PICTURING FUNCTIONS AND FUNCTIONS OF PICTURES

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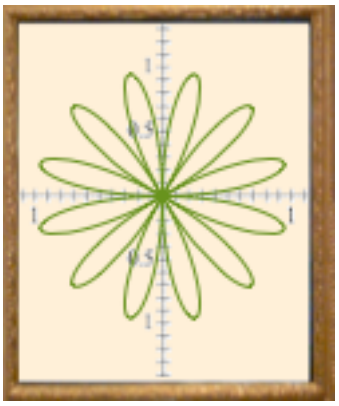
Digital pictures and graphics special effects are everywhere: the photos we take, TV, movies, the internet, and on and on, but our students seldom think about the functions that create these images. In this presentation we'll use pictures and graphs to understand functions better; we'll use functions to modify pictures and to create fractal images and other special effects; and we'll even define a function by drawing its graph on the screen.

In the digital age pictures are just collections of numbers: every photograph and every TV show exists as a collection of numbers that specify the color at every point—and the points themselves are defined as (x, y) locations on the image. Thus the geometric and numeric elements—the points, the coordinates, and the color values—are inseparably intertwined.
The Common Core addresses this connection in standard G-CO2 by stating that students should "describe transformations as functions that take points in the plane as inputs and give other points as outputs." In this presentation, that's exactly what we'll do: we'll repeatedly cross the boundaries between one-dimensional numbers and two-dimensional pictures.

PICTURING FUNCTIONS

The Cartesian graph is marvelous invention: it gives us a two-dimensional picture of the behavior of a function, even though the function's variables are one-dimensional numbers. But as a picture, it's a static object, and our students sometimes fail to grasp the dynamism, the interplay between the variables, the way in which the shape of the static graph embeds the relative rate of change of x and y.



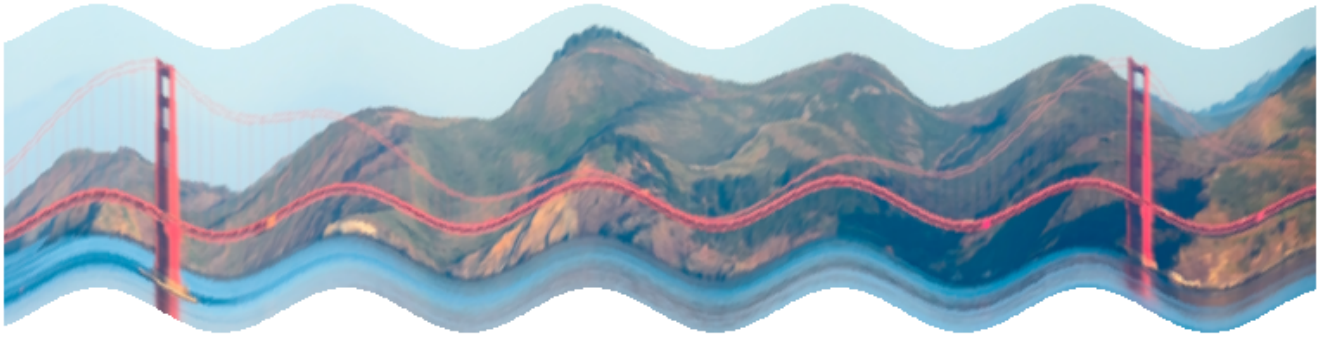
In *Parabolas in Factored Form,* in *Cartesian Graphs and Polar Graphs,* and in the *Graph Dancer* activities, we'll put the motion back into the Cartesian graph, and we'll do so literally, by having the student move the variables. Paul Foerster used to give a talk entitled *Variables Really Vary*, because students so often don't see the motion in the mathematics. These activities will help put that motion back in, by making the graph into a moving picture, physically powered by the student's own muscles as she drags the variables and modifies the parameters.

FUNCTIONS OF PICTURES

A function takes an input varible and produces an output variable, so a function of a picture must take one picture as its input and produce another as its output. The functions we'll use are true to the Common Core description of taking "points in the plane as inputs and [giving] other points as outputs." To apply such a

function to a picture, Sketchpad simply applies the function to many individual points, creating a new picture from the collective output resulting from thousands or tens of thousands of input points.

As examples we'll create several special effects. In one a picture will move and shrink, as if the object in the picture were disappearing into the distance. In another, we'll create a Swirling transformation by rotating each point by an angle that depends on its distance from the center of the picture. In yet another, we'll apply a sine function to the Golden Gate bridge.



Another example of the use of a function to modify a picture and produce a striking graphic effect is anamorphic street art. It's an example of backwards design: you begin with the final image, and use a function, point by point, to determine where on the sidewalk to actually place the chalk.

CREATE PICTURES FROM FUNCTIONS

Functions can be used not only to modify pictures, but to create pictures from scratch. Fractal images are generated entirely mathematically and are marvelous for their mix of randomness and self-similarity that leads to endless complexity. The Chaos Game activity is one example that we'll look at, and the Barnley Fern is another. Each of these activities begins with a single input point and applies a function to produce an output point. The output point is then reused as input to the function, again and again and again. (Such systems are called "iterated function systems.") By using carefully chosen functions that incorporate certain elements of randomness, truly striking images result.

RESOURCES

A blog post and the YouTube video trailer for this presentation are at blog.keypress.com. Many of the Sketchpad activities, along with the presentation sketch, will be available at www.geometricfunctions.org/2013regional. Several of the activities come from the Sample Activities available from the Learning Center in Sketchpad's Help menu, and the Swirling activities come from the Geometric Functions collection at www.dynamicnumber.org.