Cups, Ropes, and Licorice:

Making Sense of Rate of Change

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Participants will:

- interpret the slope and intercepts of a graph in the context of the situation
- recognize the relationship between rate of change and the slope of a line
- brainstorm ideas to share for teaching rate and slope in an activity format
- leave with a minimum of ten ideas for teaching rate of change in your class
- represent data in scatterplots and calculate the linear regression using the TI-84 calculator

Common Core Standards Addressed:

GRADE 6 EXPRESSIONS & EQUATIONS: Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

GRADE 7 RATIOS & PROPORTIONS: Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.2. Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.*
d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.

GRADE 8 EXPRESSIONS & EQUATIONS: Understand the connections between proportional relationships, lines, and linear equations.

8.EE.5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

GRADE 8 EXPRESSIONS & EQUATIONS: Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.8. Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

GRADE 8 FUNCTIONS: Define, evaluate, and compare functions.

8.F.1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. *[Function notation is not required in Grade 8.]*

8.F.2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

8.F.3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

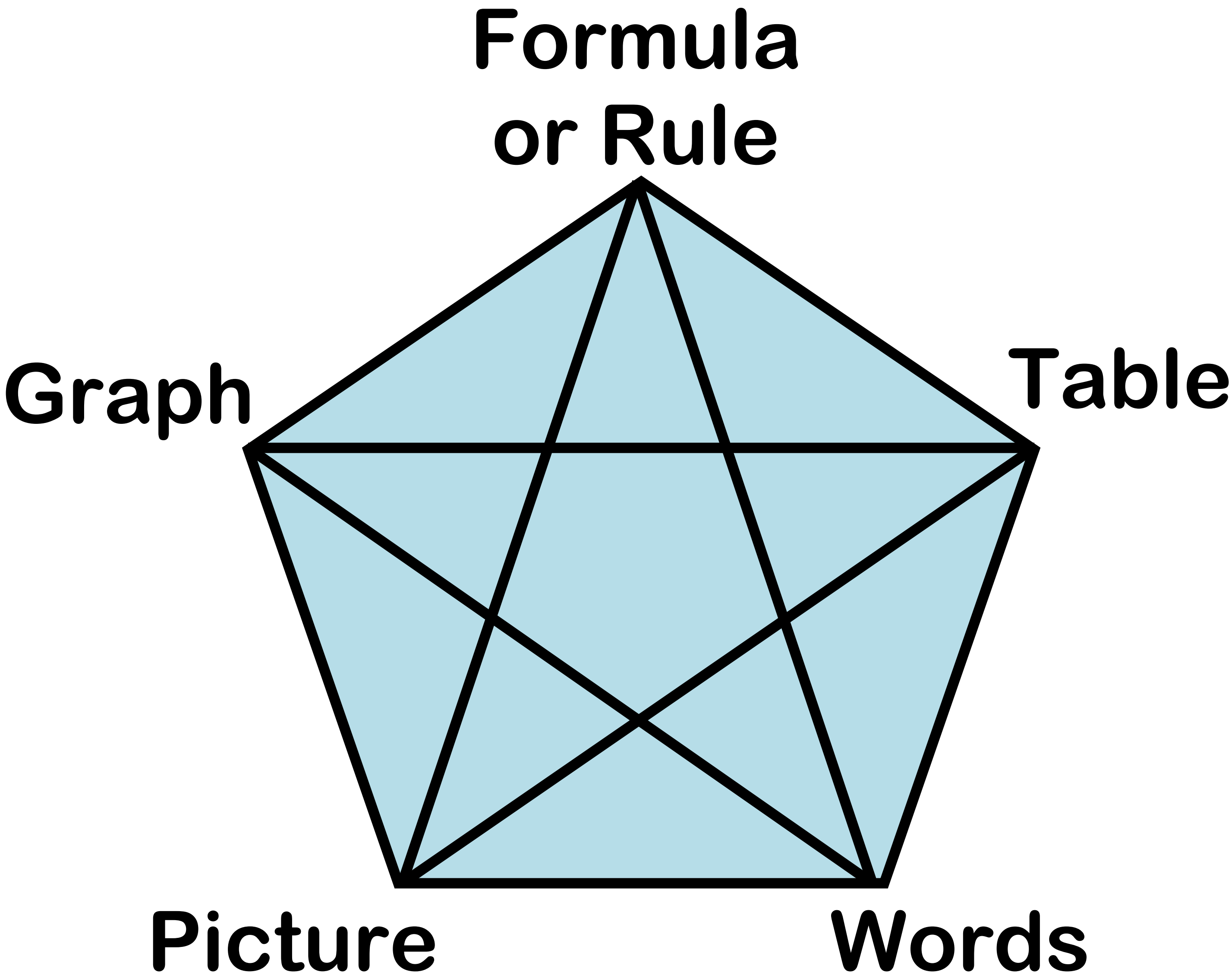
GRADE 8 FUNCTIONS: Use functions to model relationships between quantities.

