A Study of Middle School and College Students’ Misconceptions About Solving Multi-Step Linear Equations

Amber Powell

NCTM Regional Conference, Baltimore; October 18, 2013

Question
What mistake(s) do you think students will make the most when solving this equation?

\[-(x + 4) = 3(8 + 2x)\]

And this one?

\[4x + 3 = 5x - 11\]

Question
What error was made the most when solving this equation?

\[a = 19a + 18\]

And this one?

\[\sqrt{3} = 8\]

Purpose for Research
• Previous teaching experience at a high-needs, low income city school district
  – Taught AIS Math grades 6 – 8
  – Co-taught 7th grade math (special education and ESL students)
• Observations when teaching multi-step linear equations, students:
  – Were able to recall rules to solve the problem
  – Could not recall basic math facts without using strategies
  – Struggled to perform integer operations
  – Could not remember mathematical properties (i.e. distributive property)

Prior Research
Four categories of teaching/learning (Bright, 1981):
• Instrumental - students follow rules without understanding them
• Relational - students learn specific rules from more general ones
• Intuitive - students solve problems based on prior knowledge
• Formal - students have complete understanding of symbols, ideas and terminology in the problem solving process

Prior Research
Two groups of math students (Kieran, 1992):
• Arithmetical thinkers - tend to use trial-and-error substitution to solve equations because the answer is more intuitive to them
• Algebraic thinkers – use inverses and “undoing” strategies to solve equations because they recognize they are trying to find the value of the unknown

Prior Research
• Sign Confusion (Herscovics & Linchevski, 1994)
  \[6x = 7 + 11\]
  – Students forget the subtraction sign places a negative value on 7
• Equation balance (Bright, 1981)
  – “Left hand side” equals “right hand side”
• Misunderstanding of “equal” sign (Knuth, Alibali, Hattikudur, McNeil & Stephens, 2008)

Textbook Teaching Methods
• Direct instruction
• Instruction through manipulatives
• Self-discovery

Hypothesis
• 8th grade students will make the most mistakes with the distributive property and integer operations when solving multi-step linear equations
• College students in a non-majors mathematics course will make the same mistakes as 8th graders
• There will be no significant difference in scores between the two different groups in this study
Demographics
MATH 110 Students (37)
- 33 Males
- 57 Females
- 3 Freshmen
- 9 Sophomores
- 6 Juniors
- 4 Seniors
8th Grade Students (36)
- 15 Males
- 21 Females
- 2 IDPs
- 0 SENs
- 0 ESLs

Design
- Instrument contained 15 multi-step linear equations
- Participants given 15 minutes at beginning of one class period to complete the assessment
- Calculators were not permitted
- Students were also asked to answer a three question, free response survey

Data Collection
- Administered 113 assessments
- Two versions of the instrument were used – same questions, different order
- Nine criteria evaluated:
  - combining like terms
  - moving terms
  - distributing terms
  - using negative numbers in operations
  - using variables other than x
  - using the four basic operations (addition, subtraction, multiplication, and division)

Data Collection
- Three question survey
  - Favorite problem
  - Easiest problem
  - Most difficult problem
- Comments

Instrument
| 1. 7x + 7x = 14 | 2. 6 - 7x + 12 | 3. 7x - 5x = 8 |
| 4. 8x + 4x | 5. 4x + 1x + 3x = 11 | 6. 7x - 12x = 18 |
| 7. x + 3x = 20 | 8. 3x + 9x + 4x | 9. 8x - 2x = x = 4 |
| 10. 3x - 10x + 5x + 1 = 11 | 11. 7x + 8x | 12. 4x - 10 + 2x |
| 13. 7x + 4x + 8x + 9x = 12 | 14. 7x + x = 6 | 15. x + 5x - 14 |

Data Analysis
- Data comparison:
  - variables measured
  - grade level
  - gender
- Minitab used for analysis
  - Analysis of Variance
  - Tukey Test for Significance

Results
- Mistakes that occurred frequently
  - Operations using negative numbers
  - Moving numbers and variables to the opposite side of the equal sign

Results
- Eighth grade students made twice the number of mistakes that MATH 110 students did (P-value: 116.60, p-value: 0.00)

Results
- 8th graders and MATH 110 students made different types of mistakes
Results

• Males made fewer mistakes than females (F-value: 6.60, p-value: 0.01)
• There was no difference in the types of mistakes made based on gender

Student Work

\[
5. \quad \sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}
\]

\[
-4x = 3(8+2x)
\]

\[
-4x = 24 + 6x
\]

\[
x = -3
\]

\[
\frac{20}{10x} = \frac{2x}{10}
\]

\[
x = 2.6
\]

Results

Survey Results

• Mixed results
• Would reportedly perform better with calculator
Implications for Teaching

- Teachers need to continuously reinforce integer operations as a part of instruction
- Students need to understand fractions
- Students should use a calculator more effectively, and not depend on it to calculate

Application of Results

- Spiral education
  - Basic facts, integer operations and fractions incorporated into lessons and homework
- Manipulatives
  - Student generated fraction tiles
  - Bingo chips (two colors)
- Interactive White Boards
  - National Library of Virtual Manipulatives

Bingo Chips

Example: \(-7 + 3 = x\)
Let red = negative
Let blue = positive

How many are left?

Fraction Bars

What is \(1/4 + 3/8\)?

Balance

Click and drag squares from the left to balance beam pans to represent the equation.

\(2x + 4 = 2\)

Future Research

- Administer instrument to each grade level at one district
- Administer two instruments, and allow the use of a calculator for one
- Focus on manipulatives as a part of teaching

Bibliography


Rate this presentation on the conference app. www.nctm.org/confapp

Download available presentation handouts from the Online Planner! www.nctm.org/planner

Join the conversation! Tweet us using the hashtag #NCTMBaltimore

Questions?

Amber Powell
Email: powell@fredonia.edu

NCTM Regional Conference, Baltimore; October 18, 2013