



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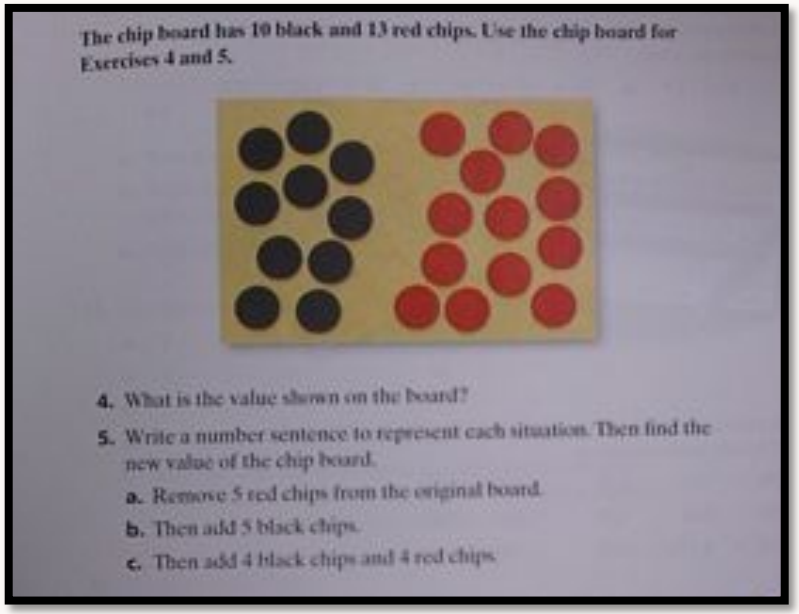
An Example for Grades 6-8

Moving towards notion of variable.



The chip board has 10 black and 13 red chips. Use the chip board for Exercises 4 and 5.

4. What is the value shown on the board?
5. Write a number sentence to represent each situation. Then find the new value of the chip board.
   a. Remove 5 red chips from the original board.
   b. Then add 5 black chips.
   c. Then add 4 black chips and 4 red chips.

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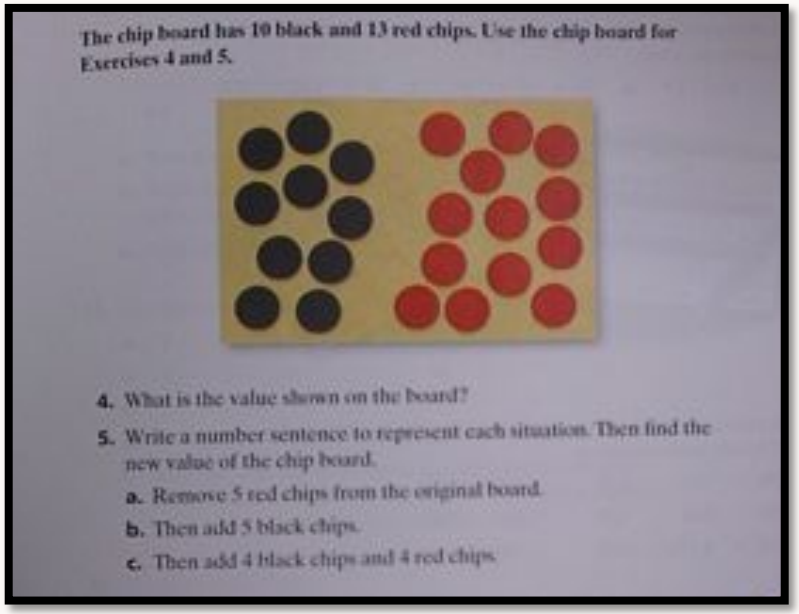
An Example for Grades 6-8

Moving towards notion of variable.



CCSS.Math.Content.6.EE.B.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

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An Example for Grades 9-12

Strategic use of appropriate tools:

Solve the following system.

$$y_1 = 3x + 4$$

$$y_2 = 4x + 7$$

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An Example for Grades 9-12

Strategic use of appropriate tools:

Solve the following system.

$$3x + 4 = 4x + 7$$

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An Example for Grades 9-12

Strategic use of appropriate tools:

Solve the following system.

$$4 - 7 = 4x - 3x$$

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An Example for Grades 9-12

Strategic use of appropriate tools:

Solve the following system.

$$y_1 = 3x + 4$$

$$y_2 = 4x + 7$$

$$-3 = x$$

$$y = -5$$

- CCSS.Math.Content.8.EE.C.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.*

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An Example for Grades 9-12

Strategic use of appropriate tools.

$$y_1 = 3x + 4$$

$$y_2 = 14x + 7$$

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An Example for Grades 9-12

Strategic use of appropriate tools.

$$(x,y) = \left(-\frac{3}{11}, \frac{35}{11} \right)$$

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An Example for Grades 9-12

Strategic use of appropriate tools.

$$z_1 = 3x + 2y + 4$$

$$z_2 = 4x + 5y + 7$$

$$z_3 = 3x - 2y - 2$$

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An Example for Grades 9-12

Strategic use of appropriate tools.

$$z_1 = 3x + 2y + 4$$

$$z_2 = 4x + 5y + 7$$

$$z_3 = 3x - 2y - 2$$

Mathematics » Solve systems of equations. » 9

(+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

http://www.corestandards.org/Math/Content/HSA/REI/C/9

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SMPs

But they won't show up on the high stakes test…

WRONG!

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SMARTER Balanced Summative Assessment Development Timeline

June 2010	Sep 2011	June 2012	Fall 2012	2013	2014-2015

Technology-enhanced Items/Tasks (TE)

Technology-enhanced items can provide evidence for mathematics practices that could not be as reliably obtained from traditional SRs and CRs. Technology-enhanced items may stand alone or may be a tool used as part of the Performance Task and/or Constructed-Response items.

Performance Tasks (PT)

Performance tasks, the most complex of all items, include the following elements:

- Integrate knowledge and skills across multiple claims.
- Measure capacities such as depth of understanding, research skills, and/or complex analysis with relevant evidence.
- Require student-initiated planning, management of information/data and ideas, and/or interaction with other materials.
- Reflect a real-world task and/or scenario-based problem.

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HW: Repeat for other SMPs

- Isolate action statements
- Integrate actions into lesson plans
- Implement instruction to support action

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Thank-You!

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