8.F.4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

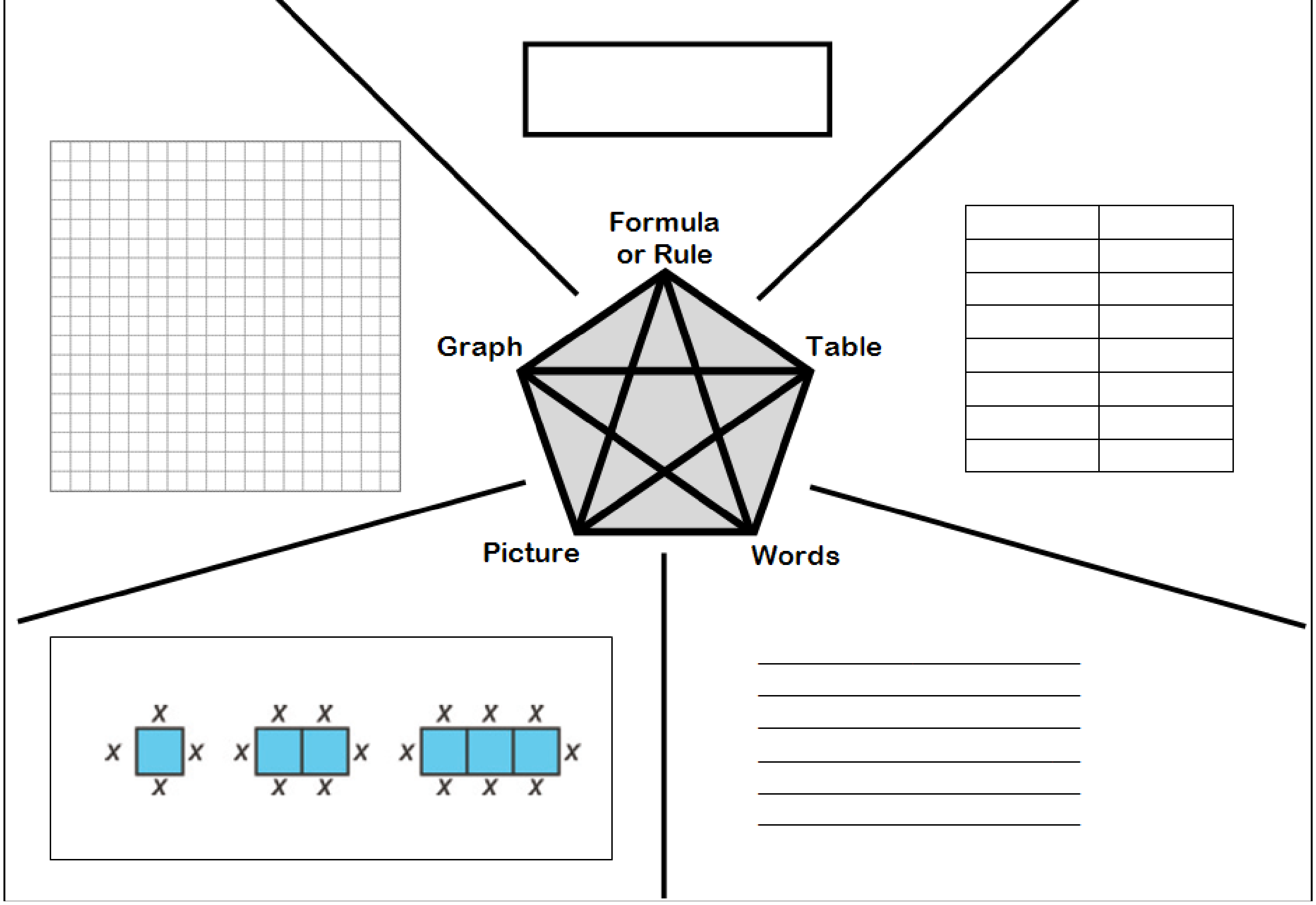
8.F.5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Key:
MAJOR CLUSTERS
SUPPORTING CLUSTERS

The Five Views of a Function



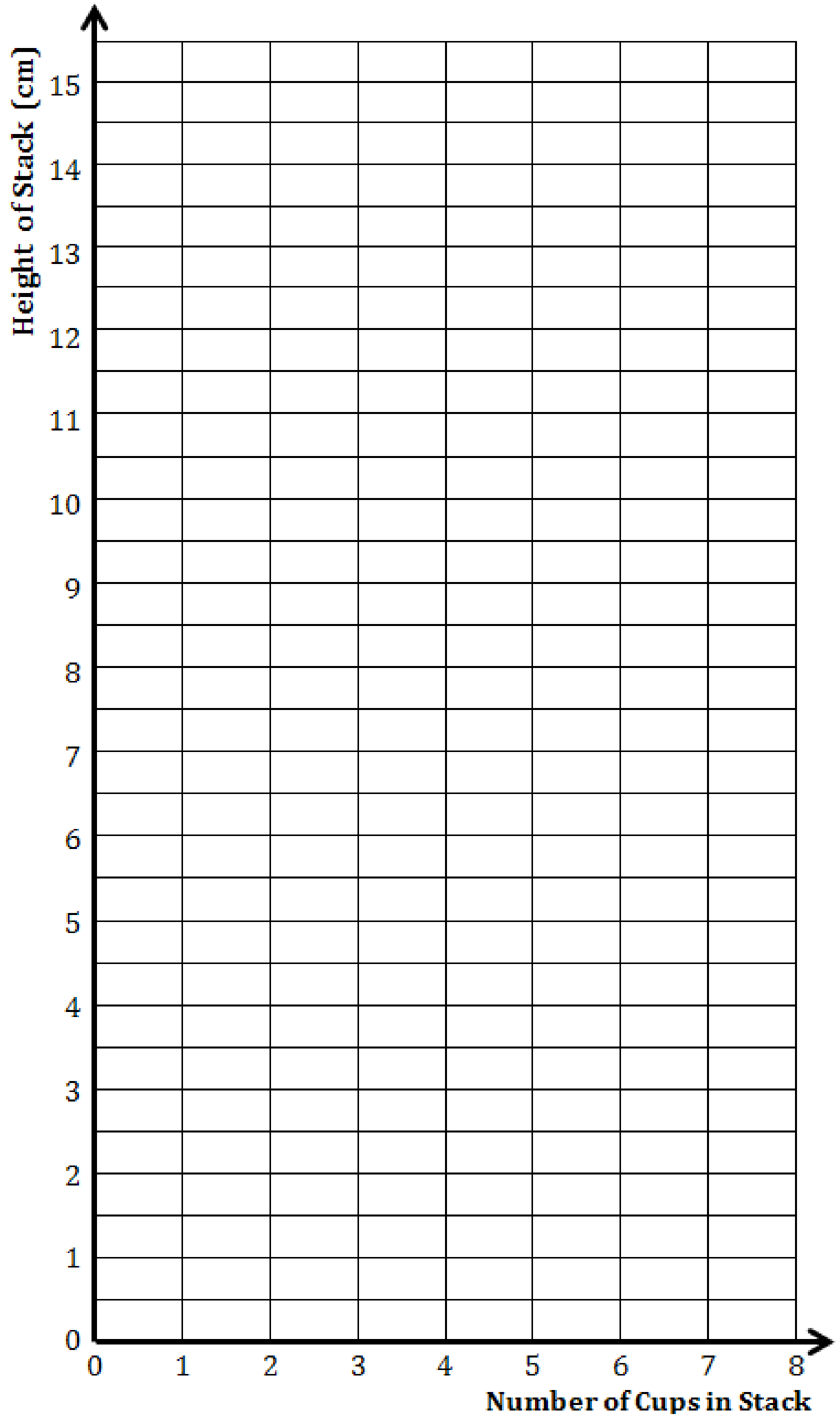
The Five Views of a Function



Stacking Cups

You have been hired by a company that makes all kinds of cups – foam hot cups, plastic drinking cups, paper cups, and more – of different sizes. For each of the kinds of cups it makes, the company needs to know the measurements of cartons that can hold a stack of 20 cups. Your task is to provide this information.

Number of Cups	Height (cm)
1	
2	
3	
4	
5	



Plot the points on the graph to the right, Draw a line through the y-axis and the points.

Here are some questions you might use in a discussion about this problem:

1. What does your graph tell you about how the stack grows?
2. How can you determine the typical amount the stack grows with each cup?
3. Is your data exactly linear? How do you know?
4. Write the equation of your line.
5. What is the slope of your line?
6. What does the slope mean in the context of this problem?
7. What is the y-intercept of the line?
8. What does it mean in the context of this problem?
9. How tall will a stack of 20 cups be? How do you know?
10. What if the maximum height of the box is 60 cm? How many cups can you fit in the box?

Other Questions:

Stacking Cups

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Number of Cups	Height (cm)
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Plot the points on your grid paper.

Draw a line through the y-axis and the points.

Answer these questions.

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Other Questions:

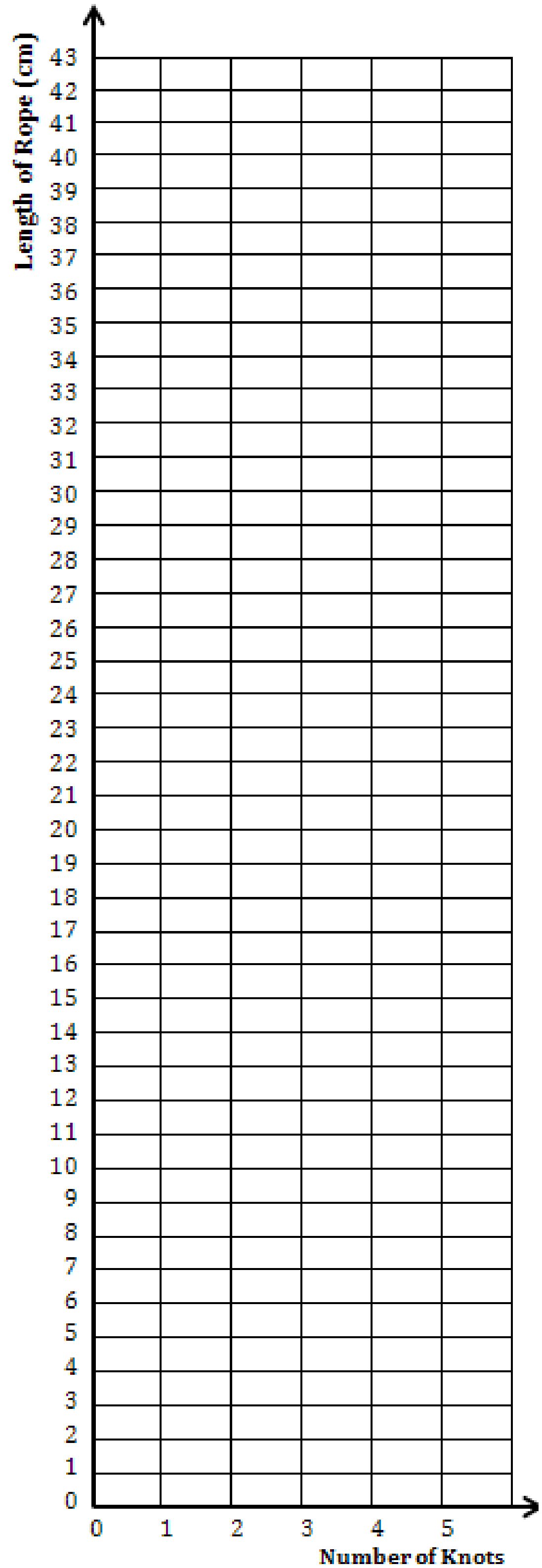
Tying Knots

In this activity we'll explore the relationship between the number of knots in a rope and the length of the rope and write an equation to model the data.

Step 1 *Choose one piece of rope and record its length in the first table. Tie five knots and measure the length of the rope after each knot. As you measure, add data to complete the table.*

Step 2 *Graph your data, plotting the number of knots on the x-axis and the length of the knotted rope on the y-axis.*

Number of Knots	Length (cm)
0	
1	
2	
3	
4	
5	



Here are some questions you might use in a discussion about this problem:

1. What is the approximate rate of change for this data set?
2. Draw a line through your points. Write an equation for the line you drew.
3. Use your equation to predict the length of your rope with 6 knots.
4. If you are able to tie a 6^{th} knot in your rope, do so. What is the difference between the actual measurement of your rope with 6 knots and the length you predicted using your equation?
5. Use your equation to predict the length of a rope with 17 knots. Explain the problems you might have in making or believing your prediction.
6. What is the maximum number of knots you can tie with your piece of rope? Explain your answer.
7. Does your graph cross the x-axis? What does this x-intercept mean in the context of the problem?
8. Substitute a value for y into the equation.
   What question does the equation ask?
   What is the answer?

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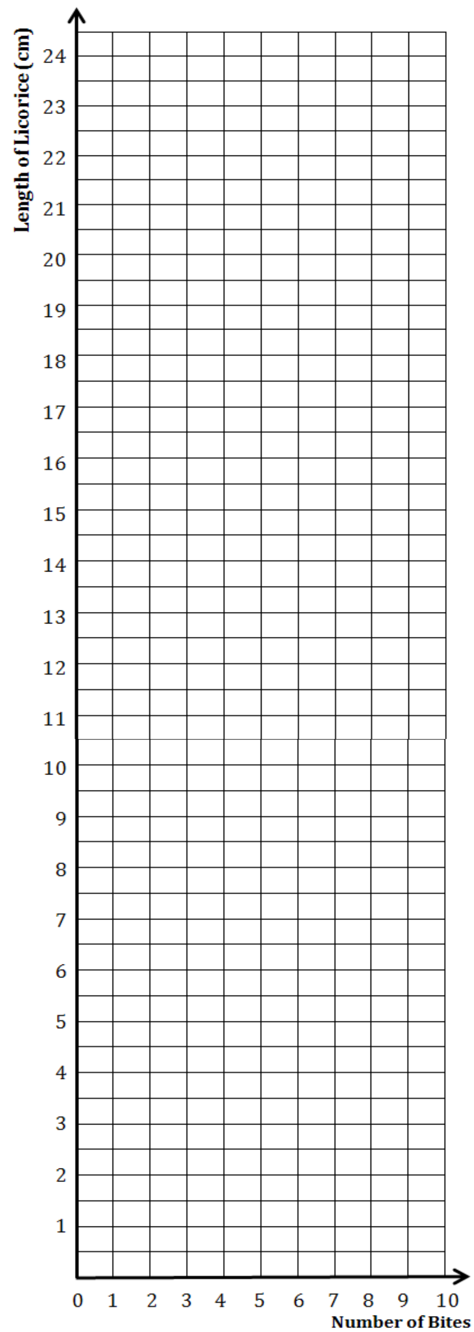
8. Substitute a value for y into the equation. What question does the equation ask? What is the answer?

Other Questions:

Licorice Bites

How can we relate the length of a licorice whip to the number of bites we have taken?

Number of Bites	Length (cm)



Plot the points on the graph to the right,

Here are some questions you might use in a discussion about this problem:

1. What does your graph tell you about what happened to the licorice as you ate it?
2. How can you determine the size of your average bite?
3. Is your data exactly linear? How do you know?
4. Write the equation of your line.
5. What is the slope of your line?
6. What does the slope mean in the context of this problem?
7. What is the y-intercept of the line?
8. What does it mean in the context of this problem?
9. You might have students make a prediction about the number of bites it will take to finish the whip after they've only finished half of it. How could you use your equation to determine how many bites are require to get a licorice whip that is 4 cm long?
10. How might your graph look different if you took bigger bites? Smaller bites? Compare your graph to the graph of someone next to you. How do they look different? What does this mean?

Other questions:

Licorice Bites

How can we relate the length of a licorice whip to the number of bites we have taken?

Number of Bites	Length (cm)

Plot the points on your graph paper,

Answer these questions:

1. What does your graph tell you about what happened to the licorice as you ate it?

2. How can you determine the size of your average bite?

3. Is your data exactly linear? How do you know?

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Other questions:

Cool Ideas for
Teaching Linear Relationships
Using Real World Examples

Proportional Relationships (Direct Variations):

- Relationship between thickness of a single book and the height of a stack of books.
- Conversion between centimeters and inches
- Heart rate vs. Elapsed time
- Height of an object vs. its shadow length
- Exchange Rates between US Dollars and other currencies
- Length of a string of paper clips vs. Number of paper clips
- Diameter of a given circle vs. the circle's circumference

Non-Proportional Linear Relationships:

- ❤ Number of people who can sit at a square table vs. number of tables lined up
- Conversions between Celsius and Fahrenheit temperatures
  - *For a system of equations, compare the actual formula, F = 1.8C + 32 to an estimate of the conversion, F = 2C + 30, to see where it's most accurate.*
- Perimeter vs. the number of pieces in a pattern block series
- Surface area vs. the number of one inch cubes stacked
  - *Or, use Cuisenaire rods and compare the surface area (in cm^2 with the volume (in cm^3)*

Relationships that fit lines, but aren't exactly linear:

- Number of times a nut is screwed into a carriage bolt vs. distance between nut and bolt head
- ❤ Number of cups stacked vs. height of stack
- ❤ Number of bites in licorice vs. Length of licorice left
- Number of sips taken of a drink vs. Height of drink left
- ❤ Number of knots tied in a section of rope vs. Length of rope left
  - *For a system of equations, use ropes of different thicknesses and lengths, and graph them on the same grid.*
- Number of times a ball is bounced vs. Time elapsed
- Bounce height of a ball vs. Height at which ball is dropped
- Height of a student vs. Jump height against a measuring tape on the wall

❤ *= Activities we did in our session today.*

Sources:

- *AIMS Looking at Lines*
- *Fulton, Brad & Lombard, Bill. The Pattern & Function Connection Key Curriculum Press, © 2001*
- *NCTM. Navigating Through Algebra in Grades 6-8.*

Online Resources that might be helpful:

- *www.nctm.org*
- *http://spacemath.gsfc.nasa.gov